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THE IMPORTANCE OF CONSIDERING REGIMES IN LONG TERM ASSET ALLOCATION TO REAL ESTATE

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The Importance of Considering Regimes in Long-term Asset Allocation to Real Estate

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Abstract

We investigate the long-term, regime-dependent asset allocation of an investor's wealth in a mixed-asset portfolio that includes publicly traded real estate. We show that augmenting standard VAR models with Markov-switching features not only improves predictive power for asset returns but also introduces economically meaningful horizon effects in optimal portfolio allocations. As the investment horizon lengthens, optimal portfolio allocations become less sensitive to the prevailing regime. Across initial states, the sensitivity of portfolio allocations to the investment horizon manifests primarily through a gradual reallocation toward risky assets relative to risk-free assets, particularly at lower levels of risk aversion. Public real estate receives economically meaningful portfolio allocations under these conditions. Out-of-sample portfolio tests further show that regime-switching models deliver higher realized utility and Sharpe ratios than linear and i.i.d. benchmarks. Overall, the results highlight the economic value of incorporating regime shifts into long-term portfolio choice and confirm the continued role of publicly traded real estate in mixed-asset portfolios.

Keywords: REITs, regime switching, asset allocation, horizon effects

The Importance of Considering Regimes in Long-term Asset Allocation to Real Estate

Introduction

Asset allocation informed by long-term investment horizons has long been of interest to both academics and practitioners. The early literature primarily focused on investigating the asset portfolio choice problem using a simple asset menu comprising stocks and bonds (see, e.g., Kandel and Stambaugh (1996), Campbell and Viceira (1999), Barberis (2000), Ang and Bekaert (2002), Guidolin and Timmermann (2007), Buraschi, et al. (2010), and Campani et al. (2021)). Meanwhile, with the development of securitized real estate, investment in real property has gained prominence among long-term investors, particularly institutional investors, who in recent years have significantly increased their allocation to Real Estate Investment Trusts (REITs). The reason is that public real estate securities have demonstrated historically counter-cyclical returns, low volatility, low correlations with other asset classes, and excellent inflation-hedging properties (see, e.g., Hoesli, et al. (2008)). Several studies have explored optimal portfolio choice when otherwise standard asset menu incorporates property stocks, empirically reporting significant diversification effects leading to strong realized performances (see, e.g., Fugazza, et al. (2007, 2009), Sa-Aadu, et al. (2010), Delfim and Hoesli (2019), Boudry et al. (2020) and Guidolin et al. (2023)).

In a classical finance framework, the asset allocation decision depends on the investor's preferences, the selection of a model capturing the joint distribution of asset returns, and the choice of a (possibly long-term) investment horizon. In the empirical finance literature focused on stocks and bonds, the standard models used to capture the predictability of returns are predominantly linear, vector autoregressive (VAR) models. In a *linear* predictability framework, future expected returns are predicted by current and lagged values of the predictor variables, as well as by lagged values of the asset returns themselves (see, e.g., MacKinnon and Al Zaman (2009), Bianchi and Guidolin (2014)). Hence, on the one hand, the application of linear predictability of asset returns to optimal asset allocation among traditional asset classes has been extensively explored. For instance, Geltner and Mei (1995), Lu and So (2001), MacKinnon and Al Zaman (2009), Rehring (2012), and Pagliari (2017) use VAR models to capture predictable time variation in the investment opportunity sets that include real estate.

On the other hand, a number of studies have focused on the predictability of asset returns using dynamic, non-linear time series models (Ang and Bakaert (2002), Guidolin and Timmermann (2007, 2008), Fugazza et al. (2009), Tu (2010), and Guidolin and Hyde (2012)). Unlike linear models that assume fixed parameters and provide forecasts for the entire time period, regime-switching models consider time-varying returns that may transition through multiple phases over time, with distinct risk-return characteristics in each state. This behavior is reflected in the considerable instability of the standard predictive relationships. Ignoring the changing nature of returns may lead to sub-optimal asset allocation decisions. Several papers have examined the returns of real estate investments under a Markov-switching (MS) framework. Liow et al. (2005), and Liow and Zhu (2007) apply a two-state non-linear model to capture the dynamics of real estate market returns and asset allocation, respectively, in six listed property markets. Similarly, Sa-Aadu et al. (2010) use a two-state Markov-switching framework to study multiple classes of assets (small-cap stocks, medium/large-cap stocks, government bonds, corporate bonds, international stocks, commodities, precious metals, and equity REITs) and examine the optimal portfolio allocation under these regimes. Case, Guidolin, and Yildirim (2014) investigate the MS dynamics in REITs, showing that their returns can be predicted by a simple two-state Markov-switching process with heteroskedastic components. In a multivariate setting, they find that stock, bond, and REIT returns can be captured by a four-state tri-variate vector autoregressive model. Demiralay and Kilincarslan (2024) fit MS models to sector-specific REIT returns when a number of uncertainty measures (VIX, EPU, etc.) enter as predictors.

In this paper, we investigate the strategic wealth allocation problem of a *long-term investment horizon* investor under a non-linear regime-switching framework. In our setup, the investment opportunity set not only includes traditional asset classes (Treasury bills, corporate bonds, and broad equity market portfolios) but also extends to publicly traded real estate investments (equity REITs), which represent an important share of the long-term positions held by many institutional investors. Importantly, to assess whether REITs provide distinct diversification benefits beyond their exposure to size and value factors, we extend the opportunity set to include Ken French's bivariate-sorted size–value portfolios (Small Neutral (ME1BM2) and Small Value (SMHiBM)). These portfolios explicitly exclude REITs and thereby allow us to test whether investors allocate to REITs because they are unique assets rather than merely capable of replicating exposures to small or value equity risks. The rationale for including the ME1BM2 category is that, during the post-2000 period, REITs typically exhibited book-to-market (B/M) ratios below the high B/M cutoff for NYSE stocks as defined by the Fama–French thresholds. On average, they predominantly fall within the ME1 BM2 group over this period.

Our research examines three related questions. *First*, we investigate whether a model featuring a multiple-state regime-switching framework can adequately capture the time-varying joint return distribution of a realistic set of asset classes. Using a number of statistical metrics, we identify the most appropriate regime structure for a system of six and ten variables spanning excess asset returns, the cash yield and, in an important extension, typical predictors (the dividend yield, the term and default yield spreads, and inflation rate changes). *Second*, we compare optimal asset allocations that include publicly traded real estate investment under non-linear regime-switching models versus traditional linear VAR specifications. While Fugazza et al. (2009) employ linear VAR models to capture the joint distribution of stocks, bonds, and REITs and investigate the diversification effect of including real estate, our research differs by examining the benefits of applying MS models in the strategic allocation. We employ a similar multiple-state setting as Case et al. (2014) to explore the dynamic asset allocation problem under MS, with a specific focus on the long-term realized risk-adjusted gains. *Third*, we examine the horizon effects in optimal portfolio composition and investor realized utility under linear and non-linear frameworks.

To the best of our knowledge, horizon effects in optimal portfolio composition under a regime-switching setup that includes publicly traded real estate remain largely unexplored in the literature. We analyze the changes in optimal portfolio composition using a buy-and-hold strategy across various investment horizons and compare the resulting welfare gains to investors. Our analysis addresses an important question concerning the attractiveness of different asset classes, including publicly traded real estate, to young vs. less young investors and portfolio managers at institutions with long vs. shorter horizon mandates. Our findings suggest that REITs retain an important role even in the presence of small- and value-stock portfolios, across the alternative regimes and under low to moderate degrees of risk aversion.[1] Moreover, in out-of-sample (henceforth, OOS) experiments, investors with longer investment horizons derive greater realized utility benefits when portfolio decisions are based on regime-switching models rather than on simpler linear specifications. These results emphasize the importance of accounting for regime dynamics when making long-term asset allocation decisions.

Our key results are as follows. We find that a three-state hybrid Markov-switching VAR model with one lag (later defined as MSIH(3,1)), in which intercepts and covariance matrices vary across regimes while the vector autoregressive coefficients are time-invariant, best characterizes the dynamic joint behavior of excess returns for the six baseline asset classes (stocks, REITs, corporate bonds, T-bills, and the two non-REIT small-value portfolios, ME1BM2 and SMHiBM). Including business-cycle predictive variables further enhances model performance. When we extend the system to ten variables by adding the dividend yield, term spread, default spread, and inflation rate changes, a three-state

[1] As one would expect, in correspondence with high/extreme relative risk aversion, we find that optimal strategic asset allocation is dominated by long positions in T-bills and long-term corporate bonds, with little room for diversification into either REIT or equity portfolios.

MSIAH(3,0) model provides the best overall fit. These results indicate that regime-dependent dynamics, combined with macroeconomic predictors, capture important nonlinearities in (excess) returns.[2]

Introducing MS meaningfully alters optimal asset allocation relative to linear models. Under the non-linear framework, investors allocate a larger share of wealth to risky assets and display greater sensitivity of portfolio weights to market states. In particular, allocations to REITs differ significantly across regimes and levels of risk aversion, falling as the investment horizon lengthens in bull regimes and rising in crash and volatile regimes, where the conditional return characteristics differ most strongly from those of traditional equities. This supports the view that REITs serve as a distinct and state-contingent source of diversification, even when small- and value-stock portfolios are present.

An interesting pattern emerges regarding the aggregate stock portfolio: under MS, its optimal weight converges to zero across all states and risk-aversion coefficients. This outcome reflects the conditional redundancy of the broad market index once investors can directly allocate to non-REIT small- and value-equity segments (ME1BM2 and SMHiBM) and to REITs. Under regime switching, these portfolios capture the equity return dynamics more effectively than the market itself, whose performance is an unconditional average of

[2] Using a sample from 1976–2023, the best-fitting model in the ten-variable system is a three-state MSIH(3,1) specification. The results are qualitatively similar to those reported here with respect to a shorter 1990–2023 sample. We focus on this period to align the analysis with the Modern REIT Era. Prior to 1990, REITs operated under a markedly different regulatory framework and market environment, with limited institutional participation and investment scale. The post-1990 period, in contrast, captures the emergence of REITs as a mainstream asset class. While a longer sample supports estimation of more complex regime structures, we prioritize economic relevance to ensure our findings reflect the behavior of contemporary REITs.

underlying state-dependent dynamics. This result does not imply that the aggregate market portfolio is intrinsically redundant, but rather that, conditional on regime switching and characteristic-based portfolios, its marginal contribution to spanning conditional equity premia becomes negligible.[3]

In contrast, VAR models, which impose a single unconditional covariance structure, lead to assigning a positive weight to the market and relatively lower weights to REITs, as a linear predictive framework cannot capture the regime-specific diversification patterns that emerge under MS. The optimal REIT allocation stabilizes around 15–18% for moderate risk aversion ($\gamma = 5$) in the six-variable model, higher than the 10% suggested by linear benchmarks. In the ten-variable system, however, the REIT allocation declines to about 5–10%, a pattern explained by the inclusion of predictive macroeconomic variables. These predictors improve the model's ability to forecast state transitions and conditional asset returns, reducing the marginal contribution of REITs to intertemporal diversification. In other words, when investors can anticipate shifts in economic and financial conditions, part of the diversification and timing benefit provided by REITs in the six-variable system is absorbed by the predictive factors themselves (see Clayton et al. (2009) and Bianchi and Guidolin (2014) in the case of single-state linear models).

Our out-of-sample performance tests confirm the economic importance of accounting for the presence of regimes in asset returns. Regime-switching models yield higher realized

[3] This result contrasts with findings in earlier literature (e.g., Lynch (2001)), which, using simple linear, single-regime frameworks that exclude real estate, but include equity style portfolios, does not find the market portfolio to be redundant.

utility and Sharpe ratios than single-regime VAR or static mean-variance frameworks, particularly over medium and long horizons. These improvements demonstrate the relevance of accounting for state-dependent return dynamics in portfolio choice. Furthermore, our results suggest significant diversification benefits for the long-term investor when adding public real estate to their portfolio, although our findings also indicate a lower optimal share of real estate assets than previously suggested (see, e.g., Fugazza et al. (2007, 2009), Sa-Aadu et al. (2010), Pagliari (2017), and Delfim and Hoesli (2019)).

Overall, our findings show that REITs remain non-redundant assets within diversified portfolios once size and value exposures are controlled for. Their contribution is regime-dependent and more moderate once predictive information is incorporated. Incorporating regime shifts not only enhances predictive accuracy but also provides a richer and more realistic framework for understanding how real estate contributes to long-term portfolio performance.

Before proceeding further, it is important to highlight that superimposing a multiple-regime, MS component onto otherwise classical VAR predictability frameworks is logically simple, economically interpretable, and, as argued below, has far-reaching implications for the resulting optimal portfolio allocations. *First*, using MS VAR models is distinct from employing complex non-linear architectures typical of machine learning (ML) approaches to predictability. The approach involves incorporating the idea that parameters can shift across different regimes, governed by a simple (i.e., Markov) law of motion with few estimable parameters (i.e., the transition probabilities) into an otherwise standard VAR model.[4] *Second*, the resulting, estimated regimes can be restricted to carry interpretable economic meaning before using them in forecasting and portfolio optimization. This stands in contrast to the criticism faced by ML methods when applied to real estate (see Viriato (2019)). *Third*, it would be imprudent to overlook the empirical evidence of such regimes in financial and real estate data, given their significant impact on optimal asset allocations and their dependence on the investor's horizon, as previously observed in terms of predictive power by Case et al. (2014).

The remainder of this paper is organized as follows. Section 2 specifies the models used to capture the joint distribution of asset returns and to perform the asset allocation exercises. Section 3 summarizes the information criteria used to determine the most suitable number of regimes for the selected models. Section 4 presents the data and descriptive statistics. Section 5 discusses the asset allocation results and investigates the asset allocation dynamics for an investor in various scenarios when considering a MS framework. Section 6 studies the economic importance of accounting for regimes by comparing the OOS realized portfolio performance with other models. Section 7 offers concluding remarks.

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[4] The only examples we are aware of ML approaches to MS VAR models are Liao (2017), who combines K-means classification with MS for GDP prediction, and Zhang et al. (2020) who use macroeconomic and financial data as inputs of a hierarchical clustering algorithm to distinguish among different economic regimes. None of these papers specifically foresee applications to real estate finance problems.

2. Model Development

A Simple Model Without Predictive Variables

We begin with a baseline six-variable model designed to capture the joint distribution of the excess returns (relative to the 30-day T-bill rate) of stocks, corporate bonds, REITs, and the two non-REIT small-value portfolios (ME1BM2 and SMHiBM), along with the 30-day T-bill rate itself. Let's consider an n×1 vector of asset returns, $r_t = (r_{1t}, r_{2t}, \dots, r_{nt})'$, where $n = 6$. Suppose that the means, variances, pairwise covariances, and (vector, own- and cross-) serial correlation coefficients of excess returns are driven by a common state variable, S_t, which takes integer values between 1 and k. A k-regimes Markov-switching vector autoregressive model (MS-VAR model) is defined as follows:

$$r_t = \mu_{S_t} + \sum_{j=1}^{p} A_{j,S_t} r_{t-j} + \varepsilon_t \tag{1}$$

where μ_{S_t} is the intercept vector in state S_t, $\sum_{j=1}^{p} A_{j,S_t} r_{t-j}$ represents the pth-order vector autoregressive lagged components in state S_t, and $\varepsilon_t \sim N(0, \Omega_{S_t})$, where Ω_{S_t} is an $(n \times n)$ state-dependent covariance matrix. The regime-switching dynamics in the state variable S_t are assumed to be governed by a transition probability matrix $\mathbb{P}$, with elements denoted as:

$$Pr(S_t = i | S_{t-1} = j) = p_{ji}, \qquad i, j = 1,2, \dots, k \tag{2}$$

Each regime is thus a realization of a first-order Markov chain with constant transition probabilities. The state S_t is not observable, but a filtered (or, when needed, smoothed) estimate can be computed from the time series data r_t (see Hamilton, 1994). Equation (1) defines a standard Gaussian reduced-form VAR(p) model. We assume the time series process is stationary, which restricts the regime-dependent VAR(p) to be stationary in at least one regime. The estimation proceeds by optimizing the likelihood function associated with equations (1) -(2) through an application of the Expectation-Maximization (EM) algorithm, as described, for instance, in Guidolin and Pedio (2018).

Adding Predictive Variables

Next, we introduce business-cycle variables as predictors in the vector autoregressive model, and we allow the coefficients of these predictors to vary depending on the common Markov regime within the multiple-state framework. To achieve this, we expand the vector r_t into an $(n+m)$-dimensional vector by adding a set of m predictor variables, denoted as $z_t = (z_{1t}, z_{2t}, \dots, z_{mt})'$.

This leads to the extended MS-VAR model:

$$\begin{pmatrix} r_t \\ z_t \end{pmatrix} = \begin{pmatrix} \mu_{S_t} \\ \mu_{z,S_t} \end{pmatrix} + \sum_{j=1}^{p} A_{j,S_t} \begin{pmatrix} r_{t-j} \\ z_{t-j} \end{pmatrix} + \begin{pmatrix} \varepsilon_t \\ \varepsilon_{z,t} \end{pmatrix}, \tag{3}$$

where $\mu_{S_t}$and μ_{z,S_t} are the state-dependent intercept vectors, $\{A_{j,S_t}\}_{j=1}^{p}$ are the $(n+m) \times (n+m)$ matrices of autoregressive coefficients in state S_t and $\begin{pmatrix} \varepsilon_t \\ \varepsilon_{z,t} \end{pmatrix} \sim N(0, \Omega_{S_t})$, where Ω_{S_t} is an $(n+m) \times (n+m)$ state-dependent covariance matrix that may also capture regime-specific correlations between shocks to excess returns and shocks to investment opportunities (see, e.g., the discussion in Fugazza et al. (2007)).

By imposing the restriction that the VAR coefficients are fixed across regimes, one may obtain from Equation (3.) a hybrid Markov-switching MSIH(*k,p*) VAR model, that nonetheless allows the intercepts and the error covariance matrix to switch across regimes.

Optimal Asset Allocation

We consider a U.S. investor facing a T-month investment horizon who can either rebalance their portfolio during the T-month period (when $T > 1$, the rebalancing case) or maintain a constant allocation across the N assets over this horizon (the buy-and-hold case). Their decision aims to maximize expected utility, which is assumed to depend only on end-of-period wealth. The asset allocation problem can be formally written as:

$$\max_{w} E_0[U(W_T)], \tag{4}$$

subject to the constraint that the portfolio weights at time t must sum to *1*, i.e., $(\omega_t^T)'\iota_n = 1$, where W_T is the wealth of the investor at the end of the period T and ω_t^T is the n x 1 vector of portfolio weights under an investment horizon T.

We consider an investor with *CRRA* (Constant Relative Risk Aversion) utility:

$$u(W_{t+T}) = \frac{W_{t+T}^{1-\gamma}}{1-\gamma}, \quad (\gamma > 1), \tag{5}$$

where γ is the coefficient of constant relative risk aversion. The investor makes asset allocation decisions at time t and maximizes expected utility by choosing a portfolio allocation to risky assets, i.e., a general stock market index , REITs, the ME1BM2 and SMHiBM portfolios, and corporate bonds, with weights $\omega_t^T = (\omega_t^S(T)\ \omega_t^{REIT}(T)\ \omega_t^{ME1BM2}(T)\ \omega_t^{SMHiBM}(T)\omega_t^{CB}(T))'$, while $1-(\omega_t^T)'\iota_5$ is the residual share of their wealth that is invested in T-bills. For simplicity, we presume that the investor has unit initial wealth, and that portfolio weights are adjusted every $\varphi = \frac{T}{B}$ months at B equally spaced points $t, t+\frac{T}{B}, t+2\frac{T}{B}, \dots, t+(B-1)\frac{T}{B}$. When $B = 1$, $(\text{i.e.}, \varphi = T)$, the investor simply implements a buy-and-hold strategy with no interim rebalancing. Let ω_b $(b = 0, 1, \dots, B-1)$ denote the portfolio weights of the risky assets at rebalancing times. Then $1-\omega_b{}'\iota_5$ is the weight in T-bills at time $t + b\frac{T}{B}$, and the utility at time B is given by:

$$u(W_B) = \frac{W_B^{1-\gamma}}{1-\gamma}\,. \tag{6}$$

Under regular rebalancing, the investor's optimization problem is:

$$\max_{\{w_j\}_{j=0}^{B-1}} E_t\left[\frac{W_B^{1-\gamma}}{1-\gamma}\right] \tag{7}$$

$$s.t.\, W_{b+1} = W_b\{(1-\omega_b{}'\iota_5)\exp(\varphi r^f) + \omega_b' \exp(R_{b+1} + \varphi r^f \iota_5)\}$$

$$R_{b+1} = r_{t_{b+1}} + r_{t_{b+2}} + \cdots + r_{t_{b+\varphi}}, \qquad b = 0, 1, \dots, B-1.$$

The wealth equation is exact when asset returns are continuously compounded, and excess returns are computed as the difference between portfolio returns and the T-bill rate.

Because the Markov states S_t are unobservable, the investor must form beliefs about the likelihood of each regime and update these beliefs at each point in time using a Bayesian updating rule:

$$\pi_{b+1}(\widehat{\theta}_t) = \frac{\left(\pi_b'(\widehat{\theta}_t)\widehat{P_t^{\varphi}}\right)' \odot \eta\left(y_{b+1};\widehat{\theta}_t\right)}{\left[\left(\pi_b'(\widehat{\theta}_t)\widehat{P_t^{\varphi}}\right)' \odot \eta\left(y_{b+1};\widehat{\theta}_t\right)\right]'\iota_k}, \tag{8}$$

where, π_b is the vector of probabilities for each of the k possible states, conditional on information at time t_b. The operator $\odot$ denotes the element-by-element product, $y_b \equiv \binom{r_b}{z_b}$, $\widehat{P_t^{\varphi}} \equiv \prod_{i=1}^{\varphi} \widehat{P}_t$, and $\eta(y_{b+1};\widehat{\theta}_t)$ is the $k \times 1$ vector whose jth element gives the density of observation y_{b+1} in the state j at time t_{b+1} conditional on $\widehat{\theta}_t$:

$$\eta_j(y_{b+1};\, \hat{\theta}_t)$$

$$= (2\pi)^{-\frac{N}{2}} |\widehat{\Omega}_j^{-1}|^{\frac{1}{2}} exp\left[-\frac{1}{2}\left(y_{b+1} - \hat{\mu}_j - \sum\nolimits_{l=0}^{p-1} \hat{A}_{j,l} y_{t_b - l}\right)' \widehat{\Omega}_j^{-1} \left(y_{b+1} - \hat{\mu}_j - \sum\nolimits_{l=0}^{p-1} \hat{A}_{j,l} y_{t_b - l}\right)\right] \quad (9)$$

This approach is consistent with the notion that an investor never observes the true state. Investors need to follow such a learning process about the regimes to allocate their wealth, because optimal portfolio choices depend not only on the future values of the asset excess returns and the predictor variables, $(r_b, z_b)'$, but also on future perceptions of the likelihood of being in each of the unobservable regimes (π_b).

Monte Carlo Methods to Solve the Portfolio Problem

Following Barberis (2000) and Guidolin and Timmermann (2007), we adopt a Monte Carlo simulation approach to solve the problem in Equation (7), incorporating regime switching, to approximate the integral in the expected utility function as follows:

$$\max_{\omega_t^T} J^{-1} \sum_{j=1}^{J} \left\{ \frac{[(1 - (\omega_t^T)'\iota_5) \exp(Tr^f) + (\omega_t^T)' \exp\left(\sum_{i=1}^{T} (r^f \iota_5 + r_{t+i,j})\right)]^{1-\gamma}}{1-\gamma} \right\}, \quad (10)$$

where $(\omega_t^T)' \exp\left(\sum_{i=1}^{T} (r^f \iota_5 + r_{t+i,j})\right)$ represents the portfolio return obtained from the *j*th Monte Carlo simulation. Each simulated path of portfolio returns is generated using draws from models (1)–(2) , allowing regimes to shift randomly as governed by the transition matrix, $\mathbb{P}$. To ensure sufficient stability of results and account for the possibility that some regimes may occur infrequently, we follow Guidolin and Timmerman (2007) and use a large number of simulations (in our case, J = 65,000). We vary the investment horizon T characterizing the tested buy-and-hold strategies values of T ranging between 6 and 120 months, with a step increment of 6 months. The initial risk aversion characterizing the investor's preferences is set at $\gamma = 5$. As a robustness check, we also test the empirical results under alternative risk aversion coefficient values of 2 and 10.

3. Model Selection

We perform a series of tests to determine the most appropriate MS model that characterizes the joint distribution of asset returns. Following Case et al. (2014), we rely on six different quantitative criteria for model selection: Akaike's information criterion, the Bayesian information criterion, Hannan-Quinn's information criterion, the Saturation Ratio, and two regime classification methods, referred to as RCM1 and RCM2. These are described in the following.

The Akaike information criterion is given by:

$$AIC = -2\ln L(\hat{\theta}) + 2d, \tag{11}$$

where $L(\hat{\theta})$ is the maximized value of the likelihood function for a given model and d is the estimated number of parameters in the vector $\hat{\theta}$, as implied by a given model specification. The AIC selects the model with the lowest value of the criterion, trading-off the goodness of fit measured by the likelihood function with the complexity of the model measured by the number of its parameters. Similarly, the Bayesian information criterion is defined as follows:

$$BIC = -2\ln L(\hat{\theta}) + d\ln(\tau). \tag{12}$$

where τ represents the number of observations used to estimate the model. Like the AIC, the BIC penalizes the complexity of the model and returns a smaller value as the model's ability to trade-off fit and complexity improves. However, BIC applies a heavier penalty to model complexity than the AIC. Finally, Hannan-Quinn's information criterion is an alternative statistic calculated as:

$$HQC = -2\ln L(\hat{\theta}) + 2d\ln(\ln(\tau))\,. \tag{13}$$

HQC provides excellent model selection performance for non-linear and regime-switching models, incorporating a performance penalty term (see Guidolin and Pedio, (2018)).

In addition to the information criteria, we also use the Saturation Ratio to determine whether each estimated parameter carries enough information from the data sample. The Saturation Ratio is computed as:

$$Saturation\ Ratio = \frac{Number\ of\ observations\ available\ for\ estimation}{Number\ of\ estimated\ parameters}. \tag{14}$$

To determine the number of regimes in an MS model, we also account for the model's ability to precisely allocate each sample observation to one of the assumed multiple regimes. To this end, we employ two regime classification methods. RCM1 is calculated as:

$$RCM1 = 100\frac{K^2}{\tau}\sum_{t=1}^{\tau}\prod_{k=1}^{K}\hat{\xi}_{t|T}^{k} \tag{15}$$

where a value of RCM1 close to zero indicates that the model precisely identifies the prevailing regime at each time t, implying an appropriate number of regimes. We select models based on their ability to minimize RCM1.

However, RCM1 may tend to zero when the number of regimes exceeds two even when the estimated model cannot always precisely identify the prevailing regime, thus leading to potentially incorrect model specification. This occurs, for instance, when out of three regimes, one regime is assigned a zero probability, but the estimation algorithm is undecided between the remaining two states. To address this issue, Guidolin (2009) proposed RCM2 as an alternative measure, which avoids such problems. RCM2 is computed as follows:

$$RCM2 = 100\left[1 - \frac{K^2 K}{(K-1)^2}\frac{1}{\tau}\sum_{t=1}^{\tau}\prod_{k=1}^{K}\left(\hat{\xi}_{t|T}^{k} - \frac{1}{K}\right)^2\right] \tag{16}$$

After estimating and selecting one or more optimized MS models using these criteria, we further implement a range of checks to ensure the reliability of model estimation and its coherence with the assumptions made to enable estimation. The EM algorithm provides time series of estimates of parameters and of smoothed probabilities $\{\hat{\xi}_{t|T}\}_{t=1}^{\tau}$ which allow the construction of smoothed residuals suitable for diagnostic checks. We use Ljung-Box type tests to check for the presence of (own- and cross-) autocorrelation in the residuals and to determine whether the estimated switching conditional means and covariances capture most of the volatility dynamics. The null hypothesis of the Ljung-Box test is that the first m lags of autocorrelations are jointly zero, H_0: $\rho_1=\rho_2=\ldots=\rho_m=0$, where the Ljung-Box test statistic is given by:

$$Q(m) = \tau(\tau+2)\sum_{h=1}^{m}\frac{\hat{\rho}_h^2}{\tau - h} \tag{17}$$

where τ is the length of observed time series and $\hat{\rho}_h$ is the estimated (sample) autocorrelation.

4. Data and Descriptive Statistics

The return data used in this paper cover the period from January 1990 to December

2023, capturing several stock market cycles, including the 2000-2002 dot-com bubble burst, the 2007-2009 financial crisis, and the 2020-2021 COVID-19 turmoil. To measure stock market returns, we use the NYSE/AMEX/NASDAQ Value-Weighted Market Index from CRSP. The returns of U.S. publicly traded real estate firms are proxied by the FTSE NAREIT All Equity REIT Index from NAREIT. For corporate bond returns, we use monthly returns of the ICE BofA US Corporate Bond Index.

To capture cross-sectional equity characteristics related to size and value, we employ two portfolios from Fama and French's (1993) six portfolios formed on size and book-to-market (2×3): the SMALL HiBM (SMHiBM) portfolio, representing small-cap firms (below the NYSE median) with high book-to-market ratios (top 30%), and the ME1 BM2 (ME1BM2) portfolio, representing below-median-sized firms with medium book-to-market ratios (middle 40%). These portfolios, obtained from Kenneth French's data library, explicitly exclude REITs and allow us to assess whether REITs provide unique diversification benefits beyond the small- and value equity segments.[5,6] As a proxy for the risk-free rate, we use the CRSP 30-day T-bill return series. Excess returns are calculated by subtracting the 30-day T-bill returns from total asset returns.

Among the predictive variables, we include commonly used variables in the empirical finance literature (see Kandel and Stambaugh (1996), Campbell and Viceira, (1999),

[5] See http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

[6] As previously noted, the rationale for including the ME1 BM2 category is that, during the post-2000 period (2000–2019), REITs typically exhibited book-to-market (B/M) ratios below the high B/M cutoff for NYSE stocks as defined by the Fama–French thresholds. On average, they predominantly fell within the ME1 BM2 group over this period.

Barberis (2000), Campbell et al. (2004) and Fugazza et al. (2009)): the aggregate equity dividend yield, the CPI inflation rate change, the default spread, and the term spread. We compute the dividend yield implicit in the returns of the CRSP NYSE/AMEX/NASDAQ Value-Weighted Market Index by taking the difference between the return series with and without dividends and scaling it by the value-weighted CRSP price index. The CPI inflation rate change is calculated as the monthly CPI growth rate from the Consumer Price Index for All Urban Consumers (CPI-U). Data on Moody's Seasoned Corporate Bond Yield minus the 10-Year Treasury Constant Maturity Rate are collected from the FRED II database and used as the default spread. The term spread is computed as the difference between the 10-Year Treasury Constant Maturity Rate and the 30-Day Treasury Bill Secondary Market Rate, also from FRED II.[7]

Table 1 presents the summary statistics of the six-return series and the four predictive variables on a monthly basis over the period January 1990–December 2023. T-bills display an average monthly return of 0.21%, corresponding to an annualized yield of approximately 2.5%. Among the risky portfolios, the SMHiBM and ME1BM2 exhibit the highest average excess returns, 0.95% and 0.90% per month, respectively, followed by equity

[7] The model could be extended to incorporate additional predictive variables, such as sentiment proxies (Clayton et al. (2009), Akinsomi et al. (2014), Bork et al. (2020), and Guidolin et al. (2023)). In unreported results, available upon request, we explore alternative variables associated with sentiment in the real estate, stock and bond markets. We have also examined the predictive power of the first difference of Economic Policy Uncertainty (EPU), a news-based measure proposed by Baker, Bloom, and Davis (2016)), to proxy for the uncertainty risk arising from policy in the United States. Similarly, we include the monthly growth rate of the Index of Consumer Sentiment (ICS), computed based on the Survey of Consumers, conducted by the University of Michigan in collaboration with Thomson Reuters. Our results remain similar to those for the 10-variable case presented in the main text.

REITs (0.67%) and the value-weighted stock index (0.66%), while corporate bonds generate the lowest mean excess return of 0.28% per month.

The standard deviation of REIT returns (5.21%) is slightly higher than that of the aggregate stock market (4.44%) and comparable to that of the ME1BM2 (5.42%) and SMHiBM (5.86%) portfolios, while corporate bonds display much lower volatility (1.39%). The resulting monthly Sharpe ratios indicate that ME1BM2 (0.13) and SMHiBM (0.12) deliver the strongest risk-adjusted performance, followed by the stock index (0.10) and REITs (0.09), whereas corporate bonds trail with a Sharpe ratio of 0.05.

All return series exhibit departures from normality. Equity REITs show negative skewness (-0.75) and fat tails (excess kurtosis is 9.08), suggesting more frequent extreme outcomes and a more asymmetric distribution than the stock index (skewness = -0.62; excess kurtosis = 4.26). The Jarque-Bera test strongly rejects normality for all portfolios and assets, consistent with the presence of heavy tails in return distributions. Turning to the predictive variables, the dividend yield, term spread, and default spread average 0.17%, 0.13%, and 0.19% per month, respectively, while the monthly change in inflation averages 0.22%. These variables exhibit markedly lower volatility compared to the return series.

Next, we examine the contemporaneous pairwise correlations between the variables, presented in Table 2. As expected, the excess returns on equity REITs are strongly correlated with the two size–value portfolios value portfolios, ME1BM2 (0.67) and SMHiBM (0.69), and with the stock market returns (0.65). These correlations indicate that REITs share exposure to the small- and value-oriented equity segments and also co-move closely with the broader equity market. The ME1BM2 and SMHiBM portfolio returns are highly correlated with each other (0.96) and with the aggregate stock index (0.87 and 0.83, respectively), confirming that they capture systematic equity risk factors related to firm size and valuation. In contrast, we observe lower correlations between the returns of the other assets. For example, the correlations between 30-day T-bill returns and the excess returns of the five different risky assets are between -0.04 and -0.07. The correlations between corporate bond excess returns and those of stocks and REITs are 0.27 and 0.40, respectively, while the correlation between T-bill and corporate bond returns is -0.05. Overall, given the generally low correlations among the assets, one would expect meaningful cross-asset diversification benefits.

We also observe strong negative correlations between the term spread and the returns of T-bills (-0.45), which is expected as the term spread tends to decline when short-term rates increase, and a similarly strong negative relationship between the default spread and T-bill returns (-0.56). According to classical analysis of hedging demands of optimal portfolio weights under CRRA preferences (see, e.g., Guidolin & Timmermann (2007)), this suggests the likely presence of strong hedging components and a non-flat portfolio demand schedule as a function of the horizon when the predictors are allowed to shift the set of investment opportunities.

5. Empirical Results

Model Selection

We next turn to the selection of a suitable MS model for the joint process followed by the returns of the assets under analysis. We begin by evaluating alternative specifications that vary in the number of regimes ($k = 1$ to 4) and in the number of autoregressive lags ($p = 0$ or 1). Following Case et al. (2014), we use the Akaike (AIC), Bayesian (BIC), and Hannan–Quinn (HQC) criteria, along with Hamilton's (1998) and Guidolin's (2009) regime classification measures (RCM1 and RCM2), to assess each model's balance between fit and parsimony. Table 3 presents the outcomes of model selection for the simple framework without predictive variables. According to these results, no single model outperforms the others under all information criteria, creating a trade-off between in-sample fit and parsimony. We begin with the simplest, one-state, no-lag (linear) model and progressively increase complexity up to the richest, four-state MS-VAR(1) case. In Panel A of Table 3, we present the model selection for a standard six-variable MSIAH(k,p) framework, where both intercepts and covariance matrices are allowed to switch across regimes. In contrast, Panel B examines model selection for a six-variable hybrid Markov-switching MSIH(k,p) framework, which features non-switching VAR coefficients, while still allowing intercepts and the error covariance matrix to switch across regimes. In Panel A, an increase in the number of regimes tends to enhance the model's log-likelihood, indicative of a better fit, and to also cause reductions in the AIC and BIC scores. However, this surge in complexity results in a notable decline in the saturation ratio, which may point to a potential risk of overfitting due to the increase in the number of estimated parameters. Despite these concerns, the two-state and three-state models stand out for their effectiveness, showing significant improvements in managing the empirical (own- and cross-) autocorrelations and providing a balance between model fitness and complexity. Nonetheless, as Case et al.

(2014) note, two regimes may not adequately capture the joint distribution of asset returns even with reference to a simpler three-asset menu due to significant synchronization between the individual regimes of REITs and equity portfolios, despite the occurrence of a notable disconnect between the regimes of REITs and bonds.

Panel B reports hybrid models that are characterized by a reduced number of switching parameters owing to the restriction on the VAR coefficients to be non-switching. These models generally lead to lower AIC and BIC values and higher saturation ratios relative to their full regime switching counterparts, suggesting a lower risk of overfitting while still retaining satisfactory model adaptation to the data. In particular, the three-state hybrid model achieves an optimal balance, making it a preferred choice for effectively minimizing overfitting while ensuring robust performance.

Overall, the empirical results in Table 3 indicate that both standard and hybrid MS-VAR models provide superior empirical fit compared to linear models featuring only one regime. Given its optimized trade-off between complexity and fit, the three-state hybrid MSIH(3,1) model is recommended for minimizing overfitting while maintaining effective in-sample predictive power for excess returns.

Next, we consider the model selection results for the MS models that include additional predictive variables, including the dividend yield, term and default spreads, and the inflation rate change. The model selection process for these ten-variable models is similar to that described for the six-variable models. We begin with simple, linear one-state models with no lags and then expand our selection up to four-state models with one lag.

Table 4 displays the outcomes of the model selection for this ten-variable framework. In Panel A of Table 4, we observe a consistent trend where the maximized log-likelihood improves as the number of regimes increases to three. However, this enhancement in model fit is accompanied by an increase in complexity, as suggested by the rising BIC values, which again emphasizes a trade-off between model adaptation to the data and parsimony. Despite higher log-likelihoods, the saturation ratio declines significantly with the addition of more states, potentially indicating overfitting. In addition, the Ljung-Box test results display significant autocorrelations in the residuals for ExCorpBond and T-bill returns across most models, suggesting that not all patterns in the data are adequately captured by the model parameters.

Among the models evaluated, the three-state MSIAH(3,0) model provides the best overall performance across the information criteria, offering a favorable balance between model fit, complexity, and residual diagnostics. Allowing both intercepts and covariances to switch across regimes, while excluding lagged dependencies, this model appears sufficient to capture the key nonlinearities and regime shifts in this larger system.

In Panel B, which displays results for hybrid models with non-switching VAR coefficients, the models generally exhibit lower log-likelihoods, indicating that standard MS-VAR models more effectively achieve a balance between complexity and fit. Despite this, the saturation ratios in these models, although decreasing with the addition of more regimes, remain higher than those in Panel A, suggesting a potentially lower risk of overfitting.

All in all, the results in Table 4 corroborate the insights from Table 3, namely that an increase in the number of regimes generally boosts model performance but also leads to greater complexity and a potential for overfitting, as emphasized by the declining saturation ratios. Based on the information criteria and test statistics reported, we conclude that the three-state MSIAH(3,0) model is the most appropriate specification to characterize the joint distribution of the ten-variable system that includes both asset returns and predictive variables.

Parameter Estimates

Table 5 shows the estimation results for the preferred model in the simple framework without predictive variables, the three-state hybrid MSIH(3,1) model, applied to the six-asset returns system. The joint distribution of the six return series is captured by three regimes with the following features.

1. Bull Regime: This regime reflects periods of sustained market appreciation, with positive mean excess returns across all risky assets and relatively low volatility, signaling steady price appreciation in a risk-on environment. The SMHiBM portfolio exhibits the highest implied unconditional average excess return (risk premium), followed by ME1BM2, REITs, and stocks, indicating that small and value-oriented portfolios generate the strongest average performance during upturns. Corporate bonds and T-bills show marginal positive returns consistent with stable credit conditions. The relative magnitudes of the estimated risk premia point to a hierarchy of expected returns consistent with risk exposure across asset classes. The filtered regime probabilities indicate that this bull regime is the most persistent, characterizing most long expansions since the early 1990s, including the post-2009 recovery and the post-COVID rebound. The transition probability matrix confirms the stability of this state, with a 91.3% estimated probability of remaining in the same regime in the following month. In unconditional terms, this regime is expected to characterize approximately 66.4% of any long sample.

2. Crash Regime: The crash regime is characterized by negative mean excess returns across all assets and increased volatility. The ME1BM2 portfolio records the lowest risk premium (most negative mean return), followed by SMHiBM, stocks, and REITs. The REIT unconditional risk premium of -2.30% in a single month indicates that real estate returns are less adversely affected, at least in relative terms, compared with the broader equity segments, emphasizing the partial resilience of REITs during steep downturns. Corporate bonds and T-bills continue to provide limited downside protection, with mean excess returns that remain negative, although smaller in absolute terms. [8] Overall, this regime represents an extreme but not persistent environment of strongly contracting asset prices and heightened cross-market correlations. This crash state carries a 50.4% probability of remaining in the same regime, implying an average duration of just two months, and about a 14.4% chance of reverting to the bull state in the next month, which increases substantially thereafter. As a result, the unconditional probability (i.e., average frequency) of this regime is 4.9%. The filtered probabilities associate this regime with major episodes of market stress, including the most pronounced peaks of the 2000–2002 dot-com collapse,

[8] The estimates of the mean excess returns reported in Table 5 are duration-adjusted, i.e., they condition on the differential duration of alternative regimes to avoid implausibly large estimates that could arise from scarcely persistent regimes.

the 2008–2009 global financial crisis, and the 2020 pandemic shock.

3. Volatile Regime: This regime captures moderately persistent periods of elevated uncertainty and high cross-asset volatility, in which returns fluctuate widely around positive means, with volatility dominating expected return dynamics rather than a persistent directional trend. All risky assets record positive or modestly positive state-specific risk premia in the range 0.16–0.35% per month. REITs perform strongly in this state, outperforming stocks and delivering returns that exceed those of the ME1BM2 and SMHiBM portfolios. This regime is infrequent, accounting for roughly 29% of the sample in terms of the implied ergodic probabilities, and moderately persistent, with a 77.6% probability of remaining in the same state and a 17.8% likelihood of transitioning into a bull regime, which acts as a sort of a stable attractor.

As already discussed, the three-state hybrid MSIH(3,1) model adopts a mixed approach in which the VAR coefficients are restricted to be constant across regimes, while the intercepts and covariance matrices are allowed to vary. This structure captures the unique characteristics and cross-asset interactions that define each market condition without introducing excessive model complexity. In Table 5, we highlight significance levels, with boldfaced coefficients indicating significance at the 5% level or better, which helps identify the most influential variables in each regime. We also report the estimated transition probability matrix, which shows the likelihood of moving from one regime to another. The results in Table 5 suggest that the model effectively distinguishes among the three market states, with robust statistical evidence supporting the reliability of the regime classifications. The conditional mean equations and covariance matrices for each state demonstrate regime-specific responses to market dynamics, reflecting distinct volatility and correlation structures across the variables.

Next, in Table 6 we present the parameter estimates for the preferred model within a framework that also includes predictive variables, namely the three-state MSIAH(3,0), applied to the returns of the six assets together with the four business-cycle predictors. Similarly to the previous specification, the joint distribution of asset returns is characterized by three distinct regimes: bull (stable growth), crash, and volatile.

1. Bull Regime: This regime features positive mean excess returns and relatively low volatility across all equity assets, while corporate bonds display modest positive excess returns consistent with stable credit conditions. Within this environment, the highest estimated risk premia are observed for the small-value portfolios (SMHiBM and ME1BM2), followed by REITs and stocks, reflecting broad risk-taking and optimism across equity sectors. The predictive variables indicate low volatility, low inflation, and a positively sloping term structure, consistent with benign funding conditions and accommodative monetary policy. The filtered regime probabilities show that this state is highly persistent and aligns with prolonged periods of economic expansion, including the growth phase of the 1990s, the post-2009 recovery, and the post-COVID rebound. The transition probability matrix confirms its dominance, with a 98.5% probability of remaining in the same regime in the subsequent period.

2. Crash Regime: This regime represents short-lived periods of broad market stress and elevated volatility, during which nearly all assets experience negative mean excess returns. Stocks, REITs, and the ME1BM2 portfolio exhibit negative risk premia, consistent with declining valuations and widespread risk aversion. Although the SMHiBM portfolio displays a large positive intercept, suggesting that small high book-to-market (value) firms may perform relatively well in downturns (see Petkova and Zhang, 2005), the coefficient is statistically insignificant. As a result, the model cannot reliably distinguish its mean return from zero in this state. Corporate bonds and the risk-free rate (T-bill) also show negative mean excess returns, indicating that credit spreads widen and short-term yields fall below inflation-adjusted levels under these conditions. REITs appear to be less adversely affected than the broader equity market, consistent with partial income support (see Chaudhry et al., 2022; Li and Zhu, 2022). Nevertheless, they still underperform, likely reflecting reduced rental demand during economic downturns and the effects of lockdown policies during the COVID-19 period (see Xie and Milcheva, 2024). This regime is infrequent and exhibits low persistence, with a 48.1% probability of remaining in the same state and roughly a 19.4% probability of transitioning to the bull regime. Its average duration is less than two months, and, in ergodic terms, it would account for less than 1% of any long sample. The filtered probabilities associate this regime to major episodes of systemic stress, such as the 2008–2009 global financial crisis and the 2020 COVID-19 pandemic, when investors faced simultaneous equity, credit, and liquidity shocks.

3. Volatile Regime: The volatile regime captures periods of market recovery and heightened uncertainty, characterized by elevated volatility but positive mean excess returns across all asset classes. In this state, small-value stocks (SMHiBM) continue to deliver the highest conditional mean returns, followed by ME1BM2, stocks, and REITs, highlighting renewed investor demand for riskier, value-oriented equity segments during transitional phases. Corporate bonds and T-bills also yield modest positive returns, consistent with improving credit conditions and normalizing short-term rates. Although all asset classes perform positively, this regime is marked by increased conditional variance and stronger cross-asset correlations, indicating that returns are achieved in a more volatile and less predictable environment. The regime is highly persistent, with a 96.5% probability of remaining in the same state and a 3.4% probability of transitioning to the bull regime, suggesting that these volatile yet expansionary conditions can endure for several months (on average, 29) before stabilizing. The filtered probabilities associate this state with re-inflationary and post-crisis periods, such as the late-1990s adjustment, the 2010–2012 recovery, and the post-pandemic normalization phase, when risk appetite returns to markets despite lingering uncertainty.

Overall, the parameters, including both intercepts and covariance matrices, vary substantially across regimes, providing a nuanced view of market dynamics under different economic conditions. The model's statistical performance is robust, with a log likelihood of -2,599 and information criteria values of AIC = 5,599, BIC = 6,869, and HQC = 6,038, all indicating a well-balanced fit relative to model complexity. However, the saturation ratio of 20.3, compared with 28.1 reported for the six-variable model reported in Table 5, reflects a lower number of observations per estimated parameter. This reduction signals *a higher potential for overfitting* in the three-state ten-variable specification, which is inherently more complex due to the inclusion of macroeconomic predictors and their interactions with asset returns. This added complexity requires more data to validate the additional parameters effectively. Moreover, the results suggest that the inclusion of predictive variables does not materially improve the model's explanatory power for REIT behavior. Consequently, given the available data, the more parsimonious six-variable MSIH(3,1) model arguably provides a cleaner and statistically more robust depiction of the regime-dependent dynamics in mixed-asset returns.

For comparison purposes, in Table 7 we display the parameter estimates for the six-variable linear VAR model with one lag, VAR6(1) (Panel A), and for the ten-variable linear VAR model without lags, VAR10(0) (Panel B). Both of these models would be selected by the information criteria if model selection were restricted to models with a single regime. The results from these VAR models and those from the regime-switching models in Tables 5 and 6 provide contrasting views of how different modeling approaches capture the dynamics of asset returns, including those of REITs. Traditional, single-state VAR models assume a single, stable relationship between the variables throughout the entire sample period. While they are simpler to estimate, since OLS can be separately applied to each equation, yielding results equivalent to maximum likelihood, and they feature a smaller number of estimated parameters, they are also more restrictive. In the VAR6(1) model, the significance of the intercept for REITs and the lack of significance of other variables suggest that the model does not effectively capture the dynamics influencing REIT returns through the included predictors.

In contrast, the MS-VAR models described in Tables 5 and 6 allow the relationship between variables to change according to different market regimes and exhibit a better fit to the data, as indicated by higher log likelihoods; this evidence also extends to the information criteria, once penalties are applied that grow with the complexity of the model specified. This suggests that regime switching models are indeed more effective in capturing the complexities and shifts in market dynamics. Overall, the enhanced ability of MS-VAR models to adjust to different regimes may lead to more informed and adaptive strategic asset allocation decisions, especially under conditions of market uncertainty or transitions across alternative economic states. Consequently, we next proceed to perform an asset allocation exercise based on the regime-switching models discussed in Tables 5 and 6, using Table 7 as a benchmark.

Asset Allocation

In this section, we analyze the buy-and-hold asset allocation strategy of a typical investor characterized by a power utility function. Based on the results from the previous section, the dynamics of excess returns of stocks, REITs, ME1BM2 and SMHiBM portfolios, corporate bonds, and T-bills returns are captured by a six-variable hybrid three-state MSIH(3,1) model. When predictor variables are incorporated, the modeling framework is expanded to a ten-variable three-state MSIAH(3,0) model.

Drawing on the methodology of Guidolin and Timmerman (2007), we employ a progressive discovery process to examine the investor's asset allocation strategies under different regime-switching configurations. Specifically, we consider both fixed and random initial regime assumptions, using historical transition probabilities to account for the persistence of market states. The resulting allocations are compared with those generated by a simpler, single-state (linear) VAR framework to evaluate the economic relevance of allowing for nonlinear dynamics. Furthermore, our analysis assesses how strategic asset allocation decisions evolve across different investment horizons and whether the inclusion of regime shifts materially alters portfolio composition, particularly the role of REITs relative to traditional asset classes and small-value equity portfolios.

Asset Allocation Results Assuming a Known Initial State

We begin by examining the change in optimal asset allocation under the assumption that the current market state is known at the start of the investment horizon, while future regimes are uncertain and stochastically driven by a three-state MS framework. This setup enables us to assess how an investor, characterized by constant relative risk aversion (CRRA), adjusts portfolio allocations given the prevailing market condition. We begin by generating 65,000 Monte Carlo simulations of future asset returns based on the six-variable MSIH(3,1) model and computing optimal buy-and-hold portfolio allocations for investment horizons ranging from 6 to 120 months, assuming risk aversion coefficients (γ) of 2, 5, and 10. The resulting simulated allocations are illustrated in Figure 1.

Bull Regime (State 1): Assuming an initial bull state, characterized by stable market growth and low volatility, the optimal portfolio for a low-risk-aversion investor (γ = 2) heavily favors small-value equities. The combined allocation to the ME1BM2 and SMHiBM portfolios accounts for approximately 70% of total wealth, while REITs receive about 30%. T-bills, corporate bonds, and the market index obtain virtually no weight. The absence of the market index reflects that, under regime switching, the two small-value portfolios already span most of the conditional equity risk premium, similarly to what is reported by Lynch (2001) in the context of asset menus that excluded bonds and public real estate. Over longer horizons, we observe pronounced horizon effects: the allocation to REITs increases slightly from around 26% to nearly 30%, while the SMHiBM share declines from about 40% to 30%. This shift indicates that longer-horizon investors place greater value on the income and diversification benefits of REITs relative to pure equity exposure. As risk aversion rises to $\gamma = 5$, the allocations to the ME1BM2 and SMHiBM portfolios and REITs decline to roughly 35%, 18%, and 18%, respectively, in favor of corporate bonds (over 25%) and a modest exposure to T-bills and stocks. For highly risk-averse investors ($\gamma = 10$), approximately 40% of the portfolio is allocated to T-bills, another 40% to the two small-value portfolios, and REITs receive less than 5%, reflecting a strong shift toward safety.

Crash Regime (State 2): In a deep, albeit scarcely persistent bear state, characterized by falling asset prices and heightened volatility, the optimal allocation becomes more conservative. In the case of $\gamma = 2$, the portfolio initially assigns about 25% to corporate bonds, but this share declines to zero for investors with horizons longer than two years, while the ME1BM2 allocation rises from slightly above 10% to around 40%. Throughout, REITs maintain a substantial weight of approximately 30%, decreasing only modestly from over 35% to just below 30% as the horizon extends. These results indicate that even in downturns, REITs retain an important role as income-producing and partially defensive assets. As risk aversion increases to $\gamma = 5$, allocations to the small-value portfolios and REITs fall to roughly 30% (ME1BM2), 18% (SMHiBM), and 16% (REITs), while exposure to corporate bonds rises to about 30%, accompanied by small positions in T-bills and stocks. T-bills initially account for more than 40% of the portfolio but decline steadily to below 5% for horizons roughly in excess of approximately 5-6 years, as near-term safety gives way to the pursuit of longer-term positive risk premia. In the case of $\gamma = 10$, approximately 50% of wealth is held in T-bills, another 30% in the two small-value portfolios, while REITs receive less than 5%, a smaller allocation than under the bull regime and yet never zero.

Volatile Regime (State 3): In the volatile regime, characterized by elevated uncertainty and strong cross-asset co-movement, the portfolio again favors exposure to small-value stocks and real estate. For $\gamma = 2$, about 60% of wealth is initially allocated to ME1BM2, declining to just over 40% for horizons in excess of two years, while the SMHiBM share increases from roughly 5% to 30%. REITs maintain a stable allocation of about 30% throughout. As risk aversion rises to $\gamma = 5$, allocations to the two small-value portfolios and REITs decrease to approximately 35% (ME1BM2), 17% (SMHiBM), and 17% (REITs), while exposure to corporate bonds decreases from over 40% to about 30%. T-bills and stocks remain minimal (1–2% each). For $\gamma = 10$, as one would expect, approximately 45% of the portfolio is invested in T-bills, about 35% in the two small-value portfolios, and REITs receive an allocation of less than 5%, which is higher than in the crash regime but below the real estate weight in the bull regime. The favor that REITs enjoy in the optimized portfolios under low and moderate degrees of risk aversion reflects the empirical evidence that REIT returns provide a good hedge against supply-shock-driven inflation, at least in this particular regime (see, e.g., Demiralay and Kilincarslan (2024)).

Asset Allocation Results with Uncertain Initial State

Within a related Markov-switching framework, Guidolin and Timmermann (2007) have emphasized the importance of considering not only uncertainty about future states but also the unknown identity of the initial state. Similarly, Veronesi (1999) highlighted the significance of understanding asset price dynamics when there is uncertainty regarding the prevailing regime. We extend our simulation exercises to account for the unknown initial state by exploring two scenarios: one assuming equal probabilities of each state and another assigning initial weights based on the steady-state probabilities estimated from historical data.[9]

The optimal asset allocation results when the initial state is uncertain and equal probabilities are assumed are depicted on the left-hand-side graphs in Figure 2. A short-term investor with low risk aversion (γ=2) leans heavily toward REITs, allocating more than 30% to them. The allocations to the small- value portfolios are also substantial: approximately 40% to ME1BM2 and 30% to SMHiBM. As the investor's risk aversion increases, the allocation to REITs declines to about 18% for γ = 5 and to 5% or lower for γ = 10, in favor of a larger position in T-bills, which rises to 50% for highly risk-averse investors. As expected, the uncertainty about the initial state appears to have a substantial impact on short-term asset allocation dynamics, while its influence diminishes for long-term investment strategies. Accordingly, REITs provide valuable hedging benefits in situations

[9] Based on the three-state, six-variable hybrid MSIH(3,1) model, the long-run (ergodic) probabilities implied by the estimates of the transition probability matrix turn out to be 66.4% for State 1, 4.9% for State 2, and 28.7% for State 3.

of increased uncertainty regarding the state of the economy (see Demiralay and Kilincarslan, 2024).

The right-hand-side plots in Figure 2 show the asset allocation schedules as a function of T in the second scenario, where steady-state probabilities determine the (uncertain) identity of the initial state. The optimal portfolio shares are similar to those reported on the left-hand side of the figure. However, this experiment may not be as meaningful as the one based on imputing equal regime probabilities, because it may overestimate, at least in the perspective of a genuinely risk-averse investor, the bull regimes probability and underestimate the crash regime likelihood, thus leading to an overallocation to risky assets.

Taken together, the results in Figure 2 reinforce the conclusion that REITs play a stabilizing and hedging role in optimal mixed-asset portfolios, even when investors face uncertainty about the prevailing market regime. Whether the initial state is known or uncertain, REITs consistently attract meaningful allocations, reflecting their ability to enhance portfolio resilience during periods of elevated macroeconomic uncertainty and shifting market dynamics. This occurs primarily at the expense of the optimal weight allocated to stocks, which is consistent with the literature on economic uncertainty in finance (see, e.g., Bloom (2014)).

Asset Allocation Dynamics under Linear Predictability

In this section, we take a step back and shift our focus from MS models to examine the structure of optimal portfolios under a single-state vector autoregressive (VAR) model, which captures linear predictability in asset returns. The strategic asset allocation implications of linear predictability in returns have been widely studied (see, e.g., Campbell and Viceira (1999), Barberis (2000), Campbell et al. (2004), Rerhing (2012), and Bianchi and Guidolin (2014)). We compare the results obtained from a VAR(1) model without predictive variables (that is, in which predictability arises solely from past returns) with those discussed in the previous section. The goal is to illustrate how far-reaching and decisive the effects of accounting for regimes are for optimal portfolio shares across different asset classes.

Figure 3 illustrates the variation in optimal portfolio weights as a function of the investment horizon (T) for investors with different risk aversion coefficients of ($\gamma = 2$, 5, and 10). Notably, the optimal weights under the VAR model differ markedly from those obtained under the MS framework. The VAR model produces flatter allocation schedules as the investment horizon changes, which indicates a limited horizon sensitivity. For instance, assuming $\gamma = 5$, a short-term investor allocates approximately 42% of wealth to corporate bonds, 33% to stocks, 15% to SMHiBM, and 10% to REITs, while ME1BM2 and T-bills receive almost no allocation. As the investment horizon lengthens, the portfolio weights remain relatively stable, and the overall allocations to risky assets are lower than those reported under the MS framework.

As the coefficient of risk aversion increases, the investor gradually reduces their exposure to risky assets and increases their allocation to corporate bonds and T-bills for all investment horizons. For investors with $\gamma = 10$, REITs are effectively excluded from the optimal portfolio, and allocations become dominated by fixed-income assets, with the value-weighted market portfolio, SMHiBM, and T-bills each receiving approximately 10%. The absence of meaningful REIT exposure contrasts with the regime-switching results, where REITs retain an allocation of roughly 5%.

Another notable contrast emerges when comparing the allocation to ME1BM2 portfolio under the MS framework with the allocation to the market portfolio in the linear VAR model. Under the VAR framework, the market index consistently attracts a positive and stable share of the portfolio, as the model imposes a single unconditional equity return distribution. In contrast, under regime switching, the market portfolio becomes almost redundant; its role in spanning the equity opportunity set is effectively replaced by the small-value portfolios (ME1BM2 and SMHiBM), which dominate equity exposure across all states. This pattern is similar to that documented by Lynch (2001), though here it relies on 20 additional years of data and proves robust to the inclusion of additional asset classes, and emphasizes that, in a nonlinear world, investors prefer to hold characteristic-based portfolios that capture conditional premia rather than less sharply characterized aggregate market exposures.

Relative to the MS framework, the VAR model produces portfolio allocations that are less responsive to changes in the investment horizon. When regimes are accounted for, portfolio weights as a function of T display steeper slopes and reflect dynamic rebalancing as the investor responds to regime-dependent investment opportunities. Thus, the MS framework introduces pronounced horizon effects in the optimal asset allocation strategy of a buy-and-hold investor. Prior studies, including Brandt (1999), Barberis (2000), Aït-Sahalia and Brandt (2008), and Jurek and Viceira (2011), suggest that, at least when conditioning variables such as the dividend yield are close to their sample means, longer investment horizons should be associated with higher equity demand. In our setting, regime switching generates horizon-dependent demand for REITs and equity portfolios that caries with the initial state, whereas such effects are largely absent in the linear VAR framework. This result suggests that investors benefit from the increased predictability captured by the MS framework, which may lead to larger and more responsive allocations to risky assets and, as we conjecture (see Section 6 for empirical evidence), to improved risk-adjusted returns.

Figure 4 shows the optimal portfolio shares as a function of the investment horizon obtained from the full-sample estimates of the three-state, ten-variable MSIAH(3,0) model, which captures the joint distribution of returns for stocks, REITs, corporate bonds, ME1BM2 and SMHiBM portfolios, and T-bills in the presence of predictors. Our simulations again cover the three alternative cases of initial regime initialization corresponding to the Bull, Crash, and Volatile states, and employ 65,000 Monte Carlo simulations to explore asset allocations under varying degrees of risk aversion (γ = 2, 5, and 10).

Consistent with the patterns documented in Figure 1, investors with low to moderate risk aversion coefficients (γ = 2, 5) allocate a larger share to riskier assets, such as small-value portfolios (ME1BM2 and SMHiBM) and REITs. Conversely, higher risk aversion (γ = 10) prompts a shift toward more conservative assets such as T-bills and corporate bonds to safeguard against potential losses in uncertain or adverse conditions. As expected, each state dictates distinct strategic asset allocations, and we observe pronounced horizon effects in each state.

Bull Regime (State 1): In the bull state, characterized by stable and positive market excess returns and low volatility, investors with low risk aversion ($\gamma = 2$) hold portfolios dominated by small-value equities. The SMHiBM portfolio accounts for about 40% of total wealth at short horizons and increases to over 70% as the horizon lengthens. REITs represent the second-largest component, with allocations starting at over 30% at short horizons, gradually decreasing with investment horizon and stabilizing at roughly 10% for horizons longer than 7 years, while aggregate stocks begin around 25% but quickly decline to near 10%, remaining below the REIT share. T-bills and corporate bonds receive negligible shares. These results highlight investors' preference for small-value premia in expansionary markets, complemented by a modest horizon-driven shift toward REITs for income and diversification. As risk aversion increases to $\gamma = 5$, the optimal portfolio shifts toward aggregate stocks, which vary from about 60% initially to 40% for horizons beyond six years, offset by a reduction in the SMHiBM share (0–25%). The REIT allocation remains stable, while corporate bonds gain importance. In the case of $\gamma = 10$, REITs decline from 25% to near zero beyond horizons of 60 months; SMHiBM receives roughly 0–10%, corporate bonds rise from 15% to more than 60%, and T-bills stabilize just below 5%, confirming a strong de-risking pattern.

Crash Regime (State 2): In this extreme but non-persistent state, when $\gamma = 2$, SMHiBM continues to command a dominant share of the portfolio, decreasing gradually from about 80% to 70% as the horizon extends. Corporate bonds, initially 20%, fall to zero for horizons approximately in excess of six years, while allocations to stocks and REITs increase from near zero to about 20% and 8%, respectively. This rebalancing pattern indicates that as extreme bearish conditions fade (also owing to their modest persistence), investors re-engage selectively with risk assets offering higher expected premia. With $\gamma = 5$, the SMHiBM share declines from 75% to below 40% for long horizons, while corporate bonds rise to about 40%. The REIT share grows slowly but remains small, around 1–2%, consistent with subdued demand for real estate cash flows under stress. For $\gamma = 10$, the portfolio becomes increasingly conservative: corporate bonds expand to roughly 60%, T-bills start near 40% and decrease to just above 10%, aggregate equities increase from approximately 0% to about 10%, while the two small value portfolios receive relatively stable allocations at about 10% each, and REITs receive no allocation.

Volatile Regime (State 3): The volatile regime exhibits the strongest rebalancing dynamics. For $\gamma = 2$, the weights of SMHiBM and stocks rise gradually over time, while corporate bonds and ME1BM2 decline. REITs, initially excluded, reach about 8% at longer horizons, reflecting moderate demand for inflation-sensitive assets as volatility persists. When $\gamma = 5$, corporate bonds dominate, declining from about 60 to 40% as the investment horizon lengthens, while allocations to all equity portfolios and REITs fall to just over 20% for SMHiBM and stocks, around 10% for ME1BM2 and 5% for REITs for long horizons. For $\gamma = 10$, T-bills absorb about 55% of wealth, the two small-value portfolios together around 35%, corporate bonds roughly 10%, and REITs between 1 and 3%, a higher share than in the crash state but lower than in the bull regime.

Across all regimes, REIT allocations are notably lower than in the six-variable specification that excluded business cycle predictors. The inclusion of predictive variables appears to affect the dynamics of investment opportunities by absorbing part of the role that REITs played as hedging or diversification assets, as documented by Demiralay and Kilincarslan (2024). Nevertheless, REITs remain non-redundant, particularly for low-risk-aversion investors and at shorter horizons, reflecting their contribution to income stability and inflation protection. Meanwhile, SMHiBM consistently anchors the equity exposure, often crowding out the market portfolio, underscoring investors' preferences for characteristic-based small-value premia within the ten-variable regime-switching framework.

The optimal asset allocations when the initial state is uncertain and equal random probabilities are assumed are depicted on the left-hand-side plots in Figure 5. A short-term investor with low risk aversion (γ=2) would heavily tilt their portfolio towards small-value stocks, allocating over 80% to them initially, with this share gradually decreasing to about 50% as the horizon lengthens. The allocation to a broad stock index starts instead at 0% and increases to approximately 30%. In contrast with the findings in Figure 2, REITs receive only a modest allocation, around 5% initially, rising to over 10% for longer horizons. As the investor's risk aversion increases, the allocation to REITs declines further, from roughly 6–8% for $\gamma = 5$ to close to zero for investors with $\gamma \geq 10$. This decline occurs in favor of higher weights on T-bills, which receive approximately 7-15% for the most risk-averse investors, depending on the horizon.

The right-hand-side set of plots in Figure 5 shows the asset allocation in the second scenario, where steady-state probabilities are assigned to capture uncertainty about the initial state. The shapes of the optimal portfolio schedules as a function of T are broadly similar to those reported in the left column of plots in the figure, though they favor a higher allocation to stocks, at the expense of lower shares in the SMHiBM portfolio. This adjustment reflects the higher long-run probability of the bull regime embedded in the steady-state configuration, which naturally favors riskier assets.

Finally, in Figure 6 we examine how the optimal portfolio depends on the investment horizon when a single-state, ten-variable vector autoregressive (VAR) model with no lags is (counterfactually) assumed to capture all predictability in asset returns. We compare the results from this linear framework with those in Figures 4 and 5. Figure 6 illustrates the dependence of the optimal portfolio weights on the investment horizon for investors with risk aversion coefficients γ = 2, 5, and 10. Consistent with the findings in Figure 3, the optimal allocations under the VAR model differ substantially from those obtained under the regime-switching framework. In particular, the VAR model produces much flatter allocation profiles as the investment horizon increases, whereas under regime switching the optimal weights exhibit sharper horizon-dependent rebalancing patterns. For longer horizons, the VAR model tends to assign relatively higher and more stable weights to stocks and, to a lesser extent, to REITs, consistent with its inability to account for regime-dependent risk–return trade-offs captured by the nonlinear specifications.

6. The Economic Importance of Regimes

In Section 5, we provided evidence that portfolio decisions should be influenced by the presence of regime switching in the joint distribution of asset returns. In this section, we assess the *economic significance of regimes* and evaluate the OOS performance of accounting for regimes in the return distribution. We compare the ex-post realized utility and realized Sharpe ratios of the optimal portfolios generated under regime-switching and contrast their realized performance with that obtained from a linear, single-state framework that ignores regimes.

One potential concern with the earlier asset allocation results under MS is that the portfolio recommendations discussed so far are based on parameter estimates obtained using the full-sample (end-of-sample) information. In practice, implementing regime-switching models in real time remains challenging, as parameter estimation uncertainty can induce implausible time variation in portfolio weights and reduce the models' practical reliability. It is therefore essential to evaluate the OOS performance of the MS framework and compare it with that of alternative predictive models to assess the robustness and economic value of its forecasts.

We conduct recursive pseudo-OOS tests of portfolio selection under alternative return-prediction models and for three investment horizons: T = 6, 12, and 120 months. To avoid any hindsight bias, we follow Guidolin and Timmermann (2007) and recursively estimate all the models under comparison. Specifically, we begin with the sample period January 1990–December 1999 and use parameter estimates and predicted state probabilities as of December 1999 to perform portfolio optimization for a T-month horizon spanning periods up to June 2000 (T = 6), December 2000 (T = 12), and December 2009 (T = 120). Next, we extend the sample to January 1990–December 2000 and repeat the estimation and portfolio optimization procedure for horizons ending in June 2001 (T = 6), December 2001 (T = 12), and December 2010 (T = 120).[10] This recursive procedure continues for each fixed investment horizon until the available data are exhausted, with the final evaluation dates corresponding to June and December 2023. This approach mitigates potential hindsight bias present in the full-sample results, as forecasts and resulting optimal portfolio weights are based solely on information available at the time of portfolio formation.

We focus on the buy-and-hold asset allocation problem for an investor with a risk aversion coefficient of 5.[11] In Table 8, we compare the performance of the three-state six-variable hybrid MS model with one lag (MSIH(3,1)), against a three-state, six-variable MS model without lags (MSIAH(3,0)), a linear VAR(1) model, and a simple independent and identically distributed (I.I.D.) benchmark that assumes constant means and covariances of excess returns. As an additional point of reference, we also report results for the classical static mean–variance tangency portfolio.[12]

[10] In the case of the 6-month horizon, we also compute the optimal portfolio weights and record realized portfolio returns for investments formed at the end of June 2000, 2001, 2002, …, 2023 and held until the end of the corresponding year (2000, 2001, …, 2023).
[11] Complete results for alternative coefficients of risk aversion are available upon request from the authors.
[12] Mean–variance portfolios are calculated using sample moments of T-period returns. In the case of T = 120, we use overlapping returns to ensure a sufficient number of observations for reliable estimation of the required moments.

Out-of-Sample Realized Utility and Sharpe Ratios

To evaluate the performance of alternative prediction models, we compute realized utility and Sharpe ratios for each model's optimal portfolios. The realized utility under each model, associated with a particular time series of portfolio weights, $\widehat{\omega}_t^T$, is computed at time t as:

$$V_t^T \equiv \frac{[W_T(\widehat{\omega}_t^T)]^{1-\gamma}}{1-\gamma} \equiv \frac{[(1-(\widehat{\omega}_t^T)'\iota_3)\exp(\mathrm{T}r^f) + (\widehat{\omega}_t^T)'\exp\left(\sum_{j=1}^{T} r_{t+j,n} + Tr^f\iota_3\right)]^{1-\gamma}}{1-\gamma}, \tag{18}$$

where the coefficient of risk aversion is $\gamma = 5$, $T = 6$, 12, and 120 months, and $\{r_{t+\tau}\}_{\tau=1}^{T}$ denotes the realized (excess) asset returns between $t + 1$ and $t + T$. For each investment horizon T and each asset allocation model, the period-t portfolio weights $\widehat{\omega}_t^T$ are obtained by maximizing the power utility objective, yielding a time series $\{V_t^T\}, t = 2000{:}01, \ldots, 2023{:}12 - T$ of realized utilities.

Following Guidolin and Timmermann (2007), we conduct inference using a block bootstrap (with 50,000 independent replications) applied to the empirical distribution of $\{V_t^T\}$. This procedure accounts for potential serial dependence in realized utilities arising from time variation in the conditional distribution of returns and, consequently, in portfolio weights. We further analyze OOS realized Sharpe ratios of the optimal portfolios, computed using the portfolio weight $(\widehat{\omega}_t^T)$ and the realized monthly returns of each asset class between t + 1 and t + T.

Table 8 reports pseudo-OOS performance results for the six-variable system, comparing the three-state hybrid MSIH(3,1) and MSIAH(3,0) models with a linear VAR(1), and static mean-variance and Gaussian i.i.d. (myopic) benchmarks. Panel A presents mean realized power utility (for $\gamma = 5$), while Panel B reports the corresponding Sharpe ratios for investment horizons of 6, 12, and 120 months.

The results confirm the superiority of the regime-switching framework over the static benchmarks. Both MSIH(3,1) and MSIAH(3,0) deliver higher (less negative) mean realized utilities than the linear VAR model and the static mean-variance and i.i.d. frameworks, indicating higher investor welfare once regime dynamics are incorporated. The VAR(1) model attains a marginally higher monthly Sharpe ratio (0.0124) than the regime-switching alternatives (0.0117 and 0.0119, respectively) only at the shortest horizon (T = 6 months), but this advantage fades over medium and long horizons. This suggests that while the VAR model may temporarily smooth short-term return volatility, it fails to capture the persistent, state-dependent shifts that enhance utility over time.

Table 9 reports analogous results for the ten-variable regime-switching models, as well as their corresponding VAR and of mean–variance i.i.d. counterparts. Consistent with the six-variable system, the MS models continue to produce higher (less negative) mean realized utilities than the linear VAR and static benchmarks, underscoring the value of incorporating regime dependence into return prediction. However, a comparison of Tables 8 and 9 reveals a clear trade-off between model parsimony and predictive power. The six-variable MSIH(3,1) model delivers more stable and superior OOS performance across horizons, whereas the inclusion of predictive variables in the ten-variable models offers only modest gains. In other words, while regime-switching dynamics are beneficial for capturing state-dependent behavior, their performance advantage diminishes when combined with an excessive number of predictors relative to sample length. These results reinforce the conclusion that parsimony enhances robustness; the simpler, six-variable regime-switching specification achieves a more reliable balance among flexibility, stability, and economic interpretability, consistent with recent evidence on the US housing market (see, e.g., Huang (2023)).

Overall, the higher OOS realized Sharpe ratios obtained by the MS framework indicate that incorporating regimes into VAR models enhances their forecasting power and adds meaningful economic value for long-term investments in mixed-asset portfolios.

7. Conclusion

We examine the asset allocation strategy of a long-horizon U.S. investor who accounts for the presence of regime shifts in the joint dynamics of excess returns across multiple asset classes. We model the return dynamics of a baseline six-asset menu, which includes stocks, REITs, corporate bonds, T-bills, and two size–value portfolios (ME1BM2 and SMHiBM), using vector autoregressive (VAR) specifications similar to Campbell et al. (2003). We then estimate several Markov-switching specifications with two to four regimes to capture the nonlinear and state-dependent behavior of asset returns, as in Ang and Bekaert (2002) and Guidolin and Timmermann (2007, 2008). Based on multiple information criteria, we identify the three-state hybrid MS-VAR model with one lag, MSIH(3,1), as providing the best balance between goodness of fit and parsimony. Although extending the framework to a ten-variable system with predictive variables yields modest improvements in in-sample fit, the added complexity increases the risk of overfitting and leads to weaker realized out-of-sample performance, confirming that the more parsimonious six-variable specification is both economically and statistically preferable.

Second, we analyze dynamic portfolio allocation decisions for a power-utility investor under alternative regime configurations and investment horizons. Using extensive Monte Carlo simulations, we evaluate optimal portfolios under both known and uncertain initial regimes, considering equal and steady-state regime probabilities, and compare the resulting allocations with those derived from linear single-state models. Across all regimes, the inclusion of regime switching produces greater sensitivity of portfolio weights to economic conditions and introduces pronounced horizon effects. Relative to the linear VAR framework, MS models generate steeper horizon-dependent portfolio adjustments, reflecting their superior ability to capture state-dependent investment dynamics.

Finally, OOS analysis demonstrates that regime-switching models deliver higher realized utility and Sharpe ratios than linear and i.i.d. benchmarks, underscoring their economic relevance to long-term investors. The MSIH(3,1) model achieves the strongest OOS performance, whereas the ten-variable system exhibits a less competitive performance due to estimation uncertainty. Importantly, the inclusion of REITs consistently enhances investor welfare across model specifications and investment horizons, although their optimal share moderates to around 10% in the MS frameworks once size–value portfolios are included. This evidence supports the view that public real estate remains a non-redundant asset class, providing meaningful diversification and inflation-hedging benefits even in the presence of alternative small-value exposures, and extends earlier findings in

Pagliari (2017), Delfim and Hoesli (2019), and Muckenhaupt, Hoesli, and Zhu (2025).

Overall, our findings indicate that incorporating regime-switching dynamics into standard portfolio optimization frameworks substantially improves both the predictive accuracy of asset return models and the economic performance of resulting portfolios. For long-term investors, especially those with moderate risk aversion, accounting for regime shifts yields significant welfare gains and highlights the enduring role of public real estate as a core, rather than peripheral, component of diversified investment strategies.

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Table 1: Descriptive Statistics

This table displays the descriptive statistics for the variables used in the analyses over the period January 1990–December 2023. All variables are measured at a monthly frequency. The main variables include the 30-day Treasury bill return, monthly excess returns on stocks, equity REITs, corporate bonds, and the Fama–French below-median-size, median-book-to-market-value (ME1BM2) and small-size, high-book-to-market-value (SMHiBM) portfolios. The four predictive variables include the dividend yield, the term spread, the default spread, and the monthly change in the CPI inflation rate. All return statistics are presented in percentage terms. The symbols *, **, and *** indicate significance at the 10%, 5%, and 1% significance levels, respectively.

Variable	Variable Description	N	Mean (%)	Std. Dev.	Min (%)	Max (%)	Median (%)	Std. Err.	Skewness	Kurtosis	Jarque-Bera	ADF Test	SR
Main Variables													
Rf	Return on 30-Day Treasury bills (risk-free rate)	408	0.21	0.19	0.00	0.69	0.18	0.01	0.37	1.89	30.21	-1.56	____
ExStock	Excess return on the value-weighted stock market index, including dividends, over the risk-free rate	408	0.66	4.44	-18.56	12.97	1.09	0.22	-0.62	4.26	52.93	-19.23***	0.10
ExREIT	Excess return on the total return of all equity REITs over the risk-free rate	408	0.67	5.21	-30.31	27.96	1.03	0.26	-0.75	9.08	668.03	-19.18***	0.09
ExCorpBond	Excess return on the ICE BofA US Corporate Bond Index over the risk-free rate	408	0.28	1.39	-9.54	4.47	0.37	0.07	-1.14	9.46	797.85	-13.66***	0.05
ExME1BM2	Excess return on the below-median-size, median-book-market value portfolios (ME1BM2)	408	0.90	5.42	-21.60	17.69	1.33	0.27	-0.46	4.40	47.63	-18.40***	0.13
ExSMHiBM	Excess return on the small-size, high-book-market value portfolios (SMHiBM)	408	0.95	5.86	-28.22	19.53	1.29	0.29	-0.61	5.16	104.61	-17.31***	0.12
Predictive Variables													
D/P	Dividend Yield (computed from the value-weighted stock index)	408	0.17	0.06	0.06	0.43	0.16	0.00	1.12	4.77	137.59	-4.40***	
TermSpr	Term Spread (10-year Treasury bond yield minus 3-month T-Bill yield)	408	0.13	0.10	-0.13	0.30	0.13	0.00	-0.22	2.40	9.52	-1.09	
DefSpr	Default Spread (Moody's Baa corporate bond yield minus 10-year Treasury yield)	408	0.19	0.06	0.11	0.49	0.18	0.00	1.73	8.26	673.85	-0.75	
ΔInf	Monthly change in CPI inflation	408	0.22	0.28	-1.77	1.38	0.21	0.01	-0.83	11.79	1361.07	-9.13***	

Table 2: Correlation Matrix

The table presents the correlation matrix of variables for the period January 1990–December 2023, including monthly excess returns on stocks, equity REITs, corporate bonds, the ME1BM2 and SMHiBM portfolios, and monthly returns on 30-day Treasury bills. The four predictive variables include the dividend yield, the term spread, the default spread, and the monthly change in the CPI inflation rate. The symbols *, **, and *** indicate significance at the 10%, 5%, and 1% significance levels, respectively.

(Obs=408)	*ExStock*	*ExREIT*	*ExCorp Bond*	*Ex ME1BM2*	*Ex SMHiBM*	*Rf*	*D/P*	*TermSpr*	*DefSpr*	*ΔInf*
ExStock	1.00									
ExREIT	0.65***	1.00								
ExCorpBond	0.27***	0.40***	1.00							
ExME1BM2	0.87***	0.67***	0.23***	1.00						
ExSMHiBM	0.83***	0.69***	0.23***	0.96***	1.00					
Rf	-0.04	-0.07	-0.05	-0.06	-0.07	1.00				
D/P	0.06	-0.02	0.05	0.07	0.03	0.06	1.00			
TermSpr	-0.01	0.06	0.03	0.04	0.08	-0.45***	0.20***	1.00		
DefSpr	-0.06	-0.04	0.10**	-0.02	-0.04	-0.56***	0.04	0.35***	1.00	
ΔInf	-0.01	0.01	-0.05	0.00	0.01	0.13**	-0.06	-0.07	-0.31***	1.00

Table 3: Model Selection for Six-Variable Markov-Switching Models

This table presents the information criteria used to select the most suitable number of regimes for a range of Markov-switching models for the six variables *ExStock*, *ExREIT*, *ExCorpBond*, *ExME1BM2, ExSMHiBM*, and *Rf*. MSIAH(k,p) denotes a k-state multivariate regime switching (MS) model, with regime-dependent intercepts (I) and covariance matrices (H), and p autoregressive (A) lags. The table summarizes the estimation outputs for various models: a single-state Gaussian i.i.d. model (k = 1) and Markov-switching models with alternative numbers of regimes (k = 2, 3, 4) and vector autoregressive lags p (0 or 1). The AIC, BIC, and HQC are information criteria that measure the trade-off between model fit and model complexity across the alternative specifications. RCM1 and RCM2 are regime classification measures proposed by Hamilton (1998) Guidolin (2009). The Saturation Ratio is the ratio of the total number of observations to the number of parameters estimated. LB Test Stat Value refers to the Ljung-Box test statistic used to examine the autocorrelations of the residuals from the models estimated. The symbols *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

Panel A: Six-Variable Markov-Switching MSIAH (*k, p*) Model Selection

										LB Test Stat Value					
States - k	Lags - p	Final LL	AIC	BIC	HQC	RCM1	RCM2	Params.	Sat. Ratio	*Ex Stock*	*Ex REIT*	*Ex CorpBond*	*Ex ME1BM2*	*Ex SMHiBM*	*T-bill*
One State	0	-4621	9254	9278	9339	____	____	27	**90.67**	0.29	0.39	43.45***	1.57	6.02**	392.96***
Two State	0	-4186	8484	**8809**	**8606**	0.11	**0.19**	56	**43.71**	1.80	4.25	36.42***	3.63	7.92**	619.96***
Three State	0	-4130	8433	8938	8623	**0.00**	15.14	87	**28.14**	0.64	4.04	54.36***	2.66	9.26**	663.83***
Four State	0	**-4056**	**8352**	9048	8613	**0.00**	8.30	120	**20.40**	1.59	1.72	51.93***	2.85	9.78**	572.01***
One State	1	-3838	7759	**7928**	**7901**	____	____	63	**38.86**	0.63	8.60**	9.26**	0.31	0.26	27.24***
Two State	1	-3738	7732	8474	8011	8.69	12.27	128	19.13	0.67	6.98*	8.95**	0.09	0.09	19.02***
Three State	1	**-3558**	**7507**	8638	7932	**0.00**	12.50	195	12.55	0.51	3.12	6.46*	0.15	0.67	22.71***
Four State	1	-3724	7976	9508	8552	**0.00**	**5.70**	264	9.27	0.71	4.56	11.88***	0.24	0.16	17.91***

Panel B: Six-Variable Hybrid Markov-Switching MSIH (*k, p*) Model Selection

										LB Test Stat Value					
States - k	Lags - p	Final LL	AIC	BIC	HQC	RCM1	RCM2	Params.	Sat. Ratio	*Ex Stock*	*Ex REIT*	*Ex CorpBond*	*Ex ME1BM2*	*Ex SMHiBM*	*T-bill*
Two State	1	-3758	7701	8235	7901	12.11	16.78	92	**26.61**	1.62	4.45	5.76*	0.62	0.27	17.30***
Three State	1	**-3054**	**6354**	**7068**	**6622**	**0.00**	33.88	123	19.90	0.37	5.27	6.56*	0.00	0.15	24.03***
Four State	1	-3598	7508	8413	7848	**0.00**	**5.11**	156	15.69	0.43	4.24	4.60	0.18	0.40	28.74***

Table 4: Model Selection for Ten-Variable Markov-Switching Models that Include Predictive Variables

This table presents the information criteria used to select the most suitable number of regimes for a range of Markov-switching models for the ten variables *ExStock*, *ExREIT*, *ExCorpBond*, *ExME1BM2, ExSMHiBM, Rf,* including four predictive variables (*D/P*, *TermSpr*, *DefSpr*, and *ΔInf*). MSIAH(*k,p*) denotes a *k*-state multivariate regime-switching (MS) model, with regime-dependent intercepts (I) and covariance matrices (H), and *p* autoregressive (A) lags. The table summarizes the estimation outputs for various models: a single-state Gaussian i.i.d. model ($k = 1$) and Markov-switching models with alternative numbers of regimes ($k = 2, 3, 4$) and vector autoregressive lags *p* (0 or 1). The AIC, BIC, and HQC are information criteria that measure the trade-off between fitness and complexity of the alternative models. RCM1 and RCM2 are the regime classification measures proposed by Hamilton (1998) and Guidolin (2009). The Saturation Ratio is the ratio of the total number of observations to the number of parameters estimated. LB Test Stat Value represents the Ljung-Box test statistic used to examine the autocorrelations of the residuals from the various models estimated. The symbols *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

Panel A: Ten-Variable Markov-Switching MSIAH (*k, p*) Model Selection Statistics

States - *k*	Lags -*p*	Final LL	AIC	BIC	HQC	RCM1	RCM2	Params.	Sat. Ratio
One State	0	-2978	5975	**6015**	6189	____	____	65	62.77
Two State	0	-2974	6213	7046	6500	**0.00**	**0.00**	132	30.91
Three State	0	**-2599**	**5599**	6869	**6038**	**0.00**	3.57	201	20.30
Four State	0	-2606	5757	7474	6350	**0.00**	3.70	272	15.00
One State	1	**-859**	**1938**	**2379**	**2310**	____	____	165	24.73
Two State	1	-1364	3392	5488	4116	**0.00**	**0.00**	332	12.29
Three State	1	-1321	3644	6807	4737	**0.00**	3.65	501	8.14
Four State	1	-1427	4198	8441	5664	**0.00**	2.58	672	6.07

LB-Test Stat Value

States - *k*	Lags -*p*	*ExStock*	*ExREIT*	*ExCorpBond*	*ExME1BM2*	*ExSMHiBM*	*Rf*	*D/P*	*TermSpr*	*DefSpr*	*ΔInf*
One State	0	0.29	0.39	43.45***	1.57	6.02**	392.96***	22.42***	389.90***	383.90***	87.71***
Two State	0	0.96	6.55*	43.57***	1.87	6.12*	773.69***	42.88***	749.74***	409.82***	89.36***
Three State	0	1.48	2.72	37.96***	0.44	3.34	761.11***	19.32***	753.59***	709.39***	63.04***
Four State	0	1.34	2.59	37.92***	0.41	3.30	772.65***	16.67***	751.00***	697.42***	64.97***
One State	1	0.70	6.96**	11.59***	0.09	0.13	31.80***	1.14	33.50***	15.82***	10.59**
Two State	1	3.89	6.98**	11.32***	5.34	5.16	15.80***	4.97	39.56***	4.13	7.72**
Three State	1	0.79	6.45*	14.15***	0.49	0.17	67.45***	1.99	90.27***	211.10***	11.56***
Four State	1	0.55	6.03*	13.56***	0.32	0.80	249.10***	0.34	321.18***	324.92***	19.45***

Panel B: Ten-Variable Hybrid Markov-Switching MSIH (*k, p*) Model Selection Statistics

States - *k*	Lags -*p*	Final LL	AIC	BIC	HQC	RCM1	RCM2	Params.	Sat. Ratio
Two State	1	**-1682**	**3827**	**5292**	**4333**	**0.00**	**0.00**	232	17.59
Three State	1	-1709	4020	5921	4677	**0.00**	**0.00**	301	13.55
Four State	1	-2938	6620	8968	7431	**0.00**	0.65	372	10.97

LB-Test Stat Value

States - *k*	Lags -*p*	*ExStock*	*ExREIT*	*ExCorpBond*	*ExME1BM2*	*ExSMHiBM*	*Rf*	*D/P*	*TermSpr*	*DefSpr*	*ΔInf*
Two State	1	0.72	8.12**	11.17***	0.63	0.79	14.70***	2.13	40.87***	6.05*	11.30***
Three State	1	1.26	6.50*	11.42***	1.08	0.07	10.36**	2.69	41.59***	5.36*	7.38**
Four State	1	2.22	7.77**	9.71**	1.15	0.29	205.10***	21.72***	361.39***	177.85***	11.51***

Table 5: Model Estimates of a MSIH (3,1) Model for the Six-Variable System

This table presents the ML parameter estimates for the three-state, six-variable MSIH(3,1) model. The parameters of the three distinct regimes—bull, crash, and volatile—are displayed from top to bottom. Boldfaced coefficients indicate significance at the 5% level or lower. The vector autoregressive coefficients are restricted to not switch across regimes. On the left-hand side of the table, the estimates of the regime-specific conditional mean equations are reported. On the right-hand side, a six-by-six regime-specific covariance matrix is displayed for each regime. The lower panel reports the unconditional (ergodic), duration-adjusted mean returns implied by the model for each asset and regime, expressed in percentage terms. The transition probability matrix is shown in the lower-right portion of the table; the significance levels of transition probabilities are denoted by *, **, and ***, indicating 10%, 5%, and 1% significance levels, respectively.

Three-State Six-Variable MSIH (3,1) Model

Sample Period: January 1990–December 2023

Log Likelihood: -3,054 Saturation Ratio: 19.90 AIC: 6,354 BIC: 7,068 HQC: 6,622 RCM1: 0.00 RCM2: 33.88

Conditional Mean Equation / **Covariance Matrix**

Regime 1 Bull

$$r_{ExStock,t} = 0.727 - 0.005\, r_{ExStock,t-1} - 0.008\, r_{ExREIT,t-1} + \mathbf{0.105}\, r_{ExCorpBond,t-1} + 0.112\, r_{ExME1BM2,t-1} - 0.075\, r_{ExSMHiBM,t-1} + \mathbf{1.170}\, r_{Rf,t-1} + \varepsilon_{stock,t}$$
$$r_{ExREIT,t} = 0.828 + \mathbf{0.184}\, r_{ExStock,t-1} - 0.112\, r_{ExREIT,t-1} + \mathbf{0.250}\, r_{ExCorpBond,t-1} - 0.214\, r_{ExME1BM2,t-1} + 0.271\, r_{ExSMHiBM,t-1} + 0.635\, r_{Rf,t-1} + \varepsilon_{stock,t}$$
$$r_{ExCorpBond,t} = 0.158 + 0.135\, r_{ExStock,t-1} + 0.032\, r_{ExREIT,t-1} + \mathbf{0.288}\, r_{ExCorpBond,t-1} - 0.039\, r_{ExME1BM2,t-1} - 0.043\, r_{ExSMHiBM,t-1} + \mathbf{0.704}\, r_{Rf,t-1} + \varepsilon_{stock,t}$$
$$r_{ExME1BM2,t} = 0.953 + \mathbf{0.159}\, r_{ExStock,t-1} - 0.043\, r_{ExREIT,t-1} + 0.0188\, r_{ExCorpBond,t-1} - 0.134\, r_{ExME1BM2,t-1} + 0.131\, r_{ExSMHiBM,t-1} + 2.361\, r_{Rf,t-1} + \varepsilon_{stock,t}$$
$$r_{ExSMHiBM,t} = 1.002 + 0.175\, r_{ExStock,t-1} - 0.027\, r_{ExREIT,t-1} - 0.072\, r_{ExCorpBond,t-1} - 0.182\, r_{ExME1BM2,t-1} + \mathbf{0.241}\, r_{ExSMHiBM,t-1} + 2.261\, r_{Rf,t-1} + \varepsilon_{stock,t}$$
$$r_{Rf,t} = \mathbf{0.002} - 0.001\, r_{ExStock,t-1} - 0.0001\, r_{ExREIT,t-1} - 0.0004\, r_{ExCorpBond,t-1} + 0.0004\, r_{ExME1BM2,t-1} + 0.001\, r_{ExSMHiBM,t-1} + \mathbf{0.981}\, r_{Rf,t-1} + \varepsilon_{stock,t}$$

15.588					
10.971	19.701				
1.415	2.196	1.132			
16.639	13.011	1.398	22.215		
16.842	14.310	1.542	22.764	25.352	
-0.034	-0.038	-0.008	-0.042	-0.048	0.001

Regime 2 Crash

$$r_{ExStock,t} = -0.739 - 0.005\, r_{ExStock,t-1} - 0.008\, r_{ExREIT,t-1} + \mathbf{0.105}\, r_{ExCorpBond,t-1} + 0.112\, r_{ExME1BM2,t-1} - 0.075\, r_{ExSMHiBM,t-1} + \mathbf{1.170}\, r_{Rf,t-1} + \varepsilon_{stock,t}$$
$$r_{ExREIT,t} = -0.487 + \mathbf{0.184}\, r_{ExStock,t-1} - 0.112\, r_{ExREIT,t-1} + \mathbf{0.250}\, r_{ExCorpBond,t-1} - 0.214\, r_{ExME1BM2,t-1} + 0.271\, r_{ExSMHiBM,t-1} + 0.635\, r_{Rf,t-1} + \varepsilon_{stock,t}$$
$$r_{ExCorpBond,t} = -0.201 + 0.135\, r_{ExStock,t-1} + 0.032\, r_{ExREIT,t-1} + \mathbf{0.288}\, r_{ExCorpBond,t-1} - 0.039\, r_{ExME1BM2,t-1} - 0.043\, r_{ExSMHiBM,t-1} + \mathbf{0.704}\, r_{Rf,t-1} + \varepsilon_{stock,t}$$
$$r_{ExME1BM2,t} = -1.111 + \mathbf{0.159}\, r_{ExStock,t-1} - 0.043\, r_{ExREIT,t-1} + 0.0188\, r_{ExCorpBond,t-1} - 0.134\, r_{ExME1BM2,t-1} + 0.131\, r_{ExSMHiBM,t-1} + 2.361\, r_{Rf,t-1} + \varepsilon_{stock,t}$$
$$r_{ExSMHiBM,t} = -0.889 + 0.175\, r_{ExStock,t-1} - 0.027\, r_{ExREIT,t-1} - 0.072\, r_{ExCorpBond,t-1} - 0.182\, r_{ExME1BM2,t-1} + \mathbf{0.241}\, r_{ExSMHiBM,t-1} + 2.261\, r_{Rf,t-1} + \varepsilon_{stock,t}$$
$$r_{Rf,t} = -\mathbf{0.197} - 0.001\, r_{ExStock,t-1} - 0.0001\, r_{ExREIT,t-1} - 0.0004\, r_{ExCorpBond,t-1} + 0.0004\, r_{ExME1BM2,t-1} + 0.001\, r_{ExSMHiBM,t-1} + \mathbf{0.981}\, r_{Rf,t-1} + \varepsilon_{stock,t}$$

23.425					
29.478	44.066				
1.615	-1.039	2.193			
22.277	21.875	3.899	36.784		
25.514	25.820	2.266	35.165	42.699	
-0.031	-0.064	-0.018	-0.047	-0.104	0.063

Regime 3 Volatile

$$r_{ExStock,t} = 0.664 - 0.005\, r_{ExStock,t-1} - 0.008\, r_{ExREIT,t-1} + \mathbf{0.105}\, r_{ExCorpBond,t-1} + 0.112\, r_{ExME1BM2,t-1} - 0.075\, r_{ExSMHiBM,t-1} + \mathbf{1.170}\, r_{Rf,t-1} + \varepsilon_{stock,t}$$
$$r_{ExREIT,t} = 0.848 + \mathbf{0.184}\, r_{ExStock,t-1} - 0.112\, r_{ExREIT,t-1} + \mathbf{0.250}\, r_{ExCorpBond,t-1} - 0.214\, r_{ExME1BM2,t-1} + 0.271\, r_{ExSMHiBM,t-1} + 0.635\, r_{Rf,t-1} + \varepsilon_{stock,t}$$
$$r_{ExCorpBond,t} = 0.324 + 0.135\, r_{ExStock,t-1} + 0.032\, r_{ExREIT,t-1} + \mathbf{0.288}\, r_{ExCorpBond,t-1} - 0.039\, r_{ExME1BM2,t-1} - 0.043\, r_{ExSMHiBM,t-1} + \mathbf{0.704}\, r_{Rf,t-1} + \varepsilon_{stock,t}$$
$$r_{ExME1BM2,t} = 0.956 + \mathbf{0.159}\, r_{ExStock,t-1} - 0.043\, r_{ExREIT,t-1} + 0.0188\, r_{ExCorpBond,t-1} - 0.134\, r_{ExME1BM2,t-1} + 0.131\, r_{ExSMHiBM,t-1} + 2.361\, r_{Rf,t-1} + \varepsilon_{stock,t}$$
$$r_{ExSMHiBM,t} = 0.842 + 0.175\, r_{ExStock,t-1} - 0.027\, r_{ExREIT,t-1} - 0.072\, r_{ExCorpBond,t-1} - 0.182\, r_{ExME1BM2,t-1} + \mathbf{0.241}\, r_{ExSMHiBM,t-1} + 2.261\, r_{Rf,t-1} + \varepsilon_{stock,t}$$
$$r_{Rf,t} = -\mathbf{0.002} - 0.001\, r_{ExStock,t-1} - 0.0001\, r_{ExREIT,t-1} - 0.0004\, r_{ExCorpBond,t-1} + 0.0004\, r_{ExME1BM2,t-1} + 0.001\, r_{ExSMHiBM,t-1} + \mathbf{0.981}\, r_{Rf,t-1} + \varepsilon_{stock,t}$$

36.976					
23.308	52.295				
2.922	4.890	2.922			
34.543	34.289	3.658	48.696		
35.628	42.654	3.768	51.866	60.435	
-0.062	-0.095	-0.017	-0.103	-0.121	0.003

Unconditional Mean (%, duration-adjusted)

	Bull	Crash	Volatile
ExStock	0.937	-2.710	0.232
ExREIT	1.015	-2.303	0.347
ExCorpBond	0.419	-1.967	0.162
ExME1BM2	1.375	-5.027	0.291
ExSMHiBM	1.502	-5.010	0.262
Rf	0.145	-1.902	0.037

Transition Probability Matrix (%)

State	Bull	Crash	Volatile
Bull	0.913***	0.144**	0.178***
Crash	0.017	0.504*	0.046**
Volatile	0.071	0.352**	0.776**

Table 6: Model Estimation of a MSIAH (3,0) Model for the Ten-Variable System

This table presents the results of parameter estimation for the three-state, ten-variable MSIAH(3,0) Model. The four predictive variables included are dividend yield (*D/P*), the term spread (*TermSpr*), the default spread (*DefSpr*), and the change in CPI inflation (*ΔInf*). The ML parameter estimates conditional on three different regimes—bull, crash, and volatile—are displayed from top to bottom. Regime-dependent intercepts and covariance matrices are allowed to vary across states. Boldfaced coefficients indicate significance at the 5% level or lower. The transition probability matrix, displayed in the lower part of the table, includes significance levels denoted by *, **, and ***, indicating 10%, 5%, and 1% significance sizes, respectively.

Three-State Ten-Variable MSIAH (3,0) Model

Sample Period: January 1990–December 2023

Log Likelihood: -2,599 Saturation Ratio: 20.30 AIC: 5,599 BIC: 6,869 HQC: 6,038 RCM1: 0.00 RCM2: 3.57

	Equations	Intercept	Covariance Matrix									
Regime 1 Bull	$ExStock_t$	**1.224**	12.082									
	$ExREIT_t$	**1.225**	8.477	15.988								
	$ExCorpBond_t$	**0.384**	0.466	1.117	1.152							
	$ExME1BM2_t$	**1.410**	14.191	10.972	0.077	21.313						
	$ExSMHiBM_t$	**1.468**	14.340	11.506	0.075	21.644	23.536					
	Rf_t	**0.192**	0.009	-0.008	0.000	-0.010	-0.001	0.030				
	D/P_t	**0.182**	0.005	-0.004	0.005	0.026	0.022	0.001	0.003			
	$TermSpr_t$	**0.150**	-0.005	0.014	0.000	0.026	0.046	-0.006	0.001	0.008		
	$DefSpr_t$	**0.180**	0.001	0.008	0.006	0.005	0.002	-0.006	0.000	0.001	0.002	
	ΔInf_t	**0.181**	0.001	-0.018	-0.018	0.002	0.002	0.004	0.000	0.001	-0.001	0.036
Regime 2 Crash	$ExStock_t$	-1.544	25.106									
	$ExREIT_t$	-0.879	7.882	23.307								
	$ExCorpBond_t$	-0.242	2.405	6.019	1.935							
	$ExME1BM2_t$	-0.624	-2.792	3.249	2.506	45.168						
	$ExSMHiBM_t$	3.335	6.697	6.019	3.490	60.304	87.469					
	Rf_t	**-0.436**	-0.016	-0.069	-0.027	-0.099	-0.074	0.032				
	D/P_t	**-0.276**	0.043	-0.014	0.003	-0.014	0.021	0.000	0.004			
	$TermSpr_t$	**0.016**	-0.007	-0.047	0.005	0.055	0.067	-0.005	0.003	0.015		
	$DefSpr_t$	**-0.209**	-0.003	-0.021	-0.001	-0.004	-0.035	-0.005	0.000	0.007	0.009	
	ΔInf_t	-0.637	-0.035	0.012	-0.030	0.005	-0.006	0.001	-0.002	-0.004	-0.018	0.133
Regime 3 Volatile	$ExStock_t$	**0.566**	35.338									
	$ExREIT_t$	0.440	28.450	49.506								
	$ExCorpBond_t$	**0.198**	4.419	7.129	3.693							
	$ExME1BM2_t$	1.103	35.814	36.079	5.597	47.557						
	$ExSMHiBM_t$	**1.143**	36.943	41.071	5.958	50.077	57.712					
	Rf_t	**0.252**	-0.107	-0.172	-0.020	-0.158	-0.250	0.043				
	D/P_t	**0.157**	0.049	-0.008	0.010	0.032	0.008	0.002	0.005			
	$TermSpr_t$	**0.102**	-0.015	0.065	0.010	0.014	0.043	-0.011	0.001	0.011		
	$DefSpr_t$	**0.216**	-0.016	0.010	0.024	-0.002	-0.019	-0.007	0.001	0.004	0.005	
	ΔInf_t	**0.288**	-0.016	0.022	-0.050	0.005	0.017	0.006	-0.001	-0.005	-0.015	0.143

Transition Probability Matrix (%)

State	Bull	Crash	Volatile
Bull	0.985***	0.194*	0.034***
Crash	0.002*	0.481**	0.002**
Volatile	0.013**	0.325	0.965***

Table 7: VAR Model Estimates for the Six- and the Ten-Variable Systems

Panel A presents ML parameter estimates of the six-variable linear vector autoregressive model with one lag (VAR6(1)). The conditional mean equations for VAR6(1) are displayed on the left-hand side of the table, while the covariance matrix is reported on the right-hand side. Boldfaced coefficients indicate significance at the 5% level or lower. Panel B displays parameter estimates for the ten-variable linear vector autoregressive model with no lags (VAR10(0)). The conditional mean equations for VAR10(0) are shown on the left-hand side, and the covariance matrix is displayed on the right-hand side. Boldfaced coefficients in both panels indicate significance at the 5% level or lower.

Panel A: Six-Variable Linear VAR (1) Model

Sample Period: January 1990–December 2023

Log Likelihood: -3,838 AIC: 7,759 BIC: 7,928 Number of Estimated Parameters: 63

VAR (1) Equations

$$r_{ExStock,t} = \mathbf{0.823} + 0.005\, r_{ExStock,t-1} - 0.007\, r_{ExREIT,t-1} + 0.038\, r_{ExCorpBond,t-1} + 0.140\, r_{ExME1BM2,t-1} - 0.116\, r_{ExSMHiBM,t-1} - 0.768\, r_{Rf,t-1} + \varepsilon_t$$
$$r_{ExREIT,t} = \mathbf{0.924} + 0.150\, r_{ExStock,t-1} - 0.127\, r_{ExREIT,t-1} + 0.145\, r_{ExCorpBond,t-1} - 0.191\, r_{ExME1BM2,t-1} + 0.244\, r_{ExSMHiBM,t-1} - 1.677\, r_{Rf,t-1} + \varepsilon_t$$
$$r_{ExCorpBond,t} = \mathbf{0.202} + \mathbf{0.113}\, r_{ExStock,t-1} + \mathbf{0.036}\, r_{ExREIT,t-1} + \mathbf{0.243}\, r_{ExCorpBond,t-1} - 0.028\, r_{ExME1BM2,t-1} - 0.043\, r_{ExSMHiBM,t-1} - \mathbf{0.087} r_{Rf,t-1} + \varepsilon_t$$
$$r_{ExME1BM2,t} = \mathbf{1.095} + 0.148\, r_{ExStock,t-1} - 0.041\, r_{ExREIT,t-1} - 0.055\, r_{ExCorpBond,t-1} - 0.114\, r_{ExME1BM2,t-1} + 0.095\, r_{ExSMHiBM,t-1} - 0.997\, r_{Rf,t-1} + \varepsilon_t$$
$$r_{ExSMHiBM,t} = \mathbf{1.185} + 0.146\, r_{ExStock,t-1} - 0.022\, r_{ExREIT,t-1} - 0.177\, r_{ExCorpBond,t-1} - 0.167\, r_{ExME1BM2,t-1} + 0.198 r_{ExSMHiBM,t-1} - 1.332\, r_{Rf,t-1} + \varepsilon_t$$
$$r_{Rf,t} = 0.003 - 0.001\, r_{ExStock,t-1} - 0.0001\, r_{ExREIT,t-1} - 0.001\, r_{ExCorpBond,t-1} + 0.0005\, r_{ExME1BM2,t-1} + 0.001\, r_{ExSMHiBM,t-1} + \mathbf{0.982}\, r_{Rf,t-1} + \varepsilon_t$$

Covariance Matrix

19.431					
14.825	26.408				
1.574	2.735	1.607			
20.739	18.533	1.590	28.885		
21.214	20.409	1.667	29.851	33.393	
-0.002	-0.001	-0.002	-0.012	-0.016	0.001

Panel B: Ten-Variable Linear VAR (0) Model

Sample Period: January 1990–December 2023

Log Likelihood: -2,978 AIC: 5,975 BIC: 6,015 Number of Estimated Parameters: 65

Equations	Intercepts	Covariance Matrix									
$ExStock_t$	**0.665**	19.624									
$ExREIT_t$	**0.668**	14.929	27.097								
$ExCorpBond_t$	**0.279**	1.662	2.907	1.925							
$ExME1BM2_t$	**0.902**	20.923	18.961	1.728	29.292						
$ExSMHiBM_t$	**0.946**	21.404	21.037	1.828	30.399	34.242					
Rf_t	**0.214**	-0.035	-0.067	-0.012	-0.057	-0.076	0.035				
D/P_t	**0.173**	0.018	-0.007	0.005	0.023	0.010	0.001	0.004			
$TermSpr_t$	**0.133**	-0.005	0.030	0.004	0.023	0.042	-0.008	0.001	0.009		
$DefSpr_t$	**0.192**	-0.016	-0.011	0.008	-0.006	-0.015	-0.006	0.000	0.002	0.003	
ΔInf_t	**0.220**	-0.016	0.015	-0.021	0.003	0.012	0.007	-0.001	-0.002	-0.005	0.076

Table 8: Recursive Pseudo-Out-of-Sample Tests under Alternative Return Prediction Models for the Six-Variable System

This table presents the recursive pseudo out-of-sample (OOS) tests for a portfolio allocation exercise under alternative return prediction models across three investment horizons: T = 6, 12, and 120 months. The monthly return series system includes the excess return of stocks, REITs, corporate bonds, the ME1BM2 and SMHiBM portfolios, and T-bills. Panel A reports OOS realized power utility (for γ=5). C.I. denotes confidence intervals, and boldfaced statistics highlight the highest realized power utility among all models for a given investment horizon. MSIH (3,1) refers to a three-state six-variable Markov-switching VAR(1) hybrid model, MSIAH (3,0) to a three-state six-variable Markov-switching VAR model with no lag, and VAR(1) to a standard linear vector autoregressive model with one lag. Differences are computed relative to the benchmark model indicated in the first column. The symbols *, **, and *** indicate significance at 10%, 5%, and 1% levels, respectively. Panel B, to the right, reports the Sharpe ratios of the portfolio returns obtained from the same recursive OOS tests. Boldfaced numbers emphasize the highest Sharpe ratios among all models for a given investment horizon.

Panel A: Recursive pseudo-OOS realized power utility for periods from 2000/1 to 2023/12						100 x Differences in out-of-sample			Panel B: Recursive pseudo-OOS Sharpe Ratio	
Six-Variable MSIH (3,1)	Mean	St. Dev	C.I. 5%		Portfolio Return St. Dev	Mean	St. Dev	T-stat	Mean	St. Dev
T = 6	**0.1949**	0.0011	0.1946	0.1952	0.0304	NA	NA	NA	0.0117	0.0028
T = 12	**0.1491**	0.0020	0.1486	0.1497	0.0237	NA	NA	NA	**0.0081**	0.0019
T = 120	**0.0019**	0.0045	0.0006	0.0032	0.0142	NA	NA	NA	**0.0055**	0.0005
Six-Variable MSIAH (3,0)	Mean	St. Dev	C.I. 5%		Portfolio Return St. Dev	Mean	St. Dev	T-stat	Mean	St. Dev
T = 6	0.1960	0.0011	0.1957	0.1963	0.0294	0.1054	0.7522	0.6862	0.0119	0.0028
T = 12	0.1538	0.0019	0.1532	0.1543	0.0242	0.4638	1.3602	1.6704*	0.0079	0.0019
T = 120	0.0022	0.0031	0.0013	0.0031	0.0149	0.0326	2.7207	0.0586	0.0054	0.0006
Six-Variable VAR (1)	Mean	St. Dev	C.I. 5%		Portfolio Return St. Dev	Mean	St. Dev	T-stat	Mean	St. Dev
T = 6	0.2096	0.0010	0.2093	0.2099	0.0248	1.4709	0.7370	9.7777***	**0.0124**	0.0036
T = 12	0.1755	0.0017	0.1750	0.1760	0.0212	2.6424	1.2934	10.0083***	0.0080	0.0022
T = 120	0.0095	0.0007	0.0093	0.0097	0.0143	0.7640	2.2767	1.6439	0.0049	0.0008
Static Mean Variance	Mean	St. Dev	C.I. 5%		Portfolio Return St. Dev	Mean	St. Dev	T-stat	Mean	St. Dev
T = 6	0.2228	0.0008	0.2225	0.2230	0.0398	2.7858	0.6701	20.3668***	0.0061	0.0025
T = 12	0.1998	0.0013	0.1995	0.2002	0.0296	5.0728	1.1705	21.2316***	0.0033	0.0018
T = 120	0.0371	0.0002	0.0371	0.0371	0.0158	3.5194	2.2516	7.6575***	0.0030	0.0004
IID/Myopic	Mean	St. Dev	C.I. 5%		Portfolio Return St. Dev	Mean	St. Dev	T-stat	Mean	St. Dev
T = 6	0.2254	0.0008	0.2251	0.2256	0.0348	3.0322	0.6734	22.0599***	0.0081	0.0033
T = 12	0.2053	0.0013	0.2049	0.2056	0.0270	5.6037	1.1781	23.3029***	0.0052	0.0020
T = 120	0.0508	0.0002	0.0507	0.0508	0.0148	4.8850	2.2525	10.6245***	0.0031	0.0005

Table 9: Recursive Pseudo-Out-of-Sample Tests for the Ten-Variable System

This table presents the recursive pseudo out-of-sample tests for a portfolio allocation exercise under alternative return prediction models across three investment horizons: T = 6, 12, and 120 months. The monthly return series system includes the excess return stocks, REITs, corporate bonds, the ME1BM2 and SMHiBM portfolios, and T-bills. The four predictive variables included are the dividend yield (*D/P*), the term spread (*TermSpr*), the default spread (*DefSpr*), and the change in CPI inflation (*ΔInf*). Panel A reports OOS realized power utility (for γ=5). C.I. denotes confidence interval, and boldfaced statistics highlight the highest realized power utility among all models for a given investment horizon. MSIAH(3,0) refers to a three-state, ten-variable Markov-switching VAR model with no lags, MSIH(3,1) to a three-state, ten-variable Markov-switching VAR(1) hybrid model, and VAR(0) to a standard linear vector autoregressive model with no lags. Differences are computed relative to the benchmark model indicated in the first column. The symbols *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively. Panel B, reported on the right-hand side, presents the Sharpe ratios of portfolio returns obtained from the same recursive OOS tests. Boldfaced numbers emphasize the highest Sharpe ratios among all models for each investment horizon.

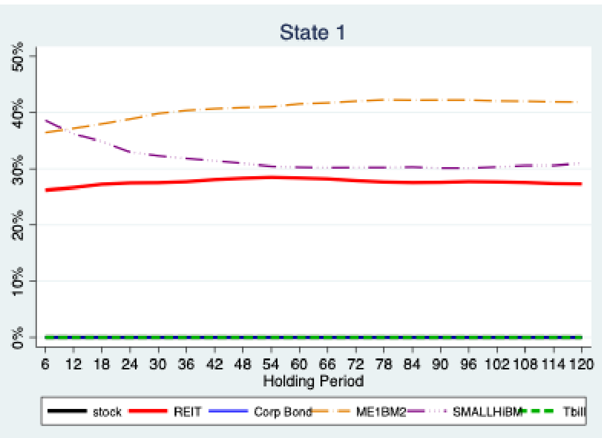
Panel A: | **Panel B:**

Recursive pseudo-OOS realized power utility for periods from 2000/1 to 2023/12						**100 x Diff in out-of-sample**			**Recursive pseudo-OOS Sharpe Ratio**	
Ten-Variable MSIAH(3,0)	Mean	St. Dev	C.I. 5%		Portfolio Return St. Dev	Mean	St. Dev	T-stat	Mean	St. Dev
T = 6	**0.2020**	0.0044	0.2007	0.2032	0.0270	NA	NA	NA	0.0132	0.0031
T = 12	**0.1668**	0.0039	0.1657	0.1679	0.0292	NA	NA	NA	**0.0127**	0.0032
T = 120	**0.0177**	0.0009	0.0174	0.0180	0.0179	NA	NA	NA	0.0069	0.0009
Ten-Variable MSIH(3,1)	Mean	St. Dev	C.I. 5%		Portfolio Return St. Dev	Mean	St. Dev	T-stat	Mean	St. Dev
T = 6	0.2046	0.0034	0.2036	0.2055	0.0347	0.4990	3.2371	0.7552	0.0132	0.0027
T = 12	0.1697	0.0032	0.1688	0.1706	0.0268	0.9614	3.2689	1.4408	0.0093	0.0020
T = 120	0.0369	0.0014	0.0365	0.0373	0.0148	1.8208	1.6391	5.4748***	**0.0071**	0.0003
Ten-Variable VAR (0)	Mean	St. Dev	C.I. 5%		Portfolio Return St. Dev	Mean	St. Dev	T-stat	Mean	St. Dev
T = 6	0.2058	0.0034	0.2049	0.2068	0.0163	0.3944	3.0795	0.6274	**0.0141**	0.0041
T = 12	0.1733	0.0039	0.1722	0.1744	0.0165	0.6552	2.7280	1.1766	0.0093	0.0042
T = 120	0.0187	0.0006	0.0186	0.0189	0.0182	-0.1606	0.5205	-8.4145***	0.0065	0.0010
Static Mean Variance	Mean	St. Dev	C.I. 5%		Portfolio Return St. Dev	Mean	St. Dev	T-stat	Mean	St. Dev
T = 6	0.2228	0.0008	0.2225	0.2230	0.0213	2.0837	2.2090	4.6212***	0.0061	0.0025
T = 12	0.1998	0.0013	0.1995	0.2002	0.0158	3.3043	2.0331	7.9619***	0.0033	0.0018
T = 120	0.0371	0.0002	0.0371	0.0371	0.0126	1.9406	0.4471	21.2656***	0.0030	0.0004
IID/Myopic	Mean	St. Dev	C.I. 5%		Portfolio Return St. Dev	Mean	St. Dev	T-stat	Mean	St. Dev
T = 6	0.2254	0.0008	0.2251	0.2256	0.0229	2.3418	2.2116	5.1873***	0.0081	0.0033
T = 12	0.2053	0.0013	0.2049	0.2056	0.0165	3.8466	2.0425	9.2260***	0.0052	0.0020
T = 120	0.0508	0.0002	0.0507	0.0508	0.0120	3.3076	0.4521	35.8381***	0.0031	0.0005

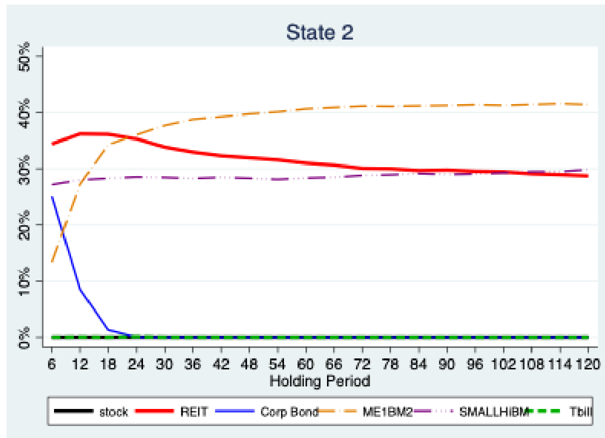
Figure 1: Asset Allocation as a Function of the Investment Horizon with Certain Initial States (MSIH(3,1) Model for the Six-Variable System)

This figure illustrates the buy-and-hold portfolio asset allocation as a function of the investment horizon across different asset classes using the MSIH(3,1) hybrid Markov-switching VAR model applied to the six-variable system, which captures the joint distribution of excess returns on stocks, REITs, corporate bonds, the ME1BM2 and SMHiBM portfolios, and T-bills. The initial state of the Markov-switching model is assumed to be known with certainty and fixed at bull, crash, or volatile, respectively. Portfolio allocations are obtained from 65,000 Monte Carlo simulations. Subfigures (a) through (i) display the optimal portfolio weights (in percentages) across asset classes for different initial states and relative risk aversion coefficients of γ = 2 (top row), γ = 5 (middle row), and γ = 10 (bottom row).

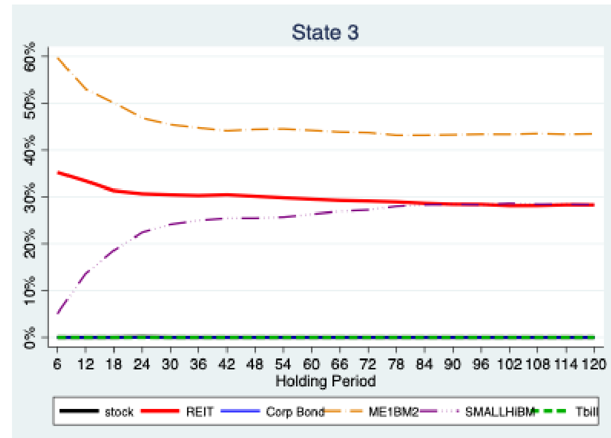
a. Asset Allocation Under State 1 (γ = 2)



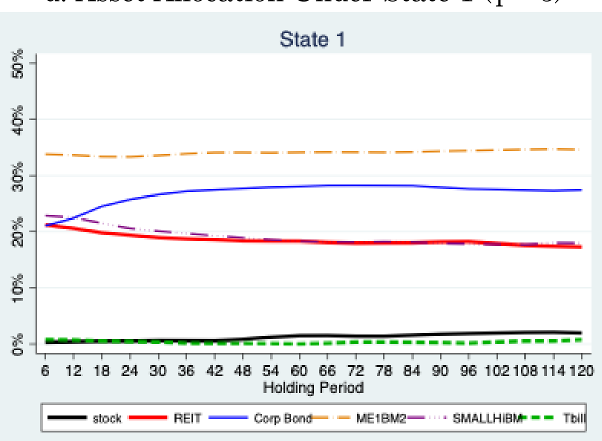
b. Asset Allocation Under State 2 (γ = 2)



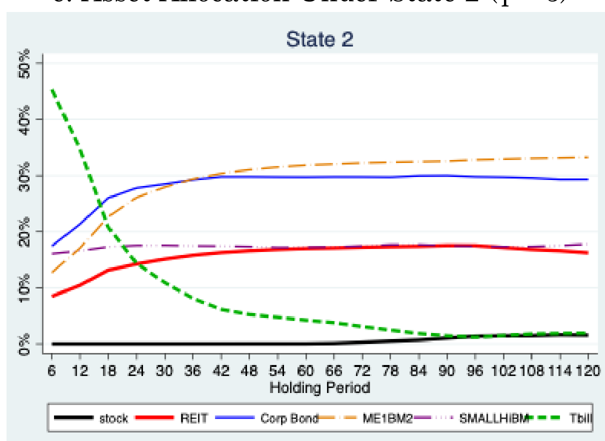
c. Asset Allocation Under State 3 (γ = 2)



d. Asset Allocation Under State 1 (γ = 5)



e. Asset Allocation Under State 2 (γ = 5)



f. Asset Allocation Under State 3 (γ = 5)

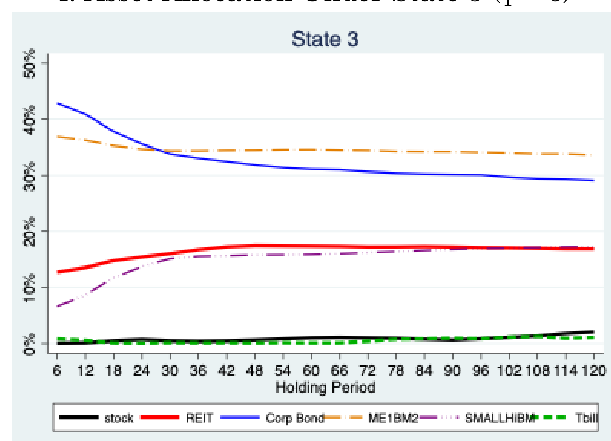
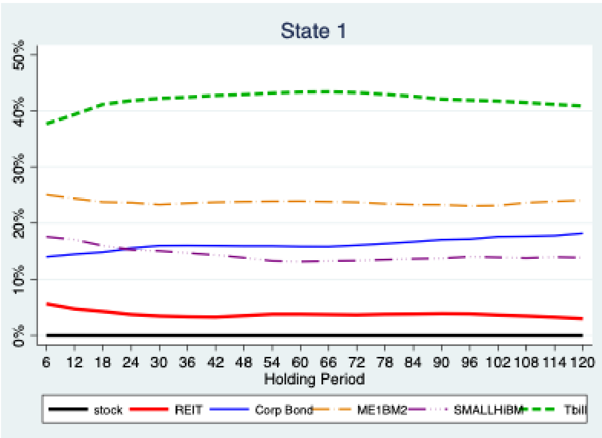
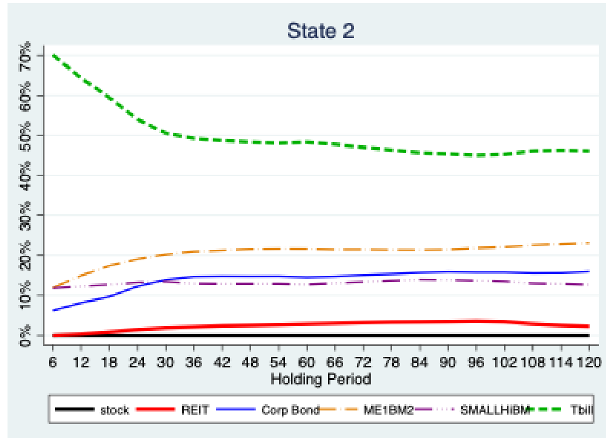


Figure 1: (Continued)

g. Asset Allocation Under State 1 (γ = 10)



h. Asset Allocation Under State 2 (γ = 10)



i. Asset Allocation Under State 3 (γ = 10)

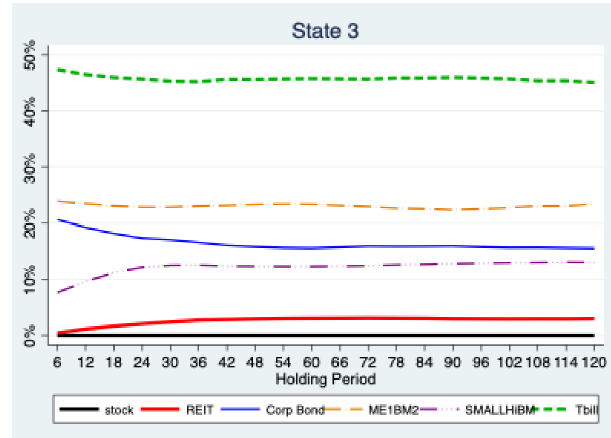
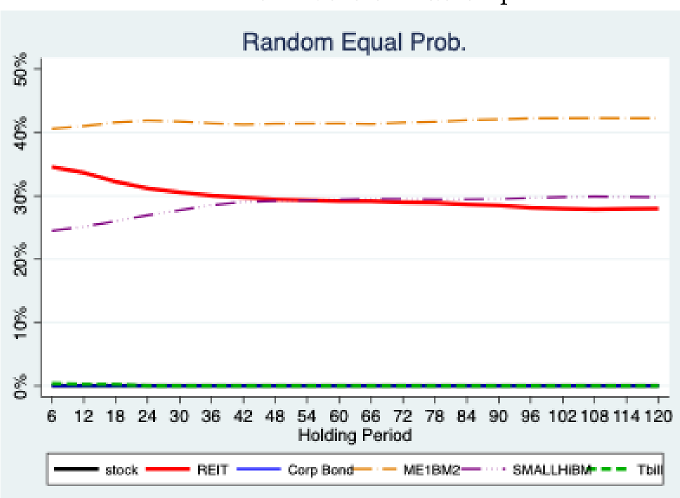


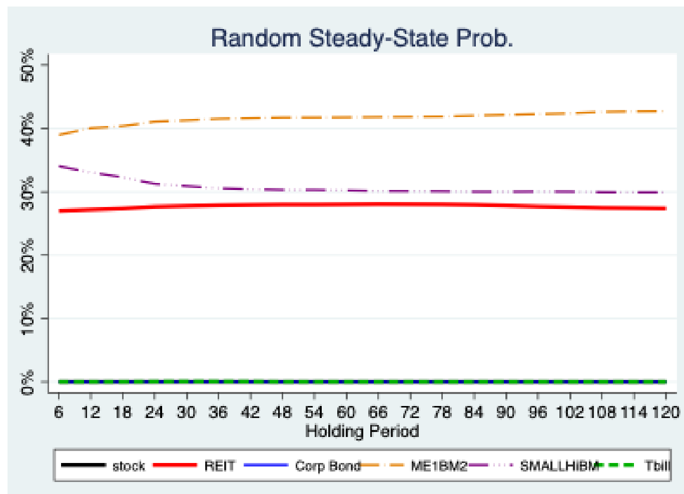
Figure 2: Asset Allocation as a Function of the Investment Horizon with Uncertain Initial States (MSIH(3,1) Model for the Six-Variable System)

This figure illustrates the buy-and-hold portfolio asset allocation as a function of the investment horizon across different asset classes using the MSIH(3,1) hybrid Markov-switching VAR model applied to the six-variable system, which captures the joint distribution of excess returns on stocks, REITs, corporate bonds, the ME1BM2 and SMHiBM portfolios, and T-bills. In contrast to Figure 1, the initial state of the Markov-switching model is uncertain. Portfolio allocations are obtained from 65,000 Monte Carlo simulations. Two alternative assumptions about the initial regime are considered: equal regime probabilities (Scenario 1, left column of plots) and steady-state (ergodic) regime probabilities (Scenario 2, right column of plots). The relative risk aversion coefficients are γ = 2 (top row) and γ = 5 (bottom row).

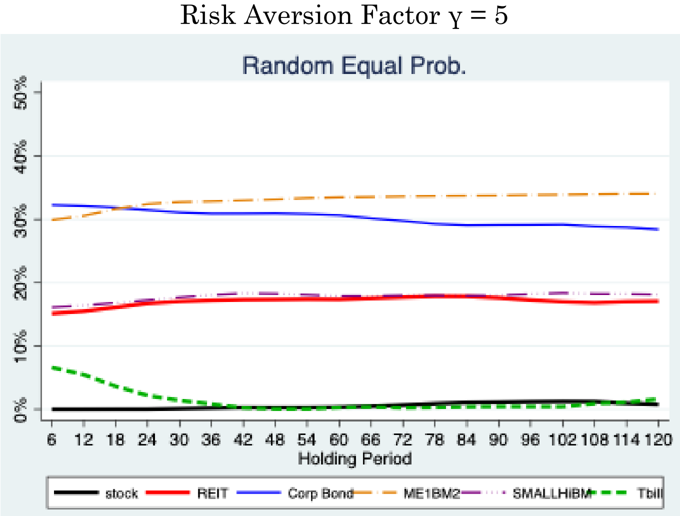
a. Asset Allocations with Random States (Scenario 1) Risk Aversion Factor γ = 2



b. Asset Allocations with Random States (Scenario 2) Risk Aversion Factor γ = 2



c. Asset Allocations with Random States (Scenario 1) Risk Aversion Factor γ = 5



d. Asset Allocations with Random States (Scenario 2) Risk Aversion Factor γ = 5

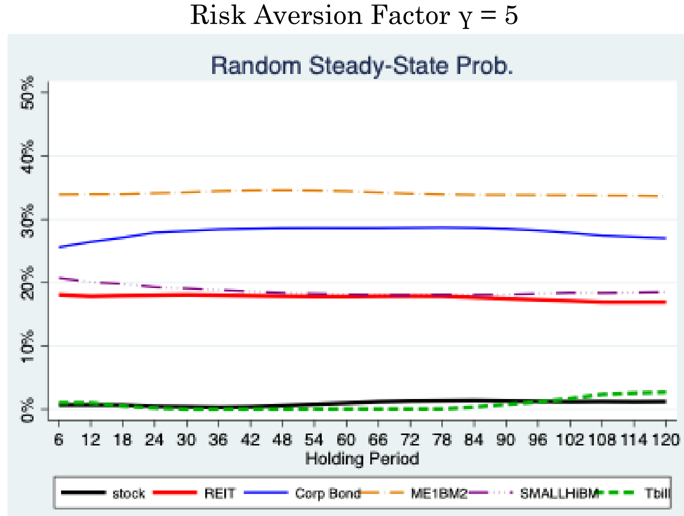
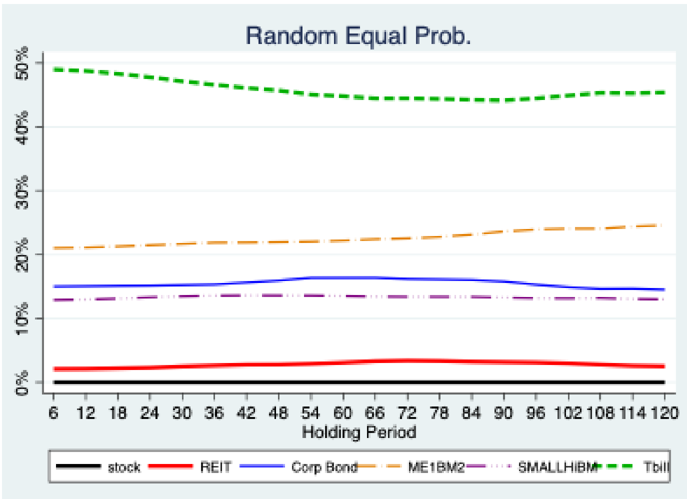


Figure 2: (Continued)

a. Asset Allocations with Random States (Scenario 1)
Risk Aversion Factor γ = 10



b. Asset Allocations with Random States (Scenario 2)
Risk Aversion Factor γ = 10

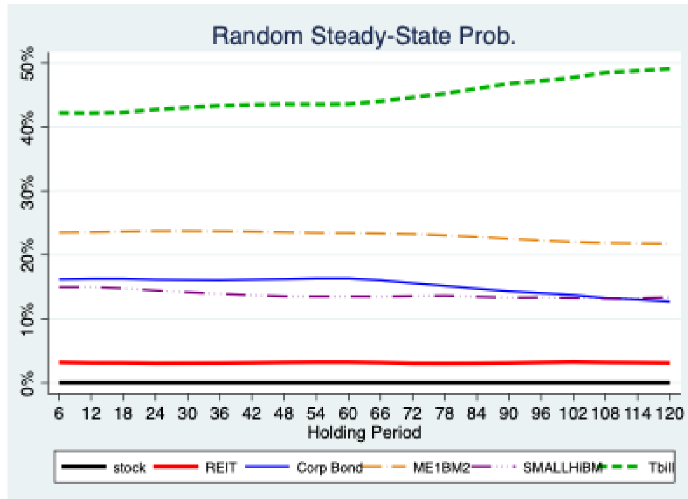
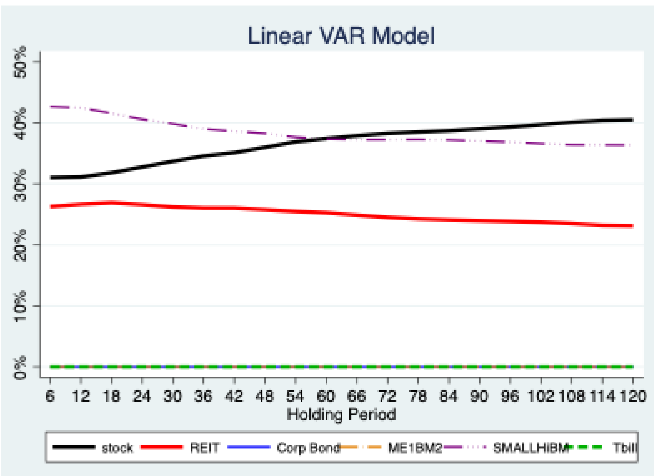


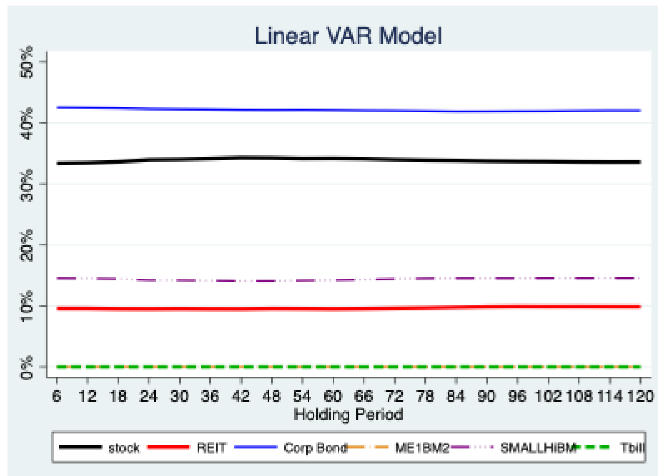
Figure 3: Asset Allocation as a Function of the Investment Horizon under a Single-State Six-Variable VAR(1) Model

This figure illustrates the buy-and-hold portfolio asset allocation as a function of the investment horizon across the different asset classes using a single-state, six-variable linear VAR(1) model, which captures the joint distribution of returns for stocks, REITs, corporate bonds, the ME1BM2 and SMHiBM portfolios, and T-bills. Portfolio allocations are obtained from 65,000 Monte Carlo simulations. The plots show the evolution of optimal portfolio weights as a function of the investment horizon for investors with relative risk aversion coefficients of γ = 2, 5, and 10.

a. Asset Allocation; γ = 2.



b. Asset Allocation; γ = 5.



c. Asset Allocation; γ = 10.

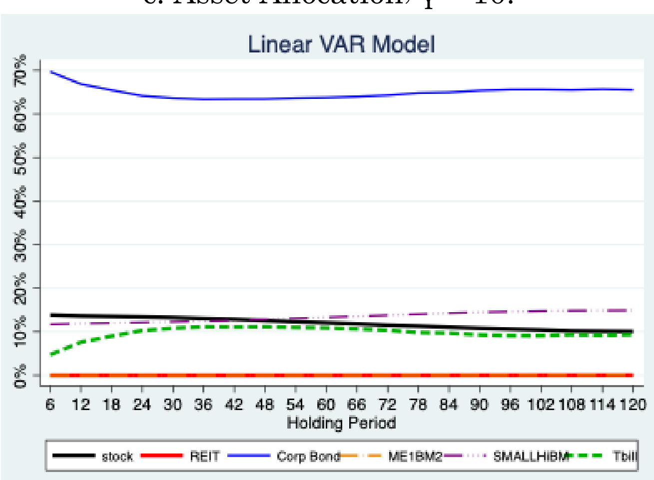
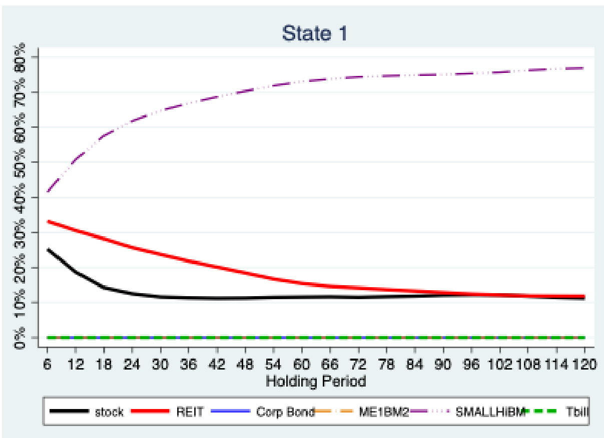


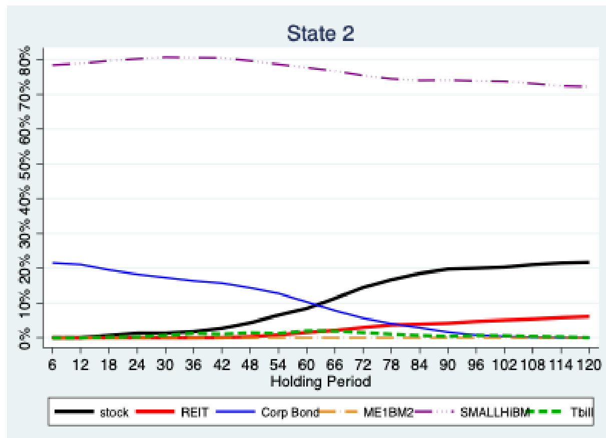
Figure 4: Asset Allocation as a Function of the Investment Horizon with Certain Initial States (MSIAH(3,0) Model for the Ten-Variable System)

This figure illustrates the buy-and-hold portfolio asset allocation as a function of the investment horizon across different asset classes using the three-state, ten-variable MSIAH(3,0) model, which captures the joint distribution of returns for stocks, REITs, corporate bonds, the ME1BM2 and SMHiBM portfolios, T-bills, conditional on the four predictive variables. The initial state of the Markov-switching model is assumed to be known with certainty and fixed to the bull, crash, and volatile regimes, respectively. Portfolio allocations are obtained from 65,000 Monte Carlo simulations. Subfigures (a) through (i) report the optimal portfolio weights for relative risk aversion coefficients of γ = 2 (top), γ = 5 (middle), and γ = 10 (bottom).

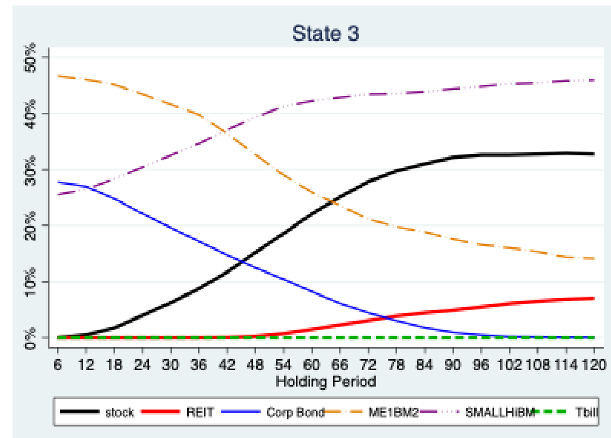
a. Asset Allocation Under State 1 (γ = 2)



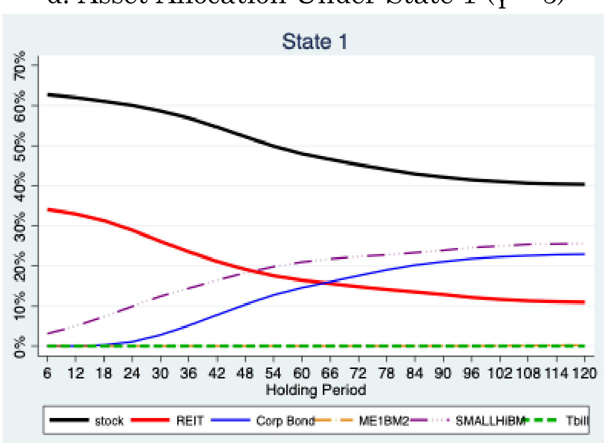
b. Asset Allocation Under State 2 (γ = 2)



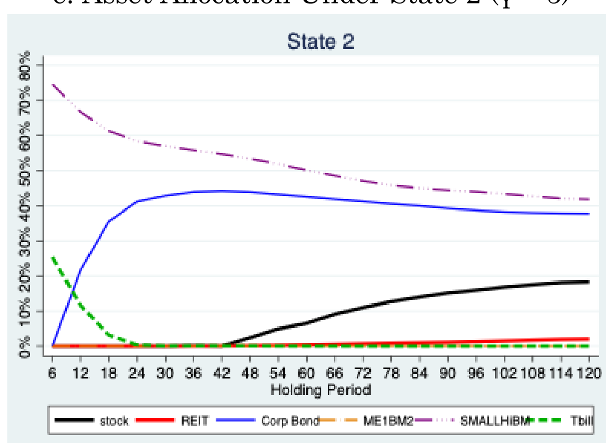
c. Asset Allocation Under State 3 (γ = 2)



d. Asset Allocation Under State 1 (γ = 5)



e. Asset Allocation Under State 2 (γ = 5)



f. Asset Allocation Under State 3 (γ = 5)

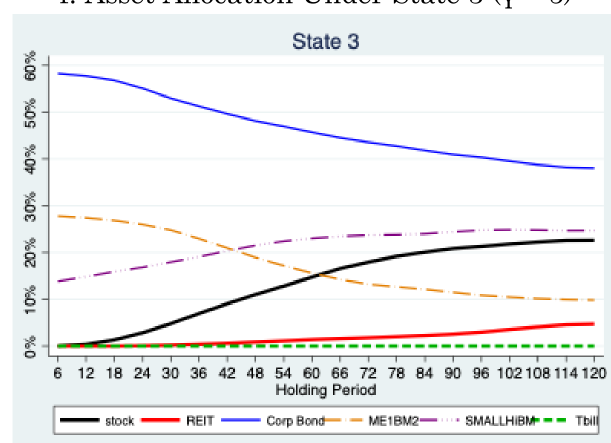
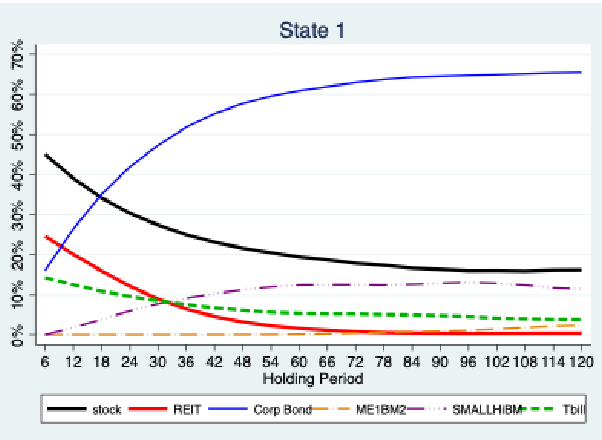
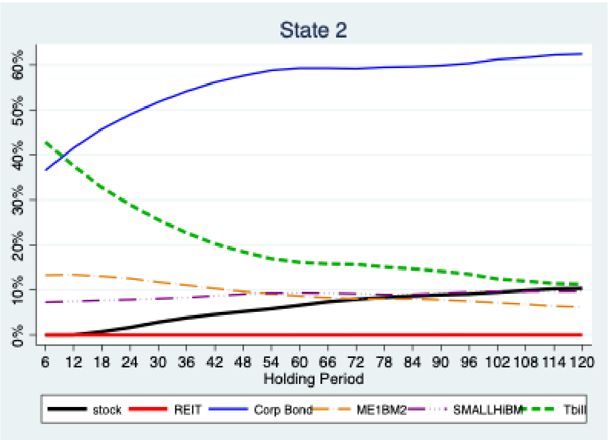


Figure 4: (Continued)

g. Asset Allocation Under State 1 (γ = 10)



h. Asset Allocation Under State 2 (γ = 10)



i. Asset Allocation Under State 3 (γ = 10)

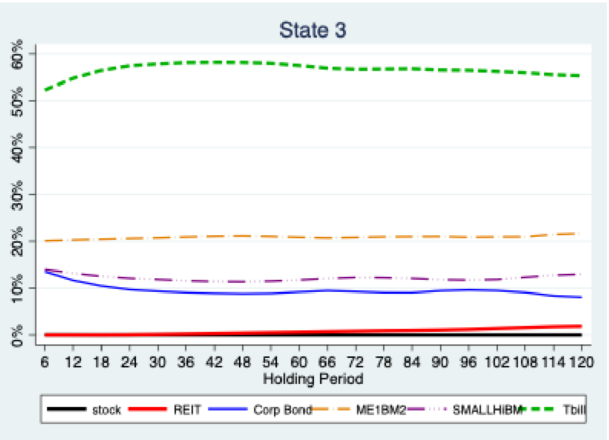
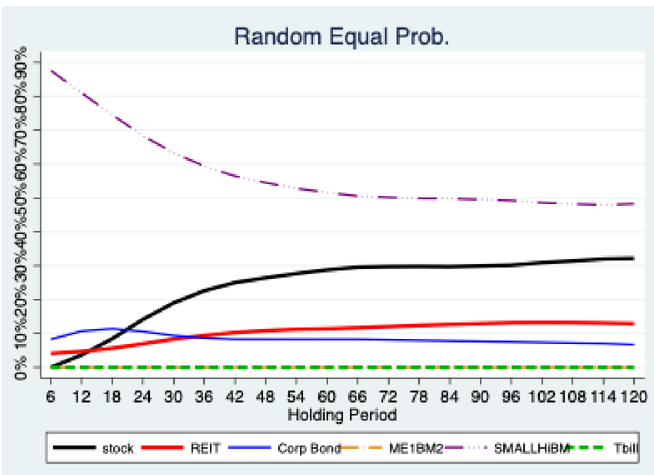


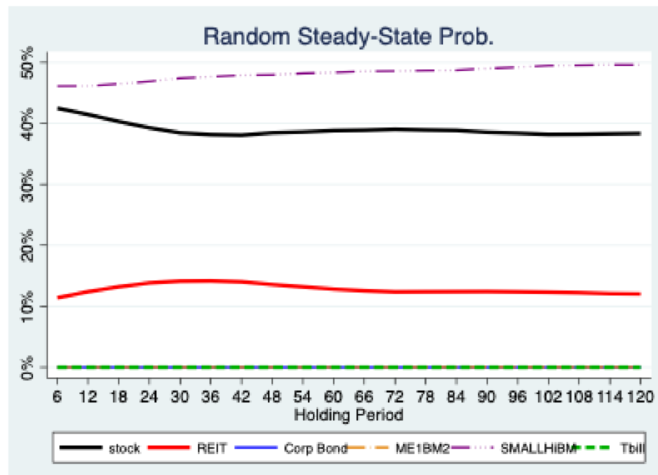
Figure 5: Asset Allocation as a Function of the Investment Horizon with Uncertain Initial States (MSIAH(3,0) Model for the Ten-Variable System)

This figure illustrates the buy-and-hold portfolio asset allocation as a function of the investment horizon across different asset classes using the three-state, ten-variable MSIAH(3,0) model, which captures the joint distribution of excess returns for stocks, REITs, corporate bonds, the ME1BM2 and SMHiBM portfolios, T-bills, conditional on the four predictive variables. Portfolio allocations are obtained from 65,000 Monte Carlo simulations. The initial state of the Markov-switching model is uncertain and is modeled using either equal regime probabilities (shown in the plots on the left) or steady-state probabilities implied by the estimated transition matrix (shown in the plots on the right). Subfigures report optimal portfolio weights for relative risk aversion coefficients of γ = 2 (top), γ = 5 (middle), and γ = 10 (bottom).

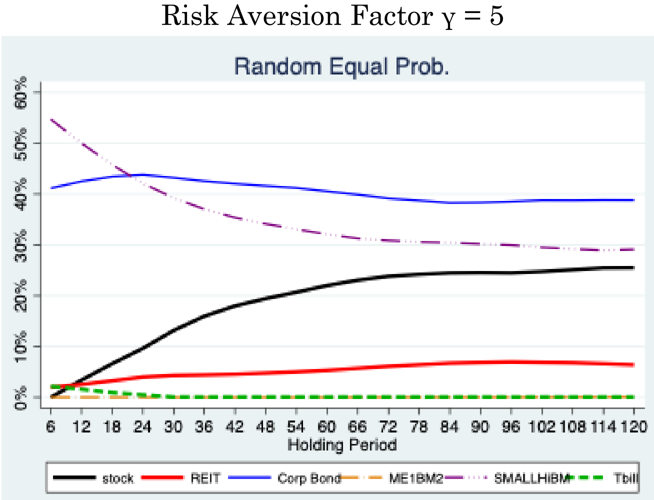
a. Asset Allocations with Random States (Scenario 1)
Risk Aversion Factor γ = 2



b. Asset Allocations with Random States (Scenario 2)
Risk Aversion Factor γ = 2



c. Asset Allocations with Random States (Scenario 1)
Risk Aversion Factor γ = 5



d. Asset Allocations with Random States (Scenario 2)
Risk Aversion Factor γ = 5

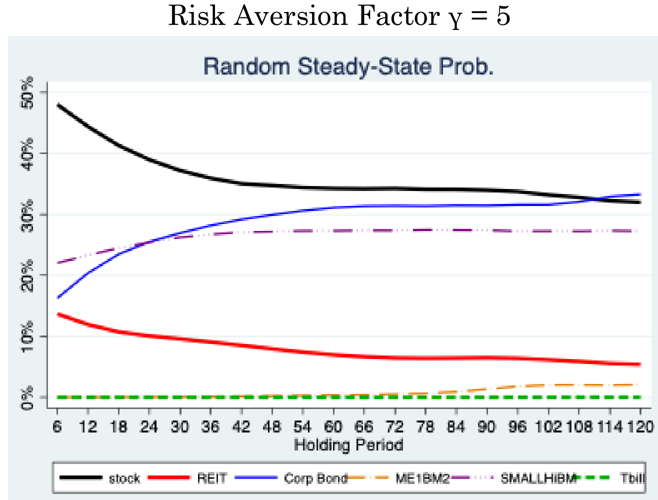
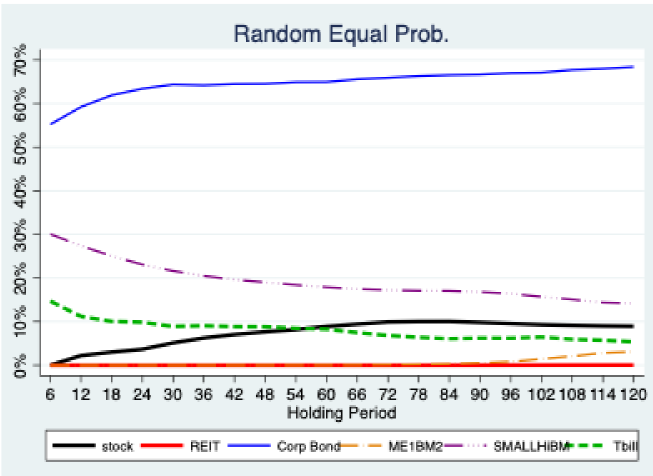


Figure 5: (Continued)

e. Asset Allocations with Random States (Scenario 1)
Risk Aversion Factor γ = 10



f. Asset Allocations with Random States (Scenario 2)
Risk Aversion Factor γ = 10

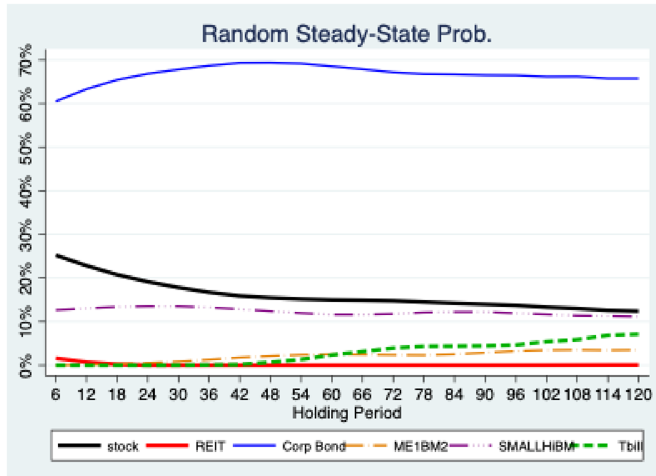
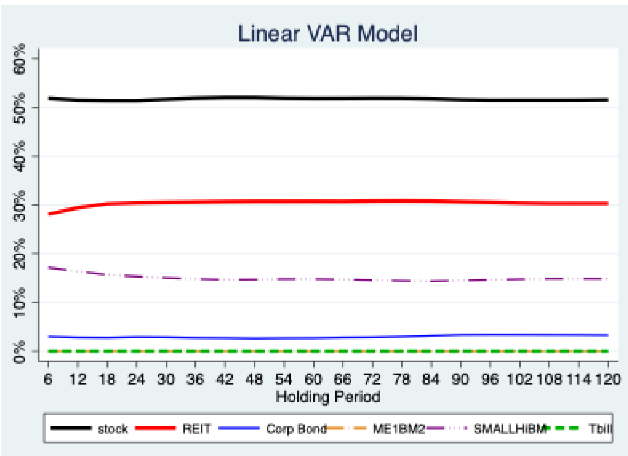


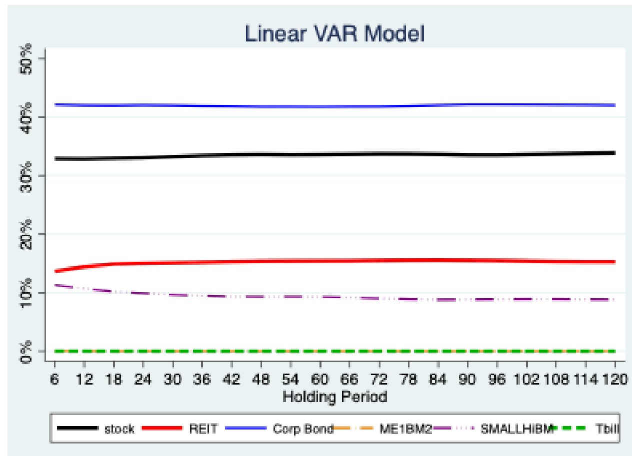
Figure 6: Asset Allocation as a Function of the Investment Horizon under a (Single-State) Ten-Variable VAR(0) Model

This figure displays the buy-and-hold portfolio asset allocations as a function of the investment horizon across the different asset classes using a ten-variable linear VAR(0) model to capture the joint distribution of returns for stocks, REITs, corporate bonds, the ME1BM2 and SMHiBM portfolios, T-bills, and four predictive variables. A total of 65,000 Monte Carlo simulations are generated to compute optimal portfolio weights. The plots illustrate the evolution of optimal portfolio allocations as a function of the investment horizon for investors with three alternative coefficients of relative risk aversion, γ = 2, 5, and 10.

a. Asset Allocation; γ = 2.



b. Asset Allocation; γ = 5.



c. Asset Allocation; γ = 10.