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Who should buy structured investment products and why?

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Abstract

Structured products in general, and investment certificates in particular, have gained increasing popularity among retail investors over the last decade, both in Europe and in the US. However, based on data on the ex-post realized gains of retail clients investing in certificates, the literature has generally concluded that the high demand of these products may be hard to rationalize within a portfolio optimization framework. In this paper, we investigate whether a rational, perfectly informed investor with standard constant relative risk aversion (CRRA) preferences who optimally allocates her wealth among risky and riskless assets can ex-ante expect to benefit from adding structured products to her portfolio. We show that the utility gains from investment certificates vary dramatically across alternative structures, investment horizons and levels of risk aversion. Therefore, a correct assessment of an investors' risk tolerance and investment horizon is crucial when advising on the relevance of structured products. We also find that the optimal demand of investment certificates as well as their benefits depend heavily on the pricing model informing the portfolio assessment and that demand is considerably higher when the joint presence of jumps in returns and volatility were to go undetected. Therefore, the high demand of structured products can be explained by the use of asset pricing models that are excessively simplistic and ignore discontinuous dynamics.

Key words: Structured products, investment certificates, retail investors, asset allocation, models with jumps.

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1. Introduction

Over the last 20 years, structured products have represented a key financial innovation for retail investors, allowing them to access far greater variety of wealth payoffs that would otherwise be precluded to them, because of high transaction costs or market restrictions (e.g., short selling constraints). In particular, among the structured products, investment certificates, which are a securitized version of complex option strategies, have gained increasing popularity since at least the turn of the millennium. According to the European Structured Product Association (EUSIPA), as of the end of September 2023, the traded volume of investment certificates stood at EUR 380 billion across nine reporting countries and there were about 427 thousand different products outstanding, an increase of approximately 50% relative to 2018.[1] To put this into perspective, the size of the structured investment product market in Europe is more than four times the size (measured in terms of assets under management, AUM) of the hedge fund industry and compares with the AUM of European private equity funds.[2] While Europe still represents the largest market for investment certificates, the US appears to be rapidly catching up. In August 2023, the total sales of structured products in the US market marked a record high at USD 13.7 billion, the highest monthly sales volume recorded in the US since 2006.[3]

Motivated by this growing market interest, in this paper, we ask whether a rational investor who maximizes a standard power utility function over terminal wealth can expect to achieve a utility gain from adding an investment certificate to her portfolio, over and above what she could obtain from optimally allocating her wealth between a riskless asset and a risky asset, represented by a standard notion of equity market portfolio. Our study is based on a classical static buy-and-hold asset allocation problem in which the investor decides the weights to be assigned to each asset class at time zero, and she selects to not rebalance her portfolio until the end of her investment horizon, which we let

[1] The nine reporting countries are Austria, Belgium, France, Germany, Italy, Netherlands, Sweden, Switzerland, and Luxemburg. EUSIPA's reports can be found at https://eusipa.org/category/market-reports/.

[2] The data have been obtained from the 2022 annual statistical report of the European Securities and Markets Authority (ESMA), which can be found at https://www.esma.europa.eu/sites/default/files/library/esma50-165-1948_asr_aif_2022.pdf. According to the report, as of the end of 2022, the size of the hedge fund industry in European Economic Area (excluding the UK) amounted to EUR 89 billion and the size of private equity funds amounted to EUR 363 billion.

[3] See https://www.structuredretailproducts.com/news/details/79337.

realistically vary between three months and three years. The maturity of the investment certificate is chosen to match the investment horizon. As noted by Hens and Rieger (2014), this framework is realistic as investors typically buy structured products with the aim of holding them until maturity. As a matter of realism, we also assume that the investor is unable to short sell or to borrow at the riskless rate. In fact, empirical evidence such as the one in Entrop et al. (2016) documents that the average holding period for structured products corresponds to the typical maturity of investment certificates (i.e., between one and two years).[4]

We report at least three novel findings. First, the optimal demand of and the expected utility gain from investment certificates varies considerably across the different products (as defined by their terminal payoffs), levels of assumed relative risk aversion, and the investment horizon. For instance, the least risk averse investor should prefer to include a Bonus Cap or a Discount certificate in her portfolio if her investor horizon were short (three to six months). Conversely, she would benefit the most from allocating her wealth to an Express certificate if her investment horizon exceeded one year. Therefore, our results seem to suggest that different structures may optimally fit different investment needs. Moreover, we show that the least risk averse investor always benefits the most from including certificates in her portfolio. Consequently, and rather sensibly, an appropriate assessment of the risk profile and investment horizon of each investor appears to be crucial when structured products are advised by wealth managers.

Second, we entertain four different asset pricing models, namely, a stochastic volatility model (SV), a stochastic volatility model with jumps in returns (SVJR), a stochastic volatility model with jumps in variance (SVJV), and a stochastic volatility contemporaneous jumps (SVCJ) model; the latter features jumps in both the return and variance dynamics. We show that the optimal demand of certificates varies dramatically across the different pricing models and hence the framework trough which investors form views on the dynamics of security prices. For instance, under the SV and the SVRJ models, an investor with a risk aversion coefficient equal to four (the least averse among the investors that we consider) and an investment horizon of one year, should optimally

[4] Because our investor is not able to dynamically rebalance her portfolio, there is no intertemporal Merton-style hedging demand arising in our analysis. However, Liu and Pan (2003) and Driessen and Maenhout (2007) have shown that empirically, any risk-adjusted performance improvements from the addition of derivatives products are mostly driven by the myopic component.

invest about 90% of her wealth in a Bonus certificate and should allocate nearly zero wealth to the riskless asset. This demand is dramatically lower when jumps in variance are considered, especially in combinations with jumps in returns. For instance, the same investor mentioned above should allocate only 25% of her wealth to a Bonus Cap certificate under a SVCJ model. Similar reductions in the optimal share to be allocated to investment certificates are obtained in the case of Discount and the Express certificates. Accordingly, also the utility gains that the investors can achieve by adding a certificate to her portfolio are notably higher under the models that do not feature jumps in variance. For instance, an investor who optimally allocates to the stock, the bond, and a Bonus Cap certificate with a 90% barrier realizes an annualized utility gain of about 4% under the SV model and 5.42% under the SVJR model. In contrast, her utility gain is only 57 basis points in a SVCJ framework. Therefore, investment certificates may look very appealing especially when (excessively) simplistic stock price and variance dynamics are assumed but much more caution is advisable when more realistic beliefs on the distribution of future returns are considered. Further analysis shows that the massive drop un the certificates' utility gain under the SVCJ model is due to the fact that under this model the estimated conditional volatility of volatility is larger than under the other models and the implied variance premium is smaller.

Finally, our findings show that in general, investors, especially those that are less risk averse, benefit from having access to products that allow them to take a short position on volatility. This is especially true when jumps in volatility are absent. On the contrary, products that provide full protection of the invested capital but limited participation to the appreciation of the underlying offer limited utility gains to all investors. While capital protection might look appealing especially to the more risk averse investors, Equity Protection certificates hardly deliver any utility gains in excess of 30 basis point on an annualized basis across all pricing models, horizons and preference parameterizations. Overall, we conclude that the main value of having access to structured products comes from the fact that they can be used by an investor who is restricted from short selling to trade volatility risk and harvest the variance premium.

1.1. Previous literature

A few papers have investigated the ex-post portfolio results realized by the investors who buy structured product and found that "(…) it is difficult to rationalize their purchases by informed rational investors." (see Henderson and Pearson, 2011).[5] For instance, Entrop, McKenzie, Wilkens, and Winkler (2016) used a rather unique dataset of investor-level trade data and portfolio holdings in structured financial products for more than 10,000 self-directed retail customers made available by a Germany-based direct bank, to investigate to what extent (if any) retail investors profit from innovative financial products. They find a negative average risk-adjusted performance for Discount and Bonus Cap certificates even when transaction costs are not considered. They attribute their findings to overpricing and retail investors' poor selection abilities. Similarly, Henderson and Pearson (2011) investigated a subset of the US equity structured products, namely, the Stock Participation Accreting Redemption Quarterly-pay Securities (SPARQS) showing that on average investors who purchase them pay a premium of about 8% over their fair market value.

While this empirical literature pointedly suggests that retail investors are on average not gaining from structured products, it remains of interest to determine whether this is due to the very nature of these financial instruments (e.g., unappealing, and sub-optimal payoffs) or to exogenous reasons such as a mis-use on the part of the investors or an overpricing on the part of issuers.[6] Therefore, in this paper, we take an *ex-ante* perspective and investigate under what circumstances different investment certificates may represent a valuable addition to the portfolio of a utility-optimizing and informed retail investor. Besides, we assume that the certificates are fairly priced. As a result, the utility gain that the investor realizes through access to investment certificates represents the maximum margin (bid-spread or placement fee) that she should be ready to pay for these products.

The papers that are most closely related to ours are Branger and Bauer (2008) and Hens

[5] In addition, Breur and Perst (2007) have claimed that the success of discount reverse convertible and reverse convertible bonds in the market could be explained only by assuming that investors were subject to some form of bounded rationality.

[6] A few papers have documented that retail structured product appear not to be fairly priced. For instance, Benet, Giannetti, and Pissaris (2006), analyzed a sample of Reverse Exchange Securities (RES) issued by ABM and found that those products were sold 300-500 basis points above their theoretical fair price. Further evidence of mispricing of retail structured products can be found in Szymanowska et al. (2009) and Stoimenov and Wilkens (2005). More recently, mispricing in the German market of discount certificates has been documented by Entrop, Fischer, McKenzie, Wilkens, and Winkler (2016) and Baule and Shkel (2021).

and Rieger (2014).[7] Branger and Bauer (2008) consider a power utility investor who follows a “buy-and-hold” strategy with an investment horizon of one year and is allowed to purchase some of the most common investment certificates in addition to investing in a stock index and in the riskless asset. They find that investment certificates generally do not generate significant utility gains. However, they only consider a relatively short investment horizon of one year and they do not include any certificate that embed early redemption features in their analysis. In our paper, we show that Bonus Cap certificates add value to an investors’ optimal portfolio exactly when we consider investment horizons that are shorter than the one considered in Branger and Bauer’s work. We also show that the early redemption feature becomes very valuable when long horizon (24 or 36 months) are considered. Moreover, Branger and Bauer (2008) only consider asset allocation problems in which the certificates are priced using a SVJR model, while we entertain additional asset pricing models, and we show the existence of remarkable differences across them.

Instead of focusing on specific types of certificates, Hens and Rieger (2014) assess whether it is possible to characterize an optimal structured product for different utility functions and proceed then to evaluate (assuming the CAPM holds) whether such an ideal certificate would be able to generate utility gains, expressed in terms of certainty equivalents. Their main finding is that, notwithstanding the presence of positive increases in utility, the benefit of solving an expected utility portfolio problem using the optimal structure only amounts to 5 to 10 basis points and is not large enough to compensate for the usual fees that such securities embed. However, their paper makes the strong

[7] More broadly, our paper fits within the literature that has explored the value of option strategies in investment portfolios. Haugh and Lo (2001) propose a "buy-and-hold" portfolio strategy that closely follows an investor's optimal dynamic investment strategy for stocks and bonds, with the criterion of maximizing expected utility. Liu and Pan (2003) tackle the dynamic asset allocation problem of a power-utility investor who uses derivatives to break down and compute optimal exposures to different risk factors. Their results indicate that disentangling such exposures through derivatives offers substantial economic benefits. Driessen and Maenhout (2007) show that, in equilibrium, power utility, CRRA investors always take short positions in OTM puts and ATM straddles, thus earning the premia for jump and volatility risks. More recent studies, like Cui, Oldenkamp, and Vellekoop (2013), use numerical methods to analyze optimal portfolios for CRRA investors, showing that incorporating equity and volatility derivatives can enhance wealth and improve pension fund performances. Faias and Santa-Clara (2017) propose a portfolio optimization method based on expected utility maximization, in which the asset menu is composed of the risk-free asset and of four S&P 500 options. Simulating asset returns, they find considerable outperformance over the S&P500 in terms of certainty equivalent and Sharpe ratios.

assumption that an investor would only invest in the optimal certificate rather than optimally allocating across the stock market, the riskless asset, and some (realistic, practically relevant) certificates. In contrast, our work follows a substantive tradition that assesses the portfolio relevance and risk-adjusted benefits of certificates when included in a portfolio that already contains the market portfolio and a riskless asset (see, e.g., Ascheberg et al., 2016). Moreover, we estimate the economic value of a number of investment certificates resorting to dynamics simulated from the state-of-art models that carry implications for the sign and size of both the diffusive equity risk and the variance risk premia.

The rest of the paper proceeds as follows. Section 2 discusses the asset pricing models used in our analysis and defines the asset allocation problem solved by the investor. It also describes the payoffs characterizing the investment certificates that are considered in the analysis. Section 3 discusses our main empirical results. Section 4 discusses the distinct role of jumps and stochastic volatility for generating utility gains when the certificates are included in the asset menu. Section 5 concludes.

2. Research Design

2.1. Asset pricing models

Our baseline, general framework is a security market model with contemporaneous jump arrivals in both returns and variance, also known as Stochastic Volatility Contemporaneous Jumps (SVCJ) model (see, e.g., Duffie, Pan, and Singleton, 2000; Broadie, Chernov, and Johannes, 2007; Dufays, Jacobs, Liu, and Rombouts, 2023), in which the "C" stands for *combined*. The dynamics of the equity index (S_t) and variance of its log-changes (V_t) under the physical probability, $\mathbb{P}$, are

$$dS_t = (r_t - \delta_t + \gamma_t - \lambda\bar{\mu}_s)S_t dt + \sqrt{V_t}S_t dZ_t + \left(e^{J_t^S} - 1\right)S_t dN_t \tag{1}$$

$$dV_t = k(\omega - V_t)dt + \sigma\sqrt{V_t}(\rho dZ_t + \sqrt{1-\rho^2}dW_t) + J_t^v dN_t, \tag{2}$$

where r_t is the risk-free rate, δ_t is the continuous dividend yield, γ_t is the total equity premium (i.e., inclusive of the diffusive and jump components), k is the speed of volatility mean reversion, ω is the unconditional, long-run variance and σ is the volatility of volatility. dZ_t and dW_t are standard Brownian motions and N_t is a Poisson process with constant intensity λ, which governs the simultaneous arrival of jumps in price and volatility. All three random shocks Z, W, and N are assumed to be independent and the constant coefficient ρ introduces correlation between the diffusive price and volatility shocks, by allowing dZ_t to enter the dynamics of the variance process. J_t^s and J_t^v are the jump size parameters related to stock price changes and its variance, respectively. We assume $J_t^v \backsim Exp(\mu_v)$ and $J_t^s \mid J_t^v \sim N(\mu_s + \rho_J J_t^v, \sigma_s^2)$, where the size of jumps in prices depends on the size of jumps in volatility through the parameter ρ_J.[8] The term $-\lambda\bar{\mu}_s S_t$ in the drift, in which $\bar{\mu}_s = \left[1 - \rho_J \mu_v\right]^{-1} e^{(\mu_s + \frac{\sigma_s^2}{2})} - 1$, compensates for the presence of a jump component.

The securities market described by equations (1) and (2) is incomplete and therefore multiple equivalent martingale measures exist. We follow the literature and parametrize the change of measure under a standard risk premium assumption (see, e.g., Eraker, 2004 and Dufays et al., 2023) that includes $\lambda\bar{\mu}_s^Q$ in the risk-neutral drift:

$$dS_t = \left(r_t - \delta_t - \lambda\bar{\mu}_s^Q\right)S_t dt + \sqrt{V_t} S_t dZ_t^Q + \left(e^{J_t^{sQ}} - 1\right) S_t dN_t^Q \tag{3}$$

$$dV_t = k^Q(\omega^Q - V_t)dt + \sigma\sqrt{V_t}(\rho dZ_t^Q + \sqrt{1-\rho^2} dW_t^Q) + J_t^{vQ} dN_t^Q, \tag{4}$$

where $\omega^Q = \frac{k\omega}{k^Q}$ and $k^Q = k - \eta_v$ with η_v representing a risk premium parameter associated with diffusive shocks in the variance process.[9] The jump intensity λ, the volatility of volatility σ, and the correlation parameter ρ are assumed to be constant across measures. The total equity premium, γ_t is given by $\gamma_t = \eta_s V_t + \lambda\left(\bar{\mu}_s - \bar{\mu}_s^Q\right)$, where $\eta_s V_t$ is the Brownian contribution to the equity premium and $\lambda\left(\bar{\mu}_s - \bar{\mu}_s^Q\right)$ is the jump contribution, with $\bar{\mu}_s^Q = \frac{e^{(\mu_s^Q + \frac{\sigma_s^2}{2})}}{1 - \rho_J \mu_v^Q} - 1$. The mean price jump risk premium parameter is $\eta_{Js} = \mu_s - \mu_s^Q$ and the volatility jump risk premium parameter is $\eta_{Jv} = \mu_v - \mu_v^Q$. When

[8] Similarly to Eraker (2004), Broadie et al. (2007), and Dufays et al. (2023) (but unlike Liu and Pan, 2003), in our model jumps have stochastic rather than deterministic amplitudes. Also, because it assumed that jump sizes follow a continuous rather than a discrete distribution (unlike, for instance, Branger et al., 2008), an infinite number of derivatives would be needed to complete the market.

[9] The difference between the volatility drift under the physical and the risk neutral measure, $-\eta_v V_t$, is the diffusive variance premium.

average jumps are more negative under $\mathbb{Q}$ than under $\mathbb{P}$, $\lambda(\bar{\mu}_s - \bar{\mu}_s^Q) > 0$, and there is a positive compensation for bearing jump risk.

The SVCJ model nests the stochastic volatility (SV), the stochastic volatility with jumps in returns (SVJR), and the stochastic volatility with jumps in variance (SVJV) models as special cases. More specifically, the SV model (Heston, 1993) is obtained by setting $\lambda = 0$, such that jumps (either in returns or variance) are no longer possible. If we shut down the jumps in variance, we obtain the SVJR model, in which $J_t^s \sim N(\mu_s, \sigma_s^2)$, $\bar{\mu}_s = e^{(\mu_s + \frac{\sigma_s^2}{2})} - 1$, and the jump risk premium turns out to be entirely attributable to the mean price jump risk premium parameter, η_{Js}. Finally, if we turn off the occurrence of jumps in log-prices, we obtain the SVJV model, where $J_t^v \sim Exp(\mu_v)$ and the jump risk premium is again entirely attributable to the volatility jump size premium, η_{Jv}.

In our baseline analysis, we rely on the parameters estimated jointly from stock returns and option data by Dufays et al. (2023).[10] These parameters are summarized in Table 1.[11] The parameter η_v is estimated to be positive, which implies that the risk neutral variance exceeds the physical one. Across the different models, λ is estimated to be between 0.51 and 0.64, implying that jumps happen on average every two years. Under the SVCJ model, the average jump size is estimated to be small and positive under the physical measure, but negative under the risk neutral measure, implying a positive compensation for mean price jump risk. The average variance jump size under the physical measure is 7.3% and the correlation between variance and price jumps is negative and large, as one would expect, also a result of a classical "leverage effect".

[10] Dufays et al. (2023) apply Markov Chain Monte Carlo (MCMC) to parameter search and filter the latent state variables using a particle filter. They perform joint estimation of the parameters of the option pricing models using a large panel of daily S&P 500 returns and option prices, over the sample period January 1, 1996 - December 31, 2019.

[11] Dufays et al. (2023) do not report numerical values for the risk-free and the dividend yields. Therefore, we follow the procedure described in their paper and we calculate the risk-free rates as the sample average of the appropriate daily zero-coupon yields over the sample period January 1, 1996 - December 31, 2019, and the dividend yield from the average of the daily S&P 500 dividends over the same period. The daily zero coupon and the daily S&P500 dividend yields are obtained from OptionMetrics and retrieved through Wharton Research Data Services (WRDS). The average risk-free rates turn out to equal 2.14% 2.39%, 2.77%, 3.01%, and 3.24% for three, six, 12, 24, and 36 months, respectively; the average dividend yield is 1.79%.

To simulate the paths of the equity index and its variance under the objective and the risk neutral probabilities, we use a Euler scheme to discretize the continuous time dynamics in (1)-(4). Applying Ito's Lemma and discretizing (1)-(2) leads to

$$R_{t+1} = \left(r - \delta - \frac{V_t}{2} + \gamma_t - \lambda\bar{\mu}_s\right)\Delta t + \sqrt{V_t \Delta t} z_{t+1} + J^s_{t+1} B_{t+1} \tag{5}$$

$$V_{t+1} = V_t + k(\theta - V_t)\Delta t + \sigma\sqrt{V_t \Delta t}(\rho z_{t+1} + \sqrt{1-\rho^2} w_{t+1}) + J^v_{t+1} B_{t+1}, \tag{6}$$

where R_{t+1} is the continuously compounded return process, such that $S_{t+1} = S_t e^{R_{t+1}}$, Δt is the length of the discretised time interval, and z_{t+1} and w_{t+1} are standard normal shocks. For simplicity, we fix the risk-free rate and the dividend yield to be constants, also in the light of the empirical literature on the small derivative pricing effects of stochastic interest rates and dividend yields (see, e.g. Bakshi et al., 2010). The discrete jump frequency, B_{t+1}, follows a Bernoulli distribution, meaning that, in each sub-period, there is either no jump or one jump. The corresponding discretized risk neutral dynamics are identical, but with the risk neutral parameters appearing in the equations matching (5) and (6). For convenience, in our simulations, we have rescaled the initial value of the index to be equal to 100, while the initial variance is set equal to its unconditional mean. The use of the Euler discretisation scheme leads to some inconsistencies, notably the fact that the stock prices or the variances might occasionally turn out to be lower than zero. Therefore, following Broadie and Kaya (2006), we force negative values to be equal to zero, although this happens rarely.[12],[13]

We use 250,000 simulations and we set the number of time steps equal to 500, following Duffie and Glynn (1995), who argue that in a first order discretization it is optimal to set the number of time steps equal to the square root of the number of simulations.[14] The

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[12] To provide a rough idea of the rate of occurrence of this problem, under the physical probability, considering the SVCJ model and the longest time horizon (T = 36 months), we have recorded 2.3% instances in which the volatility became negative and needed to be restricted to zero. The proportion declines to 1.8% when we simulate from the risk neutral probability.

[13] Heston (1993) tries to ensure the non-negativity of the simulated dynamics by means of Feller's condition, which is $2k\theta > \sigma^2$. However, under our calibration, Feller's condition is never satisfied. Nonetheless, it should be noted that compliance with Feller's condition does not ensure positive simulated values, as a small number of time steps may still lead to inconsistencies.

[14] This implies that, under the assumption that a year is composed by 252 trading days lasting 6.5 hours, Δt ranges approximately from 50 minutes to 1.5 days for maturities ranging from three to 36 months. This is a rather fine mesh of time discretization.

initial price of derivatives and structured products is obtained under the risk neutral probabilities by the simple discounted martingale expectation formula

$$p = e^{-rT} E^{Q}[f(S_T)] = e^{-rT} \tfrac{1}{N} \left(\textstyle\sum_{i=1}^{N} f(\hat{S}_{T,i}^{M}) \right), \tag{7}$$

where N is the number of simulations, f is the payoff function, and $\hat{S}_{T,i}^{M}$ denotes the simulated value of S_T over a sample path i, when M time steps are adopted.

2.2. *The investor's problem*

We consider an investor characterized by constant relative risk aversion (CRRA) coefficient γ and by an investment horizon T, who maximizes her expected utility over terminal wealth

$$\max \left[E \frac{W_T^{1-\gamma}}{1-\gamma} \right]. \tag{8}$$

The investor faces and selects optimal portfolios from several, alternative asset menus. As a benchmark for our analysis of the economic value of the investment certificate, we consider a stylized asset menu where the investor can allocate her wealth to the equity index and a risk-free zero-coupon bond only.[15] In addition, we consider asset menus in which the investor allocates her wealth to the equity index, a risk-free zero-coupon bond and one or more investment certificates. The comparison between the ex-ante, risk-adjusted returns of the optimal portfolios obtained from the two alternative asset menus—with and without structured products—represents the main source of our results. In the rest of this paper, to save space, we shall refer to the equity index and the zero-coupon bond simply as the "stock" and the "bond".

To keep our framework as realistic as possible, we consider a myopic investor who is not able to rebalance her portfolio continuously; instead, she follows a buy-and-hold strategy:

[15] We also entertain an alternative asset menu where the investor could invest in the equity index, a risk-free zero-coupon bond, one OTM call and one OTM put, and one where the investor could invest in the equity index, a risk-free zero-coupon bond, one ATM call and one ATM put. The goal is to assess whether the presence of plain vanilla puts and calls in the asset menu may render the structured products less valuable in a portfolio perspective. The results are largely similar to the ones reported in the paper and remain available upon request from the Authors. The fact that the results do not vary much when we use a richer benchmark asset menu is explained by the fact that short selling restrictions are imposed throughout the paper.

she decides her optimal allocation at the beginning of the period, and she holds it until the end of her investment horizon T.[16] When she is allowed to invest in the certificates, the maturity of the latter corresponds to the investor's investment horizon. We consider five different time horizons, namely three, six, 12, 24, and 36 months. As a matter of realism, the investor is not allowed to short-sell.[17]

To solve the problem in (8), we rely on the method outlined by Faias and Santa-Clara (2017). Namely, we simulate the price of the equity index at time T under the objective probability measure using the dynamics in (1) and (2). The simulated price of the equity index, $\hat{S}_T$, is used to compute the payoff of the investment certificate at maturity. The return on the equity investment is then simply defined as $r_E \equiv \hat{S}_T/S_0 - 1$; the return on the investment certificate is $r_c \equiv \hat{C}_T/p - 1$, where $\hat{C}_T$ is the payoff obtained by plugging $\hat{S}_T$ in the payoff function and p is the price of the investment product computed in (7). The investor's terminal wealth is given by

$$W_T = \omega_{rf} rf + \omega_C r_c + \omega_E r_E, \tag{9}$$

where rf is assumed to be constant for simplicity, the initial wealth W_0 is normalized to be equal to one and hence dropped from the equation, and ω_{rf}, ω_C, and ω_E are the weights assigned to the risk-free rate, the investment certificate, and the equity, respectively. The process is repeated over250,000 independent simulations and the weights are chosen to maximize the average utility obtained across simulations, which (under some of law of large numbers extended to weakly dependent stationary process) represents a way to accurately estimate the expected utility.

For each of our allocation frameworks, we compute the expected return and the certainty equivalent return (CER). The CER is computed as

$$CER_T = \left[(1-\gamma)E\left[U\left(1+r^*_{p,T}\right)\right]\right]^{\frac{1}{1-\gamma}} - 1, \tag{10}$$

[16] The fact that retail investors buy structured products with the aim of holding them until maturity is supported by the empirical evidence. For instance, Entrop et al. (2016) find that the average holding period for the structured products in their dataset was about 20 months, which is consistent with the typical maturity of investment certificates, which in their data was between one and two years.

[17] By their nature, investment certificates cannot be sold short. In addition, retail investors are unlikely to be able to short sell stocks or to borrow at the risk-free rate.

where $r^*_{p,T}$ is the return of the optimal portfolio and represent the riskless rate of return that makes the investor indifferent between her optimized portfolio and a purely risk-free investment in a T-maturity bond with yield CER_T. We also compute the CER gain as the difference between the CER when structured products are included in the asset menu and the CER under the benchmark model, where the investor optimally allocates between the stock and the riskless bond only.

2.3. The asset menu including investment certificates

Besides to a riskless zero-coupon bond and the stock index, the investor can allocate her wealth to four different investment certificates, as defined by their payoff structure. They all share the stock index as their underlying but, for ease of interpretation, these are allowed to enter the asset menu one at a time (with only exception of Table 7, where the asset menu consists of all the certificates, the stock, and the bond). Notably, the commercial names of different types of investment certificates vary a lot across the countries in which they are typically sold. Therefore, also to build a common knowledge background, in what follows we describe each of the four products under analysis and the conventional labels that we apply to them. For the sake of simplicity, the notional amount of all the certificates is set to \$100 and their strike is equal to the initial value of the underlying, which also fits common practice in the primary markets for these products.[18]

A *Bonus Cap* certificate (sometimes also known as a barrier reverse convertible) offers a fixed coupon (the "bonus") typically much higher than the risk-free rate. At the maturity of the certificate, the bonus is paid unless the value of the underlying is below a certain pre-specified level (the "barrier").[19] We also assume that the notional amount plus the bonus represents the maximum that the investor can ever be paid (the "cap"). Although the "bonus" and the "cap" do not have to coincide, this is very common in practice. If at expiry, the value of the underlying is below the barrier, the investor receives the notional

[18] As described in detail below, the strike of a certificate is used to compute the performance of the underlying. Although the strike can in principle be set arbitrarily, it is common to set it to 100% of the value of the underlying on the strike date, which can be reasonably assumed to be the same as the issuance date of the contract. An overview of the different products across a range of countries can be found at https://www.structuredretailproducts.com/.

[19] Some versions of this product exist in which the barrier is observed continuously and if a barrier breaching event occurs at any point between the issuance date of the certificate and its maturity, the investor is no longer entitled to receive the bonus. However, we only entertain a plain vanilla version of the bonus cap in which the barrier is uniquely observed at maturity.

amount of her investment multiplied by the gross performance of the underlying over the life of the product (that is, the gross performance with respect to the strike). In other words, the investor loses as much as she would have lost if she had invested in an equivalent amount of underlying earlier, when the Bonus Cap had been issued. In summary, the payoff at maturity is as follows:

$$X_T = \begin{cases} N + BA & \dfrac{S_T}{S_0} \geq B \\ \dfrac{S_T}{S_0} \times N & \dfrac{S_T}{S_0} < B \end{cases}. \tag{11}$$

In the formula, S_T is the stock price at maturity, S_0 is the initial stock price, N is the notional amount, BA is the bonus amount, and B is the barrier expressed as a percentage of the strike, which has been fixed to be equal to the value of the underlying at issuance. The payoff is summarized in Figure 1. A Bonus Cap can also be thought of (hence, replicated as) the combination of a long position in a call option with strike zero (i.e., the underlying) and a short position in a down-and-out barrier call option with strike equal to $BA/N\%$ of S_0. In our exercises we set the bonus amount to be equal to \$10 multiplied by the maturity expressed in years, but we experiment with different levels of the barrier.

A *Discount certificate* pays back the notional amount only if at maturity the price of the underlying is above the value set at the issuance date of the certificate, typically the strike. Otherwise, the investor receives the notional amount of her investment multiplied by the gross performance of the underlying with respect to the strike of the certificate, assumed to be equal to S_0. However, to compensate the resulting risk, the investor is offered an additional compensation vs. the underlying: this derives from the fact that at issuance, the Discount certificate is sold below the notional value (hence the name of this product). A Discount certificate resembles a Bonus Cap, but it lacks downside protection, compensated by the fact that the investor earns a discount in the purchase price of the product instead of receiving a coupon (the bonus) at maturity. The payoff is:

$$X_T = \begin{cases} N & S_T \geq S_0 \\ \dfrac{S_T}{S_0} \times N & S_T < S_0 \end{cases} \tag{12}$$

where S_T is the stock price at the maturity of the certificate, S_0 is the stock price at issuance, and N is the notional value. This structure can be seen (hence, replicated) as a combination between a long position in a call option with zero strike and a short position in an ATM call option. This strategy is sometimes also simply known as a covered long call. The payoff is summarized in Figure 2.

An *Express certificate* (sometimes simply known as an autocallable certificate) incorporates the possibility to be reimbursed early if, at an "early redemption date", the value of the underlying climbs above a certain threshold (the "trigger level"). As such, it carries a distinctive exotic nature. If the certificate redeems early, it pays back the notional amount plus a coupon. Otherwise, the structure cumulates coupons during its life. At maturity, three possible scenarios arise. If the value of the underlying is above the final trigger level, the certificate pays back the notional amount plus all the coupons that it has cumulated since emission. If the value of the underlying is below the final trigger level but above the barrier, the certificate pays back the notional amount but no coupons. Finally, if the value of the underlying is below the barrier, the payoff is equal to the notional multiplied by the gross performance of the underlying. In summary, the payoff at any observation date $\tau < n$, is given by:

$$X_\tau = \begin{cases} 0 & S_\tau < TL \\ N + P \times \tau & S_\tau \geq TL \quad with\ \tau = 1, 2, \dots, n-1. \end{cases} \tag{13}$$

Here N is the notional amount, S_τ is the value of the stock at the observation date, TL is the trigger level and P is the coupon. In case early redemption is not triggered, at maturity the payoff is:

$$X_T = \begin{cases} \frac{S_T}{S_0} \times N & S_T \leq B \\ N & B < S_T \leq TL \\ N + P \times n & S_T > TL \end{cases} \tag{14}$$

where n is the total number of early redemption dates and B is the barrier. Express certificates can be approximately decomposed as the combination of the following positions: a long zero strike call; a long down-and-out put, with strike price equal to the trigger level and barrier level B; a short call, with strike price equal to the trigger level; a long position on a strip of digital calls with knock-out feature and maturities equal to the early redemption dates.[20] The payoff is summarized in Figure 3.

[20] Clearly, such a portfolio decomposition originates an imperfect replication, as we would need to add a provision by which, should early redemption occur at the nth date, all options, but the first n digital calls, would expire worthless.

In our exercise, we consider only one early redemption date, corresponding to half of the life of the structure. Moreover, we set the trigger level to be the price of the underlying at issuance of the certificate, which is a common practice in the industry. The maximum payoff is set to be equal to the notional amount plus $10 times the maturity in years, while we extensively experiment with different barrier levels. In the asset allocation exercise, when the product is triggered early, the proceedings are then re-invested at the risk-free rate until the original maturity date.

Finally, an *Equity Protection certificate* (also known as equity-linked note) offers full protection from downside risk and some participation to the upside potential of the underlying. Specifically, if the underlying appreciates over the life of the certificate, the investor is paid back the notional and participates to the positive performance of the underlying. In general, the upside participation is capped or set to be less than 100%. Nonetheless, when the underlying depreciates, the investor recovers the invested capital in full. Therefore, the payoff that we consider is

$$X_T = Max\left(N, Min\left(\frac{S_T}{S_0}, Cap\right) \times N\right), \tag{15}$$

where *Cap x N* represents the maximum amount that the certificate can pay back. This certificate can be seen as the combination of the purchase of a zero coupon bond that pays back the notional at maturity, a long position in an ATM call and a short position in a call with strike equal to the cap level. In our empirical exercises, we set the cap level equal to 100% plus 10% multiplied by the maturity in years. The payoff is displayed in Figure 4.

3. Portfolio Results

3.1. Benchmark Asset Allocation

Table 2 reports the optimal weights and the CER for the benchmark asset menu, when the investor allocates her wealth between the stock and the bond. Our results span three different levels of risk aversion ($\gamma = 4, 8, 12$) and five investment horizons (three, six, 12, 24, and 36 months). Four distinct asset pricing models (SV, SVJR, SVJV and SVCJ) are considered.

The results under SV, SVJR and SVJV are roughly similar and largely unsurprising: the least risk averse investor ($\gamma = 4$) invests between 50% and 56% of her wealth in the stock when the shortest investment horizon is considered, and she earns an annualized CER between 3.65% and 4.35%. The weight assigned to the risky asset progressively increases as the investment horizon lengthens. For instance, when the investment horizon is three years, she invests roughly 70% of her wealth in the stock index and earns an annualized CER of approximately 6.50%. Conversely, the most risk averse investor ($\gamma = 12$) invests only between 17% and 19% of her wealth in the risky asset when the shortest investment horizon is considered and about 25% when the longest horizon is considered. Her annualized CER ranges between 2.65% (for the shortest horizon) and 4.30% (for the longest horizon). The weights and associated CERs are largely similar across the SV, SVJR and SVJV models, although for short (long) horizons the share of stocks and the CER are slightly decreasing (increasing) as we add jumps in log-prices only and then in volatility only.

Interestingly, when jumps in both returns and the volatility are simultaneously considered, the results change quite dramatically. Under the SVCJ model, the least risk averse investor with an investment horizon of 3 months only allocates 35% of her wealth to the risky asset, roughly 20% less than under the other asset pricing models. In addition, her annualized CER falls by 1.25-1.50% compared to what she could achieve under the remaining models that exclude either type of jump dynamics (or both). Similar results are also visible for the remaining time horizons and the other levels of risk aversion, despite the differences are less stark for more risk averse investors. Because in our benchmark allocation the investor cannot disentangle between diffusive and jump risks, the investor naturally decreases her equity exposure as jumps are added, especially when their effects are combined. Therefore, in the next section we study whether expanding the asset menu to include investment certificates may deliver economic benefits, in terms of realized CER over and above what can be achieved without access to these products.

3.2. Asset allocation with structured products

We now turn to analysing the optimal portfolio shares when the investor has access to one or more of the investment certificates described above in addition to the bond and the stock market. Table 3 reports the weights and the expected utility gains (as measured by the difference in CER with respect to the benchmark asset allocation presented in Section 3.1) for the optimal portfolio when a Bonus Cap is introduced in the asset menu across the different pricing models, risk aversion levels and investment horizons/maturities considered in Section 3.1. Each of the three panels considers a different level of the Bonus Cap barrier (70%, 80% and 90%, respectively).

Table 3 shows that the amount of wealth allocated to the bonus certificate varies significantly across the asset pricing models for the same level of risk aversion and investment horizon. In particular, under the SV and the SVJR models, the investor prefers to buy almost only the certificate instead of directly investing in the stock index. This is especially visible for low levels of γ and shorter maturities. As further elaborated in Section 4, this finding is likely to follow from the fact that a Bonus Cap allows the investor to harvest the variance risk premium, which is estimated to be around 3.6–2.8% on average under these models (see the discussion in Dufays et al., 2023), while at the same time retaining a positive (albeit partial) exposure to the equity risk premium.

Under SV and SVJR, the availability of a Bonus Cap certificate improves the portfolio performance yielding utility gains that range between 12.53% and 1.16% depending on the barrier level, the investment horizon and the level of risk aversion. The maximum utility gain is obtained by an investor with $\gamma = 4$, under an investment horizon of three months. For barriers of 80% and higher, this short-term, aggressive investor invests all of her wealth in the Bonus Cap certificate under the SVJR model and obtains a utility gain between 8.7 and 12.7 percent, similarly to the findings in Hsuku (2007) or options under SV (who, however, takes also into account interim consumption flows). In the rest of the parametric scenarios, the weight assigned to the certificate is less extreme but remains very high especially for the shorter horizon investors. Overall, the utility gains from adding the certificate to the asset menu increase with the level of the barrier and decrease with the investment horizon and the level of risk aversion. The fact that the utility increases with the barrier level is interesting and denotes that downside protection may be perceived as excessively expensive by power utility investors. In general, the results under SV and SVJR are quite similar.

Interestingly, the results turn much less favourable to the bonus certificate when we use asset pricing models that account for the evidence of jumps in variance, especially when there are combined jumps in both the dynamics of returns and variance. To make a comparison, under the SVJR model, an investor with $\gamma = 4$ and an investment horizon of three years will invest 84% of her wealth in the Bonus Cap certificate with an 80% barrier and realize a utility gain of 2.14%. Under the SVCJ model, the same investor should only invest 17.33% of her wealth in the Bonus Cap obtaining a meagre utility gain of 9 basis point on an annualized basis. This is likely to be the result of the combination of a negative compensation for bearing variance jump risk and a smaller variance risk premium of around 2% on average estimated under this model. We shall return to this discussion in Section 4.

The results for the Discount certificate reported in Table 4 are very similar to those outlined for the Bonus Cap (especially when the barrier is set at 90% in the latter analysis). This is unsurprising as the Discount certificate is very similar to a Bonus Cap certificate with the difference that there is no barrier and that the bonus is replaced with a discount applied to the issuance price of the certificate. A striking result is that in almost all the alternative configurations of the asset allocation exercise (with the sole exclusion of long horizons under a SVCJ model), the investor completely substitutes the stock with the certificate and simply allocates her wealth between the Discount and the riskless asset. Similarly to the case of the Bonus Cap, there are a few extreme cases in which the least risk averse investor ought to invest all her wealth in the Discount certificate, obtaining high annualized utility gains (between 12% and 1%, depending on and declining with the investment horizon as well as γ). Most of the findings that we have described for the Bonus Cap certificate also hold in the case of Table 4, including the fact that when a SVJC model is considered, the allocation to the certificate becomes less extreme and the utility gains are much more modest, ranging from 2.79% to a few basis points only when the investment horizon is long. Also in this case, the combined presence of stochastic jumps in both returns and variance has the power to considerable tame the demand for Discount certificates, also as a result of their reduced power to generate utility gains to a rational investor.

Table 5 reports the results for Express certificates under different levels of the barrier. The Express certificate offers limited downside protection (similar to a Bonus Cap) but also exposes to the risk of early redemption should the value of the underlying exceed a certain threshold at an early redemption date. In our exercise, we select the early redemption date to always correspond to the half-life of the certificate (for instance, if the maturity is three years, the early redemption date will be after one year and a half) and we set the threshold equal to the price of the underlying at issuance. Interestingly, the investor almost never demands the Express structure when the investment horizon is shorter than one year. Conversely, for investment horizons of one year or longer, the least risk averse investor allocates between 95% and 70% of her wealth to this certificate when the models without volatility jumps are considered; such weights decline to between 35% and 25% under the models that include stochastic jumps in volatility, i.e., under SVJV and SVCJ. In the case of a long term investor with $\gamma = 4$, the utility gain ranges from 18.44% (under SVJR) to 12.34% (under SVCJ), while for a one year investor with the same risk aversion the utility gain would only be around 60 basis points under the SVCJ model. It is evident that the value of the early redemption feature to the investor increases with the horizon. In fact, the longer is the duration of the certificate, the higher is the probability that the certificate redeems earlier then its maturity and then the price falls, especially in the presence of jumps, which are estimated by Dufays et al. (2023) to occur on average every two years. Such fall in no arbitrage prices of the Express structures makes them more attractive to a power utility investor as it shifts to the right the distribution of returns under both the risk-neutral and the physical measure.[21]

Table 6 shows the results when an Equity Protection certificate is part of the asset menu. As this certificate offers full downside protection and limited participation to the positive performance of the underlying, the investor treats this structure more as a substitute of the riskless bond than of the stock. For instance, under the SVCJ model, an investor with $\gamma = 4$ and an investment horizon of one year allocates 72% of her wealth to the certificate and 28% to the stock and ignores the risk-free asset. The same investor should instead allocate 58% of her wealth to the riskless asset were the certificate not available. For investment horizons of one year or less and again considering the SVCJ model, the share of wealth allocated to the Equity protection certificate is maximum for the medium risk aversion investor. This means that for $\gamma = 12$ the investor ought to buy protection from left-tail risks by investing both in the Equity Protection and also in the risk-free asset: for instance, under a one-year horizon, the weights are 6%, 45% and 49% on stocks, the riskless bond and the certificate, respectively. However, for horizons longer than one year, the weight of the structured product simply declines with risk aversion. Despite the large optimal demand for this product, the utility gains are very modest across all the

[21] Nonetheless, also in this case the occurrence of jumps in the variance has a moderating effect on the demand for the certificates (while returning some importance to stock investments) and causes the increase in CER due to the availability of Express certificates to decline, even though it remains substantial in the case of the 36-month horizon.

investment horizons, amounting to just a few basis points per year. For instance, under the SVCJ model, the utility gain ranges from 27 basis points, for an investor with $\gamma = 8$ and an investment horizon of three months, to 6 basis points in the case of an investor with $\gamma = 12$ and an investment horizon of two years. The substantial similarity between the Equity Protection and the plain vanilla riskless bond and the inability of this certificate to give access to the variance risk premium makes it hardly relevant in risk-adjusted terms.

Finally, in Table 7, we entrain an asset allocation exercise in which the investor can choose among all the structured products discussed above as well as the stock market and the riskless bond. For brevity, we only focus on a one-year investment horizon. The investor essentially treats the Bonus and Discount structures as substitutes and only invests in the Discount certificate when both are present. For instance, under the SVCJ model, an investor with $\gamma = 4$ allocates 53% of her wealth to the Discount certificate, 42% to the Equity protection, and 4% to the Express Certificate. Interestingly, the investor almost completely substitutes the stock market and the riskless bond with certificates. The allocation for the most risk averse investor ($\gamma = 12$) is somewhat less extreme, with 35.5% of wealth allocated to the Equity Protection, 13.2% to the Discount certificate, and 5.9% to the Express, with some marginal allocation also to Bonus Cap. Also in this case, the utility gain from including certificates in the asset allocation varies across the different asset pricing models and it is much smaller when jumps in volatility (by themselves or combined with log-price jumps) are present. For instance, under the simple SV model, the least risk averse investor gains 4.16% per annum from including certificates in her portfolio. Conversely, under the SVCJ model, she only gains 85 basis points. Therefore, the finding that jumps, and in particular in volatility, hurt the welfare investor of structured products to a power utility investor stands and will be further examined below.

To conclude, we have also experimented with asset menus that include an ATM straddle or an OTM put option (similar to Driessen and Maenhout, 2007) instead of the certificates, but found that the demand for plain vanilla calls and puts is nearly zero. In fact, the investor would like to sell options rather than buying them, which is not allowed in our framework (as it would not be realistic to assume that a retail investor can write options without any limitations). This is consistent, for instance, with the findings in Driessen and Maenhout (2007), who show that the investor always finds optimal to sell out-of-themoney put options and, in particular, to write an OTM straddle. [22] These results are unreported but are available upon request from the Authors. All in all, Tables 3-7 emphasize that while under the SV and SVJR models, the CER gains from Bonus, Express, and Discount certificates easily exceed 100 basis points per year even in the case of investors with intermediate risk aversion ($\gamma = 8$) and horizons of 3 years, this is not the case under the SVJV and SVCJ models. Therefore, the classical conclusion by Hens and Rieger (2014) that the utility increase of the best possible structured products over and above a simple portfolio consisting of the risk-free asset and the market portfolio would be too small (only 5–10 basis points in their paper) when compared with the usual margins (in excess of 100 basis points) paid on structured products (not to mention a compensation for the counterparty risk that the buyer of a structured product takes) seems to tightly depend on the model used in the assessment (hence on some type of incorrect beliefs that ignore variance risk induced by jumps) and that jumps in variance may come to play a key role.[23] This is assed next.

4. The role of stochastic volatility and jumps

In this section, we explore the role of stochastic volatility (SV) and jumps in determining the utility gains associated with different structured products. In this section, we focus on the SVCJ model whose parameter values were listed in the rightmost column of Table 1 and we change one parameter at a time to try and measure how changes in the features of either SV or of jumps may affect the risk-adjusted of investment certificates.

[22] Faias and Santa Clara (2017) confirm the findings in Driessen and Maenhout (2007) in that they find that the investor sells OTM puts, but in contrast they do not find evidence that the investor should optimally write straddles. Tan (2013) documents a very limited ability of long European style call or put options to improve the utility of an agent when only financial wealth is present. Earlier, but assuming dynamically complete markets, Liu and Pan (2003) found that negatively priced volatility risk induces a myopic demand for selling derivatives with positive volatility exposure per dollar, with the least risk averse investor being more aggressive in this strategy.

[23] Hens and Rieger (2014) move on to offer non-standard, behavioural type preferences (such as prospect theory) as an explanation of the large demand for investment certificates. However, ad discussed in the Introduction, their asset menu simply consists of an optimal structured production and excludes other asset classes. This implies that investment certificates cannot provide any hedging benefits to stocks, which instead we realistically take into account. Let us add that also in our paper Equity Protection structures never generate significant risk-adjusted performance benefits.

4.1. The role of stochastic volatility

First, in Table 8, we set $\lambda = 0$ such that jumps stop playing a role and we experiment with the parameters that drive the dynamic features of SV. For brevity, we only report the results that concern an investor with intermediate risk aversion of $\gamma = 8$ and an investment horizon equal to one year. We have experimented also with alternative risk aversion coefficients and investment horizons and the remarks that follow apply, subject to obvious differences and qualifications.

In the first panel, we report the results under different assumptions for the variance risk premium parameter. Obviously, because an investor who only has access to the stock and the riskless bond cannot get an exposure to the variance risk premium, the benchmark allocation does not depend on η_v. In the first row, $\eta_v = 0$ such that volatility risk is not rewarded. In this case, the demand for (especially, some of the) certificates still arises as the investor can use them for diversification purposes. For instance, an investor who can access the stock market, the riskless bond and an Equity Protection certificate, will invest 79.8% of her wealth in the latter. Yet, even for these types of certificates that trigger non-negligible demand when $\eta_v = 0$, the CER spreads are very limited and range between 14 basis point (when the Equity protection is included) to 4 basis points (when the Bonus certificate is included).

The outcomes change dramatically as the variance risk premium parameter increases. For instance, when $\eta_v = 0.75$, implying an average equity risk premium of about 1.7%, the investor who has access to a Discount certificate allocates approximately 65% of her wealth to this structured product, achieving a 6.10% CER gain per year, compared to the benchmark. [24] The results are comparable to those for the other certificates, with the only exception of the Equity Protection. The utility gain from the inclusion of this latter certificate in a portfolio remains rather modest (50 basis points at best). This is because, unlike the other certificates, the Equity Protection carries indeed only a very small exposure to volatility risk. Overall, the results clearly show that the investor benefits from

[24] Actual estimates of the variance risk premium vary dramatically across the literature. For instance, Eraker (2004) and Broadie et al. (2007) find that the volatility risk premium parameter is not statistically significant. In contrast, Pan (2002) estimates a very large and significant value of η_v (equal to 7.6). The estimates in Dufays et al. (2023) vary from 0.944 for a simple SV model to 0.395 for the SVCJ models, implying average risk premia that range from 3.4% under the SV model to less than 1% under the SVCJ model.

buying structured products that allow her to harvest the variance risk premium, provided that the latter is sufficiently high.

In the next panel of Table 8, we explore how the weights assigned to the certificates and the corresponding CER gains change when we let the volatility of volatility range from 10% to a rather extreme 60% per year. Again, the results for the Bonus, Discount, and Express certificate are qualitatively similar. Both the proportion of wealth and the utility gains monotonically decrease as the volatility of volatility increases. This is unsurprising as a higher σ is associated with higher kurtosis. These three types of certificates are rigidly capped to the upside but enjoy limited protection from downside risk, such that their benefit vs. a direct exposure to the stock market clearly declines when the tails of the stock return distribution become fatter. For instance, in the case of Bonus Caps, as σ increases, the weight assigned to the certificate declines from 41% to 32% and the spread in realized CER due to the structured product addition from 90 to 55 basis points. For what concerns the Equity Protection, the effect of an increase in the volatility of volatility is less clear-cut because the certificate is capped from the upside but also enjoys full protection from downside risk. The combination of these two effects yields a non-monotonic decrease in the utility gain of this certificate: as σ increases, the CER change associated with Equity protection structures at first increases from 15 to 18 basis points, to then decline to only 3 basis point when σ reaches 0.6. However, the overall utility gain remain very limited, with the maximum being 18 basis point when σ is equal to 20%.

In the next two panels of Table 8, we experiment with the level of the long run variance and the initial variance In the baseline exercise, the latter was in fact set to equal the long run variance but we now let it vary between 0.028 and 0.018, which are values spanning typical values in applied research. The results show that the incremental CER gain derived from adding the Bonus, Discount and Express certificates to the investor's portfolio increases both with the level of long run variance and the initial variance. The opposite happens to the Express certificate which turns out to be short on volatility, because of its overall concave payoff profile. This is not surprising as, for a given η_v, the total variance risk premium is obviously proportional to the level of volatility, which depends for short-term structured products on the initial volatility the system dynamics is started from and for longer-term certificates on the long-run volatility the system converges to. The results are also in line with Eraker (2004), who shows that puts are relatively more expensive when volatility is high because this structurally lifts the implied volatility surface up so that therefore selling them is more convenient.

Overall, the results above show that structured products such as the Bonus, the Discount and the Express can help the investor to harvest the variance risk premium; that their value is larger when the variance risk premium parameter is larger and the long run and initial volatilities are higher. On the contrary, despite an investor should optimally invest a large fraction of her wealth in the Equity protection, the utility gain from adding this certificate to the investment menu is very low.

4.2. The role of jumps

In Table 9, we experiment with a number of values for the jump parameters (λ, η_{Js}, η_{Jv}). First, we set both the mean jump risk premium parameter and the jump variance risk premium parameter equal to zero, so that jumps risk is not rewarded, and let the jump intensity parameter λ vary between 0 and 1, that is from no jumps to one jump per year, on average. It is worth noting that Dufays et al. (2023) anyway estimated the average return premia on jumps to be small and positive. Therefore, because long-term return distribution becomes slightly right-skewed as λ increases (as both the jumps in log-prices and volatility, under positivity constraints, end up inflating the right tail more than they do to the left tail), an investor who does not access the certificates as part of her asset menu modestly increases the proportion of wealth allocated to the stock, while her CER also increases, from 2.9% in the absence of jumps to 3.3% when these occur once a year on average. Nonetheless, their demand as well as the associated changes in CER for all the certificates decreases as λ increases. This is because jumps in variance induce kurtosis in the return distribution and have an effect that is similar to an increase in the volatility of volatility. As discussed above, all the certificates have a cap to the participation to the upside performance of the stock market and therefore they do not benefit from right skewness and kurtosis as such. Correspondingly, their optimal weight declines as λ increases, rather markedly in the case of Discount and Equity Protection certificates.

In the following two panels of Table 9, we set λ to its baseline value of Table 1 (0.641) and $\eta_{Jv} = 0$ and we let the mean jump risk premium parameter η_{Js} vary between 0 and 0.10. The overall jump risk premium is a positive function of η_{Js} and such a limited range of variation is compatible with the commonly reported estimates of the equity risk premium.

It is apparent that the utility gain of all the certificates but the Equity Protection decreases from 50-70 basis points to 20-40 basis points (across alternative structured products) as η_{Js} grows throughout the rows of the Table. Although the exact effect of jumps is hard to pin down in our framework as the market is incomplete and we cannot work with optimally determined exposures to the risk factors (say, differently from Liu and Pan, 2003), what we observe hints at the fact that, when mean jump risk is not rewarded, the investor holds Bonus, Discount or Express certificates to hedge against such unrewarded jump risk.[25] As the mean jump risk premium increases, for these three types of certificates, optimal exposure to jump risk becomes more attractive and the investor increases her exposure to stocks and decreases the weight assigned to the certificates, which tend to yield modest benefits from inflating the right tail of returns at expiration. The opposite appears to be the case for the Equity protection; however, the utility gain from adding this certificate in the investors' asset menu is very limited, accounting to a maximum of 12 basis points.

Finally, in the last panel of Table 9, we set $\eta_{Js} = 0$ and experiment with η_{Jv}, the parameter driving the variance jump premium, which has been suspected to represent a major driver of results early on already. Here, it important to recall that the overall jump risk premium is a negative function of η_{Jv}. Hence, the jump risk premium declines as one moves down through the rows from $\eta_{Jv} = -0.04$ to $\eta_{Jv} = 0.06$. In our baseline specification, η_{Jv} is estimated to be positive, which means that the jump in variance are more negative under Q that under P. Clearly, when jumps in variance are smaller under Q than under P, the Bonus, Express, and Discount certificates all become more expensive at issuance and hence (given their fixed payoff functions) less attractive to investors as no harvesting of a positive variance risk premium may occur. Hence, the weights assigned to these three types of certificates declines as η_{Jv} increases and similarly the CER gains arising from them tend to decay. The opposite happens to the Equity protection. However, again the utility gain in this case is very limited, as it increases from 8 to 11 basis points at best.

[25] It is worthwhile to note that this is a static hedging demand not a intertemporal hedging demand as our investor is myopic. In other words, the investor buys the certificates as they are not perfectly correlated with the stock market and they enable her to face a more favourable risk-return distribution of returns at expiry date.

Overall, the investor seems to benefit the most from certificates when the jumps are less frequent and the wedges between the jump size parameters under P and Q decreases.

5. Conclusion

This paper contributes to the literature on optimal portfolio choice by assessing the economic value of structured derivatives under different assumptions concerning model specification, the (choice of the) investment horizon, and the degree of relative risk aversion. Investors solve their asset allocation problem by maximising a power utility function over terminal wealth when the asset menu is extended to include investment certificates. To mimic the allocation process of a retail investors in reality, we do not allow for portfolio rebalancing, but instead consider the asset allocation as fixed over the chosen investment horizon, in a typical buy-and-hold scheme, as in Hens and Rieger (2014). Moreover, the dynamics of the securities markets are described by a well-known class of models that includes not only stochastic volatility, but also jumps in both the log-price and the variance dynamics.

The study reveals three key findings. Firstly, the optimal demand and the expected utility gains (measured by the change in certainty equivalent return) scored by investment certificates vary significantly based on product type, risk aversion levels, and investment horizon. Different structures suit diverse investment needs, emphasizing the importance of assessing investor risk profiles and horizons. Secondly, we show that optimal certificate demand differs drastically across asset pricing models. Notably, SV models without jumps in variance yield higher utility gains for investors. We conclude that this is likely due to the fact that, under a SVCJ model, the variance of variance parameter tends to be estimated to be larger while the estimated risk premium rewarding variance exposure turns out to be smaller than under other models. Because there is large disagreement within the literature regarding the correct asset pricing model and the hence the resulting estimates of key parameters (see, e.g., the discussion in Dufays et al., 2023), this variability needs to be carefully considered in the assessment of structured products. Lastly, the study indicates that, in general, investors, particularly those less risk-averse, benefit from products allowing to harvest the variance risk premium, especially when this is estimated to be large or when the initial volatility level is large. This implies that differently from some earlier literature (see Hens and Rieger, 2014) it is not implausible that *some types* of structured investment products may lead to an increase in risk-adjusted returns in excess of the typically large fees and trading costs of this asset class. Conversely, products ensuring capital protection with limited participation to stock appreciation offer minimal CER improvements, as these are consistently below 30 basis points annually across models, horizons, and risk aversion levels. These findings help shed light on the increasing demand for relatively more complex, retail structured products such as Bonus Caps and Express certificates. Finally, our results may provide some key insights to the regulators on the importance of assessing the risk profiles and investment objectives of the investors before allowing that some types of structured products may be advertised to investors.

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Figure 1

Payoff Diagram of a Bonus Cap Certificate

The figure shows the payoff at maturity of a Bonus Cap certificate. The dashed black line represents the underlying price while the solid red line represents the payoff of the Bonus Cap certificate. N is the notional amount of the certificate, BA is the bonus amount (in dollars), and B is the barrier, given by the barrier percentage (e.g., 70%) multiplied by the strike (set to be equal to S_0, the value of the underlying at issuance). S_T is the value of the underlying at the maturity date of the certificate.

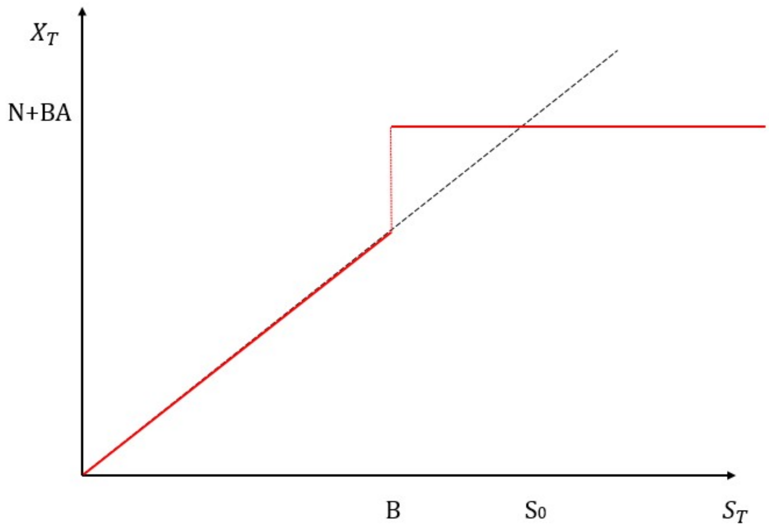

Figure 2

Payoff Diagram of a Discount Certificate

The figure shows the payoff at maturity of a Discount certificate. The dashed black line represents the underlying price while the solid red line represents the payoff of the Discount certificate. N is the notional amount of the certificate and S_0 is the value of the underlying at issuance date. S_T is the value of the underlying at the maturity date.

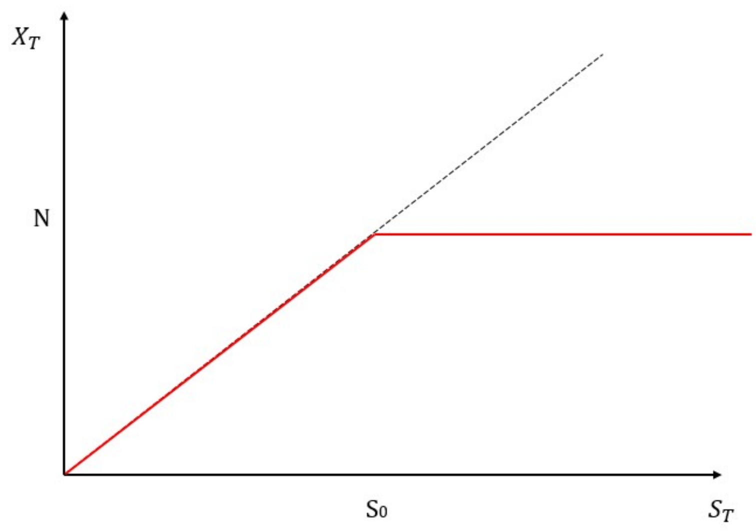

Figure 3

Payoff Diagram of an Express Certificate

The figure shows the payoff at maturity of an Express certificate. The dashed black line represents the underlying price. The solid line represents the payoff of the certificate. If the certificate redeems early, the red line represents the maximum payoff that can be achieved. If the certificate does not redeem early, the blue line is the maximum payoff that can be achieved. P is the coupon, N is the notional amount of the certificate, and B is the barrier, given by barrier percentage (e.g., 70%) multiplied by the strike (set to be equal to S_0, the value of the underlying at issuance). The trigger level is assumed to be equal to S_0. S_T is the value of the underlying at the maturity date of the certificate.

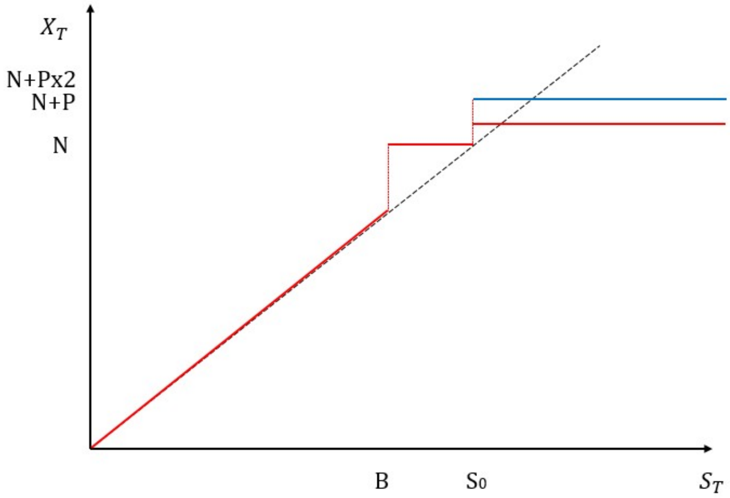

Figure 4

Payoff Diagram of a Equity Certificate

The figure shows the payoff at maturity of an Equity Protection certificate. The dashed black line represents the underlying price. The solid red line represents the payoff of the Equity certificate. N is the notional amount of the certificate. S_0 is the value of the underlying at the issuance date. Cap x N is the maximum amount that the certificate can pay. S_T is the value of the underlying at the maturity date of the certificate.

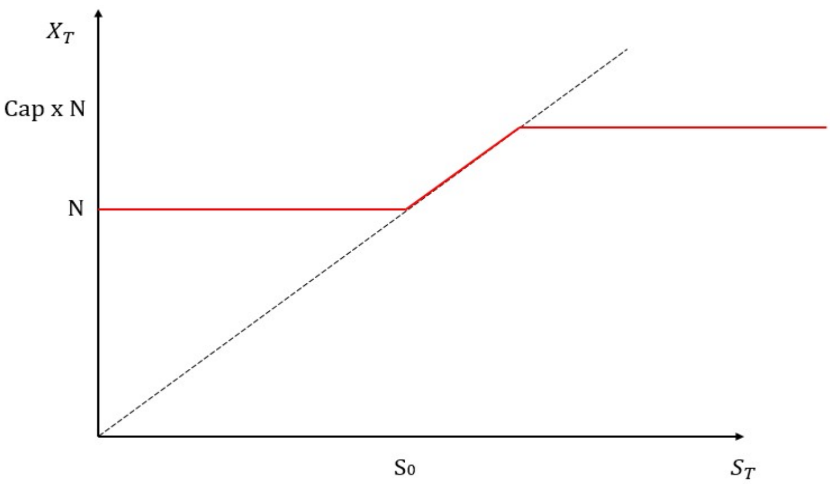

Table 1

Model Parameters Used To Perform Simulations

The table contains the parameter estimates used to simulate the log-price changes and volatility paths. The estimates are obtained from Dufays, Jacobs, Liu, and Rombouts (2023). The values reported are annualised and estimated under the physical probability measure. The Diffusive and the Jump ERP represent the average equity risk premium due to the diffusive and the jump risks, respectively.

	SV	SVJR	SVJV	SVCJ
κ	1.584	0.974	1.173	0.874
ω	0.036	0.029	0.032	0.023
σ	0.398	0.359	0.388	0.337
ρ	-0.908	-0.909	-0.931	-0.918
η_S	2.512	2.158	2.340	1.718
η_V	0.944	0.703	0.460	0.395
λ		0.513	0.598	0.641
μ_S		0.001		0.013
σ_S		0.120		0.087
η_J^s		0.064		0.047
μ_V			0.019	0.073
η_J^v			-0.020	0.040
ρ_J				-0.603
Diffusive ERP	0.0949	0.0678	0.0891	0.0657
Jump ERP		0.0326		0.009

Table 2

Asset Allocation with the Risk-Free Asset and the Stock (the Benchmark)

The table displays the results obtained for the benchmark asset allocation menu which only includes the "stock" (the S&P 500 index) and a zero-coupon riskless bond. SV, SVJR, SVJV and SVCJ denote the different option pricing models under consideration. γ is the coefficient of relative risk aversion. T is the investment horizon. The weights are expressed as percentages. The CER is expressed as percentage per year.

		T= 3 months			T= 6 months			T= 12 months			T= 24 months			T= 36 months		
Model	γ	**Weights (%)**		**CER (%)**	**Weights (%)**		**CER (%)**	**Weights (%)**		**CER (%)**	**Weights (%)**		**CER (%)**	**Weights (%)**		**CER (%)**
		Bond	Stock		Bond	Stock		Bond	Stock		Bond	Stock		Bond	Stock	
SV	4	45.32	54.68	4.16	42.16	57.84	4.55	37.26	62.74	5.18	31.21	68.79	5.83	27.67	72.33	6.42
	8	72.29	27.71	3.16	70.39	29.61	3.48	67.47	32.53	4.00	63.94	36.06	4.44	62.00	38.00	4.84
	12	81.44	18.56	2.82	80.12	19.88	3.12	78.09	21.91	3.59	75.68	24.32	3.97	74.39	25.61	4.31
SVJR	4	44.44	55.56	4.35	43.08	56.92	4.67	39.58	60.42	5.27	34.57	65.43	5.91	31.12	68.88	6.51
	8	71.90	28.10	3.25	70.99	29.01	3.54	68.86	31.14	4.04	65.94	34.06	4.47	64.06	35.94	4.88
	12	81.20	18.80	2.88	80.55	19.45	3.16	79.07	20.93	3.62	77.08	22.92	3.98	75.84	24.16	4.33
SVJV	4	50.84	49.16	3.65	47.51	52.49	4.07	41.97	58.03	4.80	34.56	65.44	5.63	29.72	70.28	6.37
	8	75.12	24.88	2.90	73.18	26.82	3.24	69.96	30.04	3.80	65.66	34.34	4.34	62.91	37.09	4.83
	12	83.35	16.65	2.65	82.00	18.00	2.96	79.77	20.23	3.46	76.82	23.18	3.90	74.97	25.03	4.30
SVCJ	4	64.80	35.20	2.95	61.78	38.22	3.43	58.11	41.89	4.19	53.40	46.60	5.08	50.27	49.73	5.86
	8	82.24	17.76	2.55	80.64	19.36	2.91	78.69	21.31	3.48	76.25	23.75	4.04	74.73	25.27	4.53
	12	88.13	11.87	2.41	87.04	12.96	2.74	85.73	14.27	3.25	84.11	15.89	3.70	83.12	16.88	4.10

Table 3

Asset Allocation with the Risk-Free Asset, the Stock and Bonus Certificates

The table displays the results obtained for an asset menu that includes the "stock" (the S&P 500 index), a zero-coupon riskless bond and one of three different Bonus Cap certificates. SV, SVJR, SVJV and SVCJ denote the different option pricing models under consideration. γ is the coefficient of relative risk aversion. T is the investment horizon, set equal to the maturity of the certificate. The weights are expressed as percentages. The weight assigned to the riskless asset can be obtained as the difference between 100% and the sum of the weights on the stock and the certificate. *dCER* is the CER gain vs. the benchmark, expressed as an annualized percentage. In Panel A, the barrier is set at 70% of the initial underlying value. In Panel B, the barrier is set at 80%. In Panel C, the barrier is set at 90%. The bonus amount is $10 multiplied by the maturity expressed in years.

Panel A : Barrier 70%																
		T= 3 months			T= 6 months			T= 12 months			T= 24 months			T= 36 months		
Model	γ	**Weights (%)**		**dCER (%)**	**Weights (%)**		**dCER (%)**	**Weights (%)**		**dCER (%)**	**Weights (%)**		**dCER (%)**	**Weights (%)**		**dCER (%)**
		Stock	Certificate		Stock	Certificate		Stock	Certificate		Stock	Certificate		Stock	Certificate	
SV	4	27.80	72.19	1.81	6.50	93.50	2.59	0.00	90.78	2.30	5.25	81.33	1.59	17.66	68.71	1.24
	8	14.08	60.82	1.32	3.22	55.39	1.52	0.00	51.74	1.35	2.46	47.18	0.98	8.28	41.48	0.81
	12	9.38	43.19	0.93	2.13	38.66	1.07	0.00	35.92	0.95	1.60	32.86	0.70	5.37	29.17	0.58
SVJR	4	15.22	84.78	3.23	0.02	99.98	4.51	0.00	93.26	3.66	0.01	86.25	2.45	9.50	75.27	1.72
	8	7.71	74.62	2.54	0.00	63.72	2.70	0.00	53.49	2.12	0.00	49.53	1.46	4.27	44.53	1.09
	12	5.13	53.10	1.81	0.00	44.47	1.88	0.01	37.16	1.47	0.00	34.36	1.03	2.73	31.09	0.78
SVJV	4	39.45	59.52	0.43	29.69	53.22	0.65	25.08	51.43	0.64	30.18	47.07	0.55	36.50	43.31	0.51
	8	19.84	35.48	0.26	14.84	30.95	0.39	12.39	30.07	0.40	14.67	28.80	0.37	17.47	27.94	0.37
	12	13.25	24.98	0.18	9.88	21.66	0.27	8.22	21.06	0.28	9.66	20.41	0.27	11.42	20.05	0.28
SVCJ	4	30.69	25.91	0.08	26.83	29.63	0.20	27.97	25.46	0.18	35.58	17.39	0.10	40.03	14.80	0.08
	8	15.40	14.68	0.04	13.37	16.78	0.11	13.78	14.80	0.11	17.16	11.25	0.08	18.97	10.52	0.07
	12	10.27	10.20	0.03	8.91	11.64	0.08	9.13	10.35	0.08	11.28	8.09	0.06	12.39	7.73	0.06
Panel B : Barrier 80%																
		T= 3 months			T= 6 months			T= 12 months			T= 24 months			T= 36 months		
Model	γ	**Weights (%)**		**dCER (%)**	**Weights (%)**		**dCER (%)**	**Weights (%)**		**dCER (%)**	**Weights (%)**		**dCER (%)**	**Weights (%)**		**dCER (%)**
		Stock	Certificate		Stock	Certificate		Stock	Certificate		Stock	Certificate		Stock	Certificate	
SV	4	0.01	99.98	5.58	0.00	100.00	4.77	0.00	91.38	3.36	0.00	86.84	2.17	0.02	86.56	1.62
	8	0.01	71.50	3.45	0.00	57.29	2.62	0.00	51.02	1.87	0.00	48.87	1.25	0.01	48.95	0.98
	12	0.01	49.38	2.38	0.00	39.43	1.80	0.00	35.13	1.28	0.01	33.66	0.87	0.00	33.71	0.68
SVJR	4	0.01	99.99	8.68	0.00	99.99	7.25	0.00	94.07	4.87	0.00	86.50	3.03	0.01	85.09	2.14
	8	0.00	79.38	5.80	0.00	62.35	4.08	0.00	52.88	2.69	0.00	48.83	1.72	0.01	48.18	1.27
	12	0.00	55.09	4.00	0.00	42.98	2.80	0.00	36.43	1.84	0.00	33.63	1.19	0.00	33.16	0.88
SVJV	4	13.78	83.79	1.49	0.03	82.30	1.45	0.01	76.93	1.18	10.34	67.13	0.84	21.68	58.15	0.70
	8	6.85	45.89	0.82	0.10	44.39	0.80	0.01	42.01	0.67	4.88	38.12	0.51	10.10	34.69	0.47
	12	4.56	31.52	0.57	0.10	30.31	0.55	0.00	28.80	0.46	3.18	26.39	0.36	6.55	24.31	0.34
SVCJ	4	11.21	57.56	0.69	7.85	51.78	0.63	16.84	37.24	0.34	31.04	21.83	0.13	37.24	17.33	0.09
	8	5.58	30.93	0.37	3.88	27.70	0.34	8.21	20.47	0.20	14.77	13.38	0.09	17.40	11.75	0.08
	12	3.71	21.11	0.26	2.57	18.88	0.24	5.42	14.06	0.14	9.67	9.47	0.07	11.30	8.52	0.06
Panel C : Barrier 90%																
		T= 3 months			T= 6 months			T= 12 months			T= 24 months			T= 36 months		
Model	γ	**Weights (%)**		**dCER (%)**	**Weights (%)**		**dCER (%)**	**Weights (%)**		**dCER (%)**	**Weights (%)**		**dCER (%)**	**Weights (%)**		**dCER (%)**
		Stock	Certificate		Stock	Certificate		Stock	Certificate		Stock	Certificate		Stock	Certificate	
SV	4	0.00	99.99	9.02	0.00	100.00	6.22	0.00	92.83	4.05	0.00	87.63	2.58	0.00	87.22	1.99
	8	0.00	70.41	5.27	0.00	57.63	3.34	0.00	51.26	2.19	0.00	48.79	1.42	0.00	48.76	1.13
	12	0.00	48.04	3.57	0.00	39.39	2.27	0.00	35.11	1.49	0.00	33.44	0.97	0.01	33.39	0.78
SVJR	4	0.00	100.00	12.73	0.00	99.99	8.73	0.00	95.46	5.55	0.00	87.14	3.37	0.00	85.61	2.47
	8	0.00	78.70	8.02	0.00	62.59	4.81	0.00	53.16	2.99	0.00	48.74	1.85	0.00	48.01	1.40
	12	0.00	53.91	5.45	0.00	42.82	3.26	0.00	36.44	2.04	0.00	33.41	1.27	0.01	32.87	0.96
SVJV	4	0.00	98.59	3.47	0.00	84.51	2.44	0.00	78.61	1.75	0.01	78.77	1.25	0.38	80.76	1.02
	8	0.01	52.07	1.82	0.00	44.98	1.29	0.00	42.49	0.95	0.01	43.31	0.71	0.10	45.04	0.62
	12	0.01	35.30	1.23	0.00	30.57	0.88	0.00	28.99	0.65	0.00	29.66	0.49	0.03	30.91	0.43
SVCJ	4	0.01	76.98	2.33	0.01	63.58	1.36	0.71	55.63	0.60	24.08	29.63	0.20	33.05	21.90	0.12
	8	0.01	40.44	1.21	0.01	33.39	0.72	0.30	29.39	0.33	11.25	17.24	0.13	15.16	14.07	0.10
	12	0.01	27.38	0.82	0.00	22.61	0.49	0.21	19.92	0.23	7.32	12.02	0.09	9.79	10.05	0.07

Table 4

Asset Allocation with the Risk-Free Asset, the Stock and Discount Certificates

The table displays the results obtained for an asset menu that includes the "stock" (the S&P 500 index), a zero-coupon riskless bond and a Discount certificate. SV, SVJR, SVJV and SVCJ denote the different option pricing models under consideration. γ is the coefficient of risk aversion. T is the investment horizon, set equal to the maturity of the certificate. The weights are expressed as percentages. The weight assigned to the riskless asset can be obtained as the difference between 100% and the sum of the weights on the stock and the certificate. *dCER* is the CER gain vs. the benchmark, expressed as an annualized percentage.

		T= 3 months			T= 6 months			T= 12 months			T= 24 months			T= 36 months		
Model	**γ**	**Weights (%)**		**dCER (%)**	**Weights (%)**		**dCER (%)**	**Weights (%)**		**dCER (%)**	**Weights (%)**		**dCER (%)**	**Weights (%)**		**dCER (%)**
		Stock	Certificate		Stock	Certificate		Stock	Certificate		Stock	Certificate		Stock	Certificate	
SV	4	0.00	99.99	8.97	0.00	99.99	6.24	0.00	99.99	4.16	0.00	99.36	2.65	0.01	99.72	2.01
	8	0.00	79.37	5.69	0.00	65.98	3.58	0.00	60.19	2.36	0.00	60.19	1.62	0.01	62.44	1.36
	12	0.00	54.28	3.86	0.00	45.39	2.44	0.00	41.72	1.62	0.00	42.09	1.14	0.01	43.98	0.97
SVJR	4	0.00	99.99	12.24	0.00	100.00	8.46	0.00	100.00	5.59	0.00	97.45	3.44	0.00	96.13	2.50
	8	0.00	87.67	8.42	0.00	71.22	5.08	0.00	61.71	3.19	0.00	59.06	2.08	0.00	60.19	1.66
	12	0.00	60.28	5.72	0.00	49.10	3.46	0.00	42.85	2.19	0.00	41.32	1.45	0.00	42.38	1.18
SVJV	4	0.00	99.98	4.13	0.00	97.58	2.81	0.00	90.67	1.89	0.01	91.50	1.27	11.89	80.65	0.97
	8	0.00	60.47	2.22	0.00	52.80	1.50	0.00	50.61	1.06	0.01	53.45	0.80	1.74	55.09	0.71
	12	0.00	41.03	1.50	0.00	36.03	1.02	0.00	34.82	0.73	0.01	37.18	0.57	0.49	39.54	0.52
SVCJ	4	0.00	88.10	2.85	0.00	73.35	1.57	0.01	65.91	0.67	23.67	36.68	0.22	32.18	29.10	0.16
	8	0.00	46.54	1.48	0.00	38.96	0.83	0.00	35.36	0.38	10.61	22.46	0.15	14.33	19.78	0.13
	12	0.00	31.55	1.00	0.00	26.47	0.56	0.00	24.09	0.26	6.81	15.88	0.11	9.16	14.34	0.10

Table 5

Asset Allocation with the Risk-Free Asset, the Stock and Express Certificates

The table displays the results obtained for an asset menu that includes the “stock” (the S&P 500 index), a zero-coupon riskless bond and an Express certificate. SV, SVJR, SVJV and SVCJ denote the different option pricing models under consideration. γ is the coefficient of relative risk aversion. T is investment horizon which is set equal to the maturity of the certificate. The weights are expressed as percentages. The weight assigned to the riskless asset can be obtained as the difference between 100% and the sum of the weights on the stock and the certificate. $dCER$ is the CER gain vs. the benchmark, expressed as an annualized percentage. In Panel A, the barrier is set at 70% of the initial underlying value. In Panel B, the barrier is set at 80%. In Panel C, the barrier is set at 90%. The coupon amount is $5 per period multiplied by the maturity expressed in years.

Panel A : Barrier 70%

		T= 3 months			T= 6 months			T= 12 months			T= 24 months			T= 36 months		
Model	γ	**Weights (%)**		**dCER (%)**	**Weights (%)**		**dCER (%)**	**Weights (%)**		**dCER (%)**	**Weights (%)**		**dCER (%)**	**Weights (%)**		**dCER (%)**
		Stock	Certificate		Stock	Certificate		Stock	Certificate		Stock	Certificate		Stock	Certificate	
SV	4	54.68	0.00	0.00	57.84	0.00	0.00	0.00	95.24	2.49	3.58	88.14	9.14	21.06	70.22	16.49
	8	27.71	0.00	0.00	29.61	0.00	0.00	0.01	54.93	1.47	0.78	53.15	6.54	9.09	45.20	12.19
	12	18.56	0.00	0.00	19.88	0.00	0.00	0.00	38.24	1.03	0.35	37.31	5.46	5.73	32.26	10.43
SVJR	4	55.56	0.00	0.00	56.92	0.00	0.00	0.00	96.56	3.60	0.00	90.29	10.63	12.13	77.11	18.09
	8	28.10	0.00	0.00	29.01	0.00	0.00	0.00	56.04	2.10	0.01	53.05	7.38	4.69	48.41	13.10
	12	18.80	0.00	0.00	19.45	0.00	0.00	0.00	39.03	1.46	0.00	37.01	6.04	2.84	34.27	11.05
SVJV	4	49.16	0.00	0.00	52.49	0.00	0.00	16.30	65.00	0.81	27.72	54.42	6.94	38.08	46.71	14.51
	8	24.88	0.00	0.00	26.82	0.00	0.00	7.55	38.00	0.50	13.01	34.11	5.22	17.91	31.69	10.95
	12	16.65	0.00	0.00	18.00	0.00	0.00	4.89	26.60	0.35	8.46	24.30	4.54	11.63	22.99	9.56
SVCJ	4	35.20	0.00	0.00	38.22	0.00	0.00	21.56	37.97	0.34	30.92	27.91	5.55	36.68	23.93	12.34
	8	17.76	0.00	0.00	19.36	0.00	0.00	10.43	21.79	0.21	14.82	17.29	4.36	17.41	15.85	9.52
	12	11.87	0.00	0.00	12.96	0.00	0.00	6.87	15.17	0.15	9.71	12.29	3.93	11.37	11.46	8.53

Panel B : Barrier 80%

		T= 3 months			T= 6 months			T= 12 months			T= 24 months			T= 36 months		
Model	γ	**Weights (%)**		**dCER (%)**	**Weights (%)**		**dCER (%)**	**Weights (%)**		**dCER (%)**	**Weights (%)**		**dCER (%)**	**Weights (%)**		**dCER (%)**
		Stock	Certificate		Stock	Certificate		Stock	Certificate		Stock	Certificate		Stock	Certificate	
SV	4	54.68	0.00	0.00	57.84	0.00	0.00	0.00	95.07	3.08	0.00	91.67	9.58	16.39	74.72	16.68
	8	27.71	0.00	0.00	29.61	0.00	0.00	0.00	53.91	1.74	0.01	53.18	6.72	6.88	46.71	12.22
	12	18.56	0.00	0.00	19.88	0.00	0.00	0.00	37.28	1.20	0.00	36.94	5.57	4.31	33.01	10.43
SVJR	4	55.56	0.00	0.00	56.92	0.00	0.00	0.00	96.49	4.20	0.00	90.11	11.04	7.81	81.42	18.35
	8	28.10	0.00	0.00	29.01	0.00	0.00	0.00	55.05	2.37	0.00	52.36	7.53	2.61	50.02	13.17
	12	18.80	0.00	0.00	19.45	0.00	0.00	0.00	38.08	1.63	0.00	36.35	6.12	1.50	35.13	11.08
SVJV	4	49.16	0.00	0.00	52.49	0.00	0.00	0.05	81.70	1.16	21.12	61.05	7.14	35.23	49.38	14.63
	8	24.88	0.00	0.00	26.82	0.00	0.00	0.01	45.15	0.67	9.79	36.92	5.32	16.55	32.51	10.99
	12	16.65	0.00	0.00	18.00	0.00	0.00	0.01	31.04	0.47	6.35	26.00	4.60	10.75	23.37	9.58
SVCJ	4	35.20	0.00	0.00	38.22	0.00	0.00	14.00	45.97	0.49	28.87	29.93	5.62	35.91	24.50	12.38
	8	17.76	0.00	0.00	19.36	0.00	0.00	6.70	25.49	0.28	13.79	18.12	4.39	17.03	15.96	9.53
	12	11.87	0.00	0.00	12.96	0.00	0.00	4.40	17.55	0.19	9.04	12.79	3.95	11.11	11.48	8.54

Panel C : Barrier 90%

		T= 3 months			T= 6 months			T= 12 months			T= 24 months			T= 36 months		
Model	γ	**Weights (%)**		**dCER (%)**	**Weights (%)**		**dCER (%)**	**Weights (%)**		**dCER (%)**	**Weights (%)**		**dCER (%)**	**Weights (%)**		**dCER (%)**
		Stock	Certificate		Stock	Certificate		Stock	Certificate		Stock	Certificate		Stock	Certificate	
SV	4	54.68	0.00	0.00	57.84	99.99	0.29	0.00	95.22	3.32	0.01	91.60	9.76	13.85	77.22	16.75
	8	27.71	0.00	0.00	29.61	0.00	0.00	0.00	53.56	1.85	0.01	52.82	6.78	5.68	47.60	12.22
	12	18.56	0.00	0.00	19.88	0.00	0.00	0.00	36.90	1.27	0.00	36.59	5.60	3.54	33.47	10.42
SVJR	4	55.56	0.00	0.00	56.92	99.99	1.17	0.00	96.58	4.40	0.00	90.03	11.16	5.56	83.69	18.44
	8	28.10	0.00	0.00	29.01	63.09	0.19	0.00	54.68	2.44	0.00	52.02	7.55	1.52	50.89	13.16
	12	18.80	0.00	0.00	19.45	0.00	0.00	0.00	37.69	1.67	0.01	36.02	6.12	0.83	35.56	11.06
SVJV	4	49.16	0.00	0.00	52.49	0.00	0.00	0.01	81.95	1.39	16.84	65.51	7.28	33.56	51.03	14.71
	8	24.88	0.00	0.00	26.82	0.00	0.00	0.01	44.93	0.78	7.73	38.87	5.38	15.77	33.07	11.01
	12	16.65	0.00	0.00	18.00	0.00	0.00	0.00	30.80	0.54	5.00	27.21	4.64	10.25	23.65	9.59
SVCJ	4	35.20	0.00	0.00	38.22	0.00	0.00	8.00	52.92	0.61	27.48	31.52	5.67	35.41	25.00	12.41
	8	17.76	0.00	0.00	19.36	0.00	0.00	3.76	28.81	0.34	13.11	18.83	4.42	16.79	16.12	9.55
	12	11.87	0.00	0.00	12.96	0.00	0.00	2.45	19.72	0.24	8.58	13.24	3.97	10.96	11.55	8.55

Table 6

Allocation with the Risk-Free Asset, the Stock and Equity Protection Certificates

The table displays the results for an asset menu that includes the "stock" (the S&P 500 index), a zero-coupon riskless bond and an Equity protection certificate. SV, SVJR, SVJV and SVCJ denote the different option pricing models under consideration. γ is the coefficient of risk aversion. T is investment horizon, set to be equal to the maturity of the certificate. The weights are expressed as percentages. The weight assigned to the riskless asset can be obtained as the difference between 100% and the sum of the weights on the stock and the certificate. *dCER* is the CER gain vs. the benchmark, expressed as an annualized percentage. The capital is fully protected and the cap is set at \$100 plus \$10 multiplied by the maturity expressed in years.

		T= 3 months			T= 6 months			T= 12 months			T= 24 months			T= 36 months		
Model	γ	**Weights (%)**		**dCER (%)**	**Weights (%)**		**dCER (%)**	**Weights (%)**		**dCER (%)**	**Weights (%)**		**dCER (%)**	**Weights (%)**		**dCER (%)**
		Stock	Certificate		Stock	Certificate		Stock	Certificate		Stock	Certificate		Stock	Certificate	
SV	4	48.05	51.94	0.18	47.40	52.60	0.21	50.69	49.30	0.17	54.65	45.34	0.15	55.19	44.80	0.19
	8	18.88	81.11	0.25	16.31	83.68	0.27	15.60	80.58	0.16	21.86	48.67	0.09	19.52	52.32	0.12
	12	8.91	91.07	0.24	5.45	94.54	0.23	10.56	53.39	0.10	14.95	31.57	0.05	13.31	34.11	0.07
SVJR	4	47.22	52.75	0.23	46.03	53.95	0.27	47.97	52.02	0.22	51.66	48.33	0.14	52.75	47.24	0.17
	8	18.68	81.28	0.33	15.93	84.04	0.35	13.85	86.12	0.22	21.94	44.12	0.08	21.60	43.26	0.09
	12	8.98	91.00	0.33	5.57	94.42	0.32	8.58	61.28	0.14	14.87	28.78	0.05	14.58	28.28	0.05
SVJV	4	40.59	59.40	0.17	41.41	58.56	0.20	44.68	55.30	0.19	48.98	51.01	0.23	51.08	48.92	0.28
	8	14.84	85.14	0.21	13.05	86.94	0.22	12.81	81.97	0.17	15.07	66.71	0.17	12.90	69.71	0.22
	12	6.05	93.94	0.19	4.47	87.81	0.16	8.67	54.38	0.11	10.29	43.87	0.10	8.78	45.89	0.14
SVCJ	4	25.21	74.74	0.25	26.04	73.96	0.26	28.10	71.87	0.20	30.32	69.66	0.18	31.05	68.93	0.21
	8	7.45	92.54	0.27	6.10	93.89	0.25	9.31	67.03	0.13	13.25	44.95	0.09	13.49	43.00	0.10
	12	1.50	98.37	0.24	1.27	85.30	0.18	6.24	44.61	0.08	8.84	29.91	0.06	8.92	28.70	0.07

Table 7

Asset Allocation with the Risk-Free Asset, the Stock and All Types of Certificates

The table displays the results obtained for an asset menu that includes the "stock" (the S&P 500 index), the riskless bond, a Bonus Cap, an Express, a Discount, and an Equity protection certificates. The investment horizon and the maturity dates of all certificates are set to one year. The barriers of the Bonus and the Express certificates are set at 70% of the initial underlying value. The bonus amount is \$10, while the coupon amount of the Express certificate is \$5 per period. For the equity protection, the capital is fully protected and the cap is set at \$110. The weight assigned to the riskless asset can be obtained as the difference between 100% and the sum of the weights on the stock and the certificate. *dCER* is the CER gain vs. the benchmark, expressed as an annualized percentage.

Model	γ	**Weights (%)**					**dCER (%)**
		Stock	Bonus	Express	Discount	Eq. Protection	
SV	4	0.00	0.00	0.00	99.99	0.00	4.16
	8	0.00	0.07	0.34	57.48	16.00	2.37
	12	0.00	0.11	0.80	39.65	8.11	1.63
SVJR	4	0.00	0.00	0.00	99.99	0.00	5.59
	8	0.00	0.04	0.02	61.18	3.55	3.19
	12	0.00	0.16	0.03	42.64	0.24	2.19
SVJV	4	0.00	0.01	0.01	85.91	14.06	1.95
	8	0.00	0.07	3.10	40.24	46.32	1.16
	12	0.00	0.09	2.53	27.51	29.66	0.79
SVCJ	4	0.01	0.01	4.09	53.08	42.80	0.85
	8	0.00	0.00	8.33	19.43	54.13	0.52
	12	0.00	0.02	5.87	13.19	35.51	0.35

Table 8

Asset Allocation under Alternative Choices of $\boldsymbol{\eta_v}$, $\boldsymbol{\sigma}$, $\boldsymbol{\omega}$ and $\boldsymbol{V_0}$

The table shows the weights assigned to the stock and different certificates and the annualized utility gains under different assumptions for the variance risk premium parameter η_v, the volatility of volatility $\boldsymbol{\sigma}$, the long run variance ω and the initial volatility V_0. Certificate prices are also reported. We consider the SVCJ model but set $\lambda = 0$ so that the jumps are ruled out. For the sake of comparison, we also report the weight assigned to the stock and the annualized CER in the benchmark model (no certificates). The investment horizon is one year, $\gamma = 8$ and the barrier level of the Bonus and Express certificate is set equal to 80%.

	Benchmark		Asset Allocation with Bonus				Asset Allocation with Express				Asset Allocation with Discount				Asset Allocation with Eq. Protection			
η_v	**Weight (%)**	**CER%**	**Price Cert**	**Weights (%)**		**dCER (%)**	**Price Cert**	**Weights (%)**		**dCER (%)**	**Price Cert**	**Weights (%)**		**dCER (%)**	**Price Cert**	**Weights (%)**		**dCER (%)**
	Stock			Stock	Cert			Stock	Cert			Stock	Cert			Stock	Cert	
0.00	15.50	2.93	103.30	6.17	14.39	0.04	99.75	0.13	24.30	0.11	92.49	0.01	26.98	0.15	102.07	0.00	79.79	0.14
0.15	15.50	2.93	102.64	0.02	25.08	0.19	99.22	0.01	28.65	0.25	91.95	0.00	31.95	0.33	102.19	0.00	71.25	0.12
0.30	15.50	2.93	101.71	0.00	30.55	0.45	98.47	0.00	33.56	0.50	91.19	0.00	37.69	0.63	102.29	0.00	64.78	0.11
0.45	15.50	2.93	100.30	0.00	36.94	0.94	97.34	0.00	39.42	0.94	90.03	0.00	44.58	1.18	102.31	0.01	63.09	0.12
0.60	15.50	2.93	97.78	0.00	45.10	2.04	95.34	0.00	46.97	1.88	87.99	0.00	53.15	2.36	102.19	0.00	71.61	0.20
0.75	15.50	2.93	91.53	0.00	57.12	5.69	90.45	0.00	58.16	4.85	83.00	0.00	64.91	6.10	101.78	0.00	99.28	0.51
σ																		
0.10	13.25	2.92	101.04	0.00	41.43	0.90	97.58	0.00	43.97	0.85	89.87	0.00	52.75	1.15	101.40	0.00	62.48	0.15
0.20	14.17	2.93	100.86	0.00	37.17	0.79	97.60	0.00	39.73	0.78	90.07	0.00	46.52	1.04	101.78	0.00	66.06	0.18
0.30	15.12	2.93	100.84	0.00	35.01	0.74	97.74	0.00	37.67	0.76	90.38	0.00	42.91	0.96	102.19	0.00	64.75	0.14
0.40	16.20	2.94	100.98	0.00	33.86	0.69	97.95	0.00	36.60	0.73	90.75	0.00	40.56	0.88	102.47	2.20	54.54	0.07
0.50	17.50	2.97	101.19	0.00	32.96	0.63	98.18	0.00	35.99	0.69	91.08	0.00	38.90	0.80	102.61	7.95	39.47	0.04
0.60	19.01	3.00	101.38	0.00	32.23	0.55	98.41	0.00	35.65	0.64	91.34	0.00	37.71	0.71	102.66	11.19	34.20	0.03
ω																		
0.02	13.68	2.88	101.56	0.00	34.43	0.67	91.10	0.01	52.64	0.71	98.35	0.00	41.99	0.87	102.33	0.00	37.25	0.08
0.04	19.97	3.27	97.85	0.00	34.76	0.89	88.00	1.71	87.17	0.86	95.42	0.00	41.64	1.14	102.10	0.00	36.95	0.18
0.06	21.76	3.71	94.93	0.00	35.03	1.01	85.68	7.95	80.79	0.92	93.14	0.00	41.52	1.30	101.89	0.01	36.97	0.15
0.08	22.54	4.14	92.48	0.01	35.48	1.12	83.75	11.57	74.28	0.96	91.26	0.00	41.53	1.42	101.74	0.00	37.16	0.13
0.10	22.94	4.58	90.39	0.01	35.90	1.22	82.06	13.64	70.50	1.00	89.62	0.00	41.58	1.54	101.62	2.57	34.46	0.12
0.12	23.16	5.01	88.53	0.20	36.06	1.30	80.55	14.85	68.99	1.04	88.17	0.00	41.66	1.64	101.52	4.80	32.02	0.12
V_0																		
0.028	16.86	2.99	100.16	0.00	34.89	0.80	97.19	0.00	37.42	0.81	89.90	0.00	42.25	1.03	102.29	0.00	71.54	0.15
0.025	16.22	2.96	100.51	0.00	34.74	0.76	97.50	0.00	37.31	0.78	90.21	0.00	42.10	0.98	102.30	0.01	67.36	0.13
0.021	14.65	2.91	101.27	0.00	34.23	0.68	98.14	0.00	37.02	0.71	90.84	0.00	41.68	0.88	102.31	0.01	58.08	0.10
0.018	13.67	2.88	101.67	0.00	33.90	0.63	98.48	0.00	36.83	0.67	91.18	0.00	41.39	0.82	102.31	0.01	52.99	0.08

Table 9

Asset Allocation under Alternative Choices of λ, η_{Js} and η_{Jv}

The table shows the weights assigned to the stock and different certificates and the annualized utility gains under different assumptions for the jump intensity parameter λ, the mean jump risk premium parameter η_{Js} and the jump variance risk premium parameter η_{Jv}. In the first panel, as λ changes, η_{Js} and η_{Js} are set to zero. The prices of the various certificates are also reported to support the commentaries expressed in the main text. We consider the SVCJ model and change one parameter at a time. For the sake of comparison, we also report the weight assigned to the stock and the annualized CER in the benchmark model (no certificates). The investment horizon is one year, $\gamma = 8$ and the barrier level of the Bonus and Express certificate is set equal to 80%.

	Benchmark		Asset Allocation with Bonus				Asset Allocation with Express				Asset Allocation with Discount				Asset Allocation with Eq. Protection			
λ	**Weight (%)**	**CER%**	**Price Cert**	**Weights (%)**		**dCER (%)**	**Price Cert**	**Weights (%)**		**dCER (%)**	**Price Cert**	**Weights (%)**		**dCER (%)**	**Price Cert**	**Weights (%)**		**dCER (%)**
	Stock			Stock	Cert			Stock	Cert			Stock	Cert			Stock	Cert	
0.00	15.50	2.93	100.89	0.00	34.47	0.72	97.81	0.00	37.15	0.74	90.52	0.00	41.91	0.93	102.31	0.00	62.91	0.12
0.20	16.25	3.02	100.11	0.00	32.12	0.65	97.31	0.00	35.13	0.68	89.88	0.00	38.78	0.84	102.25	2.43	61.65	0.10
0.40	16.52	3.10	99.40	0.00	30.22	0.58	96.88	0.00	33.42	0.61	89.29	0.00	36.32	0.76	102.20	4.76	58.64	0.10
0.60	16.75	3.18	98.68	0.01	29.16	0.54	96.42	0.00	32.28	0.56	88.69	0.00	34.90	0.71	102.15	7.17	52.16	0.08
0.80	17.01	3.26	98.02	0.00	28.34	0.49	96.00	0.06	31.32	0.50	88.14	0.00	33.93	0.67	102.10	8.54	49.42	0.07
1.00	17.18	3.34	97.38	0.51	27.28	0.45	95.59	1.82	28.76	0.45	87.62	0.00	33.19	0.63	102.05	9.77	45.77	0.06
η_{Js}																		
0.00	16.78	3.19	98.54	0.00	28.90	0.52	96.34	0.00	31.98	0.54	88.57	0.00	34.64	0.70	102.14	7.40	51.86	0.08
0.02	20.51	3.43	98.41	1.33	29.57	0.43	96.27	2.21	31.80	0.45	88.48	0.00	37.12	0.66	102.17	10.49	56.23	0.09
0.04	23.89	3.71	98.23	6.14	26.72	0.36	96.16	7.21	28.52	0.38	88.36	0.00	39.54	0.59	102.19	13.44	59.72	0.10
0.06	26.95	4.02	98.01	10.68	24.03	0.29	96.01	11.75	25.70	0.32	88.20	0.14	41.71	0.51	102.20	16.26	62.31	0.11
0.08	29.71	4.37	97.78	14.94	21.51	0.24	95.86	15.98	23.05	0.27	88.02	4.31	38.84	0.44	102.21	18.90	64.37	0.11
0.10	32.20	4.73	97.51	18.63	19.54	0.20	95.69	19.73	20.87	0.23	87.81	8.45	35.91	0.38	102.20	21.29	66.52	0.12
η_{Jv}																		
-0.04	28.38	4.19	97.16	7.56	30.13	0.47	95.55	18.97	55.56	0.40	87.55	0.00	44.79	0.77	102.25	11.24	28.41	0.08
-0.02	26.77	4.00	97.64	7.65	28.08	0.40	95.81	16.72	58.52	0.38	87.92	0.00	42.71	0.67	102.22	9.98	28.16	0.09
0.00	25.04	3.82	98.15	7.73	25.88	0.34	96.11	14.47	60.83	0.35	88.30	0.00	40.40	0.56	102.20	8.90	27.49	0.10
0.02	23.19	3.65	98.71	8.14	22.94	0.26	96.42	12.21	62.43	0.32	88.71	0.00	37.81	0.46	102.17	7.80	26.65	0.11
0.04	21.21	3.48	99.28	8.51	19.82	0.19	96.74	9.99	63.08	0.29	89.14	0.01	34.86	0.36	102.14	6.66	25.66	0.11
0.06	19.07	3.33	99.90	9.06	16.07	0.12	97.09	7.82	62.72	0.25	89.60	0.15	31.24	0.25	102.11	5.68	24.17	0.11