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Taylor Rule and Shadow Rates: theory and empirical analysis

Camilla Lupiani♠

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In view of the persistent zero lower bound, which has dominated the European financial landscape since December 2012, the European Central Bank (ECB) has implemented unconventional monetary policies. However, the effects of these unconventional policies have not been fully captured by the traditional reference rates, which have remained anchored at values close to or below the zero lower bound. In order to assess the impact of these measures in more detail, the concept of "shadow rates" was introduced. These shadow rates, often based on financial indicators, provide a more comprehensive view of the overall macroeconomic situation. The present study aims to compare the predictive accuracy of a Taylor rule based on shadow rates with that based on the reference rate, the €str, in an out-of-sample period. The results of this analysis highlight that a Taylor rule based on shadow rates offers a more accurate representation of the stance of the monetary policy, and is even used by monetary analysts to form expectations, especially when the central bank does not provide clear guidance. This study suggests that incorporating the shadow rate into the Taylor rule could provide valuable insights for guiding monetary policy and get a deeper understanding of the macroeconomic landscape.

Keywords: central bank, ECB, interest rate, shadow rate, GMM, efficiency, zero lower bound, effective lower bound

JEL classification: E02, E43, E52, E58, F01,

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1. Introduction

Monetary policy plays a crucial role in ensuring the proper functioning and balance of the economy by acting on key market measures. The entities responsible for the execution of this policy are the central banks, which do not have direct control over inflation, growth, or employment rates, but can influence these factors by implementing changes in key interest rates. This is the reason for their importance and relevance.

This study focuses on the monetary policies implemented in the European context by the European Central Bank, whose goal is to keep the level of inflation stable at around 2%.

In the European Union, in the period between December 2012 and December 2021, interest rates were very close to the effective lower bound, i.e. the lower limit of nominal interest rates below which it becomes difficult or impossible for central banks to further stimulate the economy through conventional monetary policies. This situation has necessitated unconventional measures such as quantitative easing and forward guidance.

In order to assess the effects of these unconventional policies, many 'shadow rates' have been developed, including the one elaborated by Wu and Xia (2016) and the one constructed by Volpi (2023) specifically for the European Union.

Reaching the effective lower bound has led monetary analysts to abandon the use of the Taylor rule, a function that seeks to minimise differences between inflation and output targets. Especially in recent months, however, the structural change in the macroeconomic environment - from stagflation scenarios to the risk of high inflation - has led major central banks to pursue the normalisation of monetary policy and to return to thinking in terms of reaction functions. (Masciandaro, 2022).

The key point of this study is to determine whether the use of the Taylor rule with shadow rates produces better estimates than the Taylor rule with traditional reference rates.

The analysis begins with a review of the literature on shadow rates, focusing on their construction and methods of use. Next, an overview of the European Central Bank with its responses to the 2007 financial crisis and the subsequent liquidity trap is given. This is followed by an econometric analysis using in-sample data, covering the period of low interest rates, and the projection of these results onto the out-of-sample period.

The conclusions of the analysis show that the use of shadow rates in the Taylor rule produces superior results in terms of out-of-sample performance. This is confirmed in terms of both root mean squared error (RMSE) and mean absolute error (MAE) when comparing the accuracy of the two models over the out-of-sample period. An analysis is also conducted to check whether monetary analysts already take the shadow rate into account when elaborating expectations, although not being officially implemented by the ECB.

This type of analysis is of great importance since shadow rates are constructed considering financial factors that are not reflected in traditional reference rates, thus improving the performance of macroeconomic models. Moreover, in light of the possibility of a return to the zero lower bound in the future, shadow rates can play a significant role in the formulation of monetary policies.

2. Literature Review

2.1. Taylor Rule

The Taylor rule, formulated by J.B. Taylor in 1993, is a reaction function widely adopted by central banks. This function delineates the optimal way in which the interest rate should fluctuate with respect to exogenous variation in GDP and inflation. Whilst it has been further developed with new and distinct assumptions, the initial Taylor rule, as proposed by Taylor for the Federal Reserve, can be expressed as follows:

$i = r^* + \pi + \alpha(\pi - \pi^*) + \beta(y - y^*)$ with $\alpha, \beta > 0$

In this context, the variable i denotes the reference rate, while r^*represents the natural rate. The difference between the inflation rate (π) and the inflation target (π^*), also known as the inflation gap, is denoted by ($\pi - \pi^*$). Similarly, the difference between the actual output (y) and the potential output (y^*) is referred to as the output gap. The central bank's response to these fundamental macroeconomic gaps is captured by the parameters α and β. Monetary policy may be defined as active when it serves as a means of stabilisation with the goal of minimising gaps. Conversely, it may be deemed neutral if the measures implemented are neither expansionary nor restrictive. The magnitude of the parameters is a crucial aspect. It can be observed that in the scenario where both α and β are greater than 1, the reference rate exhibits a propensity to respond to variations in inflation and output for the purpose of maintaining stability of the underlying economic fundamentals. If both α and β are smaller than one, the reference rate undergoes a shift to accommodate fluctuations in inflation. As posited by Bernanke and Woodford (1997) and Clarida et al. (1998), the implementation of an accommodative regime may give rise to instances of inflation and output explosions. Henceforth, a precise evaluation of said parameters may facilitate comprehension of the stance of monetary policy. The magnitude of the coefficients of the Taylor Rule, however, remains a topic of debate among scholars.

Nevertheless, the Golden Taylor Rule posits that the federal funds rate, denoted by r, can be expressed as: $r = p + 0.5y + 0.5(p - 2) + 2$

The variable p denotes the inflation rate observed during the course of four consecutive quarters. The variable y represents the percentage deviation of actual Gross Domestic Product (GDP) from the predetermined target.

Thus, following this policy rule, the federal funds rate increases if the inflation exceeds the 2% threshold or if the GDP exceeds the trend. Assuming that both inflation and real GDP are operating at their designated targets, the federal funds rate will remain around the steady-state growth rate, specifically at 4% or 2% in real terms, as posited by Taylor (1993).

According to Taylor's (1993) conclusions, it is imperative not to mechanically adhere to the rule, but rather to use it as a means to improve economic performance. As posited by Blanchard and Fischer (1989), a policy rule can be seen as a resolution to a dynamic optimisation dilemma.

2.2. Monetary Inertia

Based on the specifications provided, it can be inferred that the central bank promptly modifies the interest rate with no regard to past monetary policy measures. (Masciandaro 2023). Hence, the phenomenon of monetary persistence or inertia does not appear to be a noteworthy concern. However, many academics, such as Castelnuovo (2007) and Clarida et al. (1998), claim that prior determinations retain their importance. In fact, the presence of a smoothing parameter could help in establishing credibility and preventing capital market disruptions[1]. Monetary policy persistence or inertia manifests itself in situations where current interest rate decisions are influenced by past decisions. According to this specification, the policy rule takes the general form $i_t = (1-\rho)i_t^* + \rho i_{t-1}$, where i_t is the policy interest rate level in quarter t and is set as a weighted average of the previous quarter's actual value i_{t-1}, and the current desired level, i_t^*, as determined by the Taylor Rule. Indeed, according to Clarida et al. (2000), the smoothing rate parameter ρ is high, suggesting the presence of substantial monetary inertia. From historical data, estimates of ρ are often in the range of 0.8 (Rudebusch 2001), so these

[1] See McCallum, B. T. (1999). Issues in the design of monetary policy rules. In *Handbook of macroeconomics* (p. 1, 1483-1530.), Levin, A., Wieland, V., & Williams, J. (1999). Robustness of simple monetary policy rules under model uncertainty. *Monetary policy rules. University of Chicago Press.*, 263-318 and Clarida, R., Gali, J., & Gertler, M. (2000). Monetary policy rules and macroeconomic stability: evidence and some theory. *Q. J. Econ.*, 115 (1), 147–180.

empirical rules seem to imply a very slow adjustment speed of the policy rate to its fundamental determinants.

Then, given $i^* = r^* + \pi + \alpha(\pi - \pi^*) + \beta(y - y^*)$

And $i_t = (1-\rho)i_t^* + \rho i_{t-1} + \varepsilon_t$,

The new optimal Taylor Rule with interest rate smoothing is going to be:

$$i_t = (1-\rho)\big(r^* + \pi + \alpha(\pi - \pi^*) + \beta(y - y^*)\big) + \rho i_{t-1} + \varepsilon_t,$$

$$i_t = \rho i_{t-1} + r^{*\prime} + \pi' + \alpha'(\pi - \pi^*) + \beta'(y - y^*) + \varepsilon_t$$

Where the reference rate i* follows the standard Taylor rule (2), 0<ρ<1 is the inertia factor and ε_t is an independent and normally distributed random variable with zero mean and finite variance (Masciandaro 2023).

2.3. Forward Looking Taylor Rule

Another debate between academics is whether we should consider a backward- or forward-looking Taylor Rule. The one presented by Taylor (1993) is a backward-looking monetary rule. However, as stated by Clarida et al. (2000), economists rely on expected information, particularly expected inflation, when determining the reference rate. Therefore, they developed a forward-looking monetary rule by employing a Generalized Method of Moments (GMM) estimation. Their assumption is that the central bank exhibits forward-thinking behaviour and possesses perfect control over the interest rate (Clarida, Gali e Gertler 2000). The authors believe that a forward-looking Taylor Rule allows central banks to take into account also additional factors that may impact the market, such as inflation in commodity prices or the spread between short-term and long-term market interest rates. This specification, as a matter of fact, allows the monetary rule to describe the conduct of central banks more accurately. Furthermore, they argue that in the scenario in which lagged inflation, or the linear combination of lagged inflation and output serve as an adequate measure for forecasting expected inflation, then their model can collapse into the original Taylor rule. However, in this case, the coefficients should be interpreted cautiously, as they will also capture the ability of forecasting the economy. Moreover, prior empirical research, including Fourçans and Vranceanu (2004) and Sauer and Sturm (2007), has highlighted the significance of incorporating a forward-looking Taylor rule in the evaluation of the European Central Bank's monetary policy.

2.4. Non linearities in Taylor Rule

Non linearities in the Taylor rule can come both from asymmetric preferences of the policy makers and from non-linear macroeconomic cycles (Caporale, et al. 2018). In line with Caporale's (2018) argument, the actions of central banks can be influenced by the economic phase. The central bank's response is oriented towards inflation management during economic expansions, whereas during economic contractions, its focus shifts towards stabilising output. This has been noted by Cukierman and Gerlach (2003) as well as Ahmad (2016). Surico's analysis (2003) supports the notion that inflation targeting is symmetrical, meaning that the central bank (CB) is equally attentive to both inflation and deflation. However, during economic booms, the CB's response to inflation appears to be more robust than during recessions, as a result of the economy's convex structure. Moreover, the findings obtained by Castro (2011) are noteworthy. The author demonstrated that the European Central Bank (ECB) adheres to the non-linear Taylor rule, whereby it responds to inflation solely when it exceeds 2.5% and reacts to the business cycle only after inflation has stabilised, i.e., significantly below 2.5%. Taylor and Davradakis (2006) argue that the Bank of England employs a non-linear Taylor rule to determine interest rates, despite the institution's symmetrical inflation target. The assertion is further substantiated by the research conducted by Martin and Milas (2013), which provided empirical evidence in favour of a non-linear Taylor rule in the United Kingdom. According to Fourçans and Vranceanu (2004), in the event that the central bank assigns varying weights to negative and positive deviations of inflation and output in its loss function, it may be more suitable to utilise a non-linear Taylor rule to elucidate the conduct of monetary policy.

2.5. Estimation problems

The Taylor rule has been extensively employed in various theoretical and empirical investigations to analyse monetary policy in terms of both descriptive and prescriptive aspects. The research has primarily focused on the efficacy of basic rules in resolving the issue of inconsistency bias[2], their level of optimality[3], and the performance of these rules in different macroeconomic models[4].

According to academics such as Svensson (2003) and Woodford (2001), the effectiveness of simple rules in addressing complex tasks, such as those encountered by central banks, may not always be optimal. Furthermore, their robustness is not always guaranteed owing to the uncertainty surrounding the factual model and potential output levels. There is still an open debate regarding the appropriate course of action for the policy maker when confronted with measurement inaccuracies within the framework of the Taylor rule. According to Orphanides (2001), certain scholars advocate for a more prudent strategy, whereas others, such as Onatski and Stock (2002), argue for a more assertive approach.

Furthermore, as per Rudebusch's (2001) findings, an additional challenge is to differentiate between a policy rule that employs interest rate smoothing and one that incorporates policy shocks that are serially correlated.

The selection of variables to incorporate in the Taylor rule can pose a significant challenge. This holds true from a qualitative perspective, as evidenced by the choice of the CPI index over the core CPI, among other factors, as well as from a methodological standpoint. Indeed, the utilisation of real-time data as opposed to ex-post data is a problem that has been thoroughly examined by Orphanides (2001). The author argues that using real-time data yields disparate outcomes and may result in inaccurate representation of proposed policies. Taylor's rule, as originally proposed in 1993, suggests that the federal funds rates should be determined by the current quarter's output gap and inflation, which is based on the output deflator. However, due to the unavailability of accurate data for these variables until later, the implementation of such a described Taylor rule is not feasible for the Federal Reserve and cannot be considered a

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[2] McCallum, B. (1999a). Issues in the Design of Monetary Policy Rules. *Issues in the Design of Monetary Policy Rules, 1C*.

[3] See McCallum, B. (1999a). Issues in the Design of Monetary Policy Rules. *Issues in the Design of Monetary Policy Rules, 1C*; Svensson, L. E. (2003). What is Wrong with Taylor Rules? Using Judgment in Monetary Policy through Targeting Rules. *Journal of Economic Literature, 41*, 426–77; Woodford, M. (2001). The Taylor Rule and Optimal Monetary Policy. *American Economic Review, Paper and Proceedings , 91*, 232–37.

[4] See Taylor, J. (1999). Monetary Policy Rules. *Chicago: University of Chicago Press*; Isard, P., Douglas, L., & Ann-Charlotte, E. (1999). Simple Monetary Policy Rules Under Model Uncertainty. *International Tax and Public Finance, 6*, 537–77

practical policy to follow. The primary issue at hand pertains to the reliance on ex-post revised data for analytical purposes, which may result in erroneous attempt to identify the past trajectory of policy.

Orphanides and Williams (2002) have identified an additional issue pertaining to the estimation of the long-term interest rate. As a matter of fact, an inaccurate specification of this variable can potentially result in erroneous outcomes.

2.6. Taylor Rule and Central Banks

Prior to delving deeper, it is imperative to understand the way in which the major central banks implemented the Taylor rule. Academics, including Clarida et al. (2000), have conducted an analysis of the historical development of the Taylor rule and the Federal Reserve's adoption of this rule. They focused on the disparities between the pre- and post-Volcker era. A general rule was estimated that considers the federal funds rate as an instrument of monetary policy. The authors confirmed that the Federal Reserve had pursued a highly accommodative monetary policy during the pre-Volcker era, as evidenced by their decision to increase interest rates at a rate lower than the anticipated rise in inflation. In contrast, the Federal Reserve adopted a proactive approach to manage inflation during the Volcker-Greenspan era by methodically increasing short-term interest rates in response to an upsurge in inflation. The circumstances prior to Volcker's appointment, resulted in the accumulation of disparities and the rapid escalation of both inflation and output. Broadly speaking, it is evident that prior to Volcker's tenure, the Federal Reserve did not adhere to a genuine Taylor rule. Subsequently, during the Volcker-Greenspan era, the conduct of the Federal Reserve can be explained by the Taylor rule.

It is important to analyse the conduct of other central banks, with a particular focus on the European Central Bank (ECB). Performing an analysis akin to that carried out on the Federal Reserve is challenging due to the insufficiency of available data, as the European Central Bank (ECB) has been active since 1999. However, as per the findings of Clarida et al. (2000), during the same period, the Bundesbank adhered to a Taylor rule, and the central banks of France, Italy, and the UK followed a monetary policy that was in line with that of Germany. Sauer and Sturm (2003), scholars in the field, conducted a study on the development of monetary policy in the European region. Their findings indicate that the European Central Bank's response, when utilising a forward-looking Taylor rule, can be deemed stabilising and comparable to that of the Bundesbank. However, it is worth noting that the output gap is given a higher degree of significance.

Paloviita et al. (2017) conducted a study that examined the monetary effects of the European Central Bank (ECB). The study centred on examining the degree of adherence of the European Central Bank (ECB) to the Taylor rule, including the specific weighting given to inflation and the output gap, as well as the precise inflation target. They arrived at strong findings by employing reaction functions that considered several factors, including the effective inflation target at varying levels, cyclical variables, a natural rate that changes over time, varying degrees of historical and predictive information in the real-time data, and asymmetry.

They found that the European Central Bank (ECB) adheres to the Taylor rule and its response mechanism considers previous inflation trends and expected inflation prospects. Furthermore, it can be seen that the European Central Bank's de facto inflation target is significantly below the 2% threshold, with estimates ranging from 1.6% to 1.8%. The results indicate that the European Central Bank's (ECB) policy response adheres to the fundamental principles of optimality, in line with its mandate.

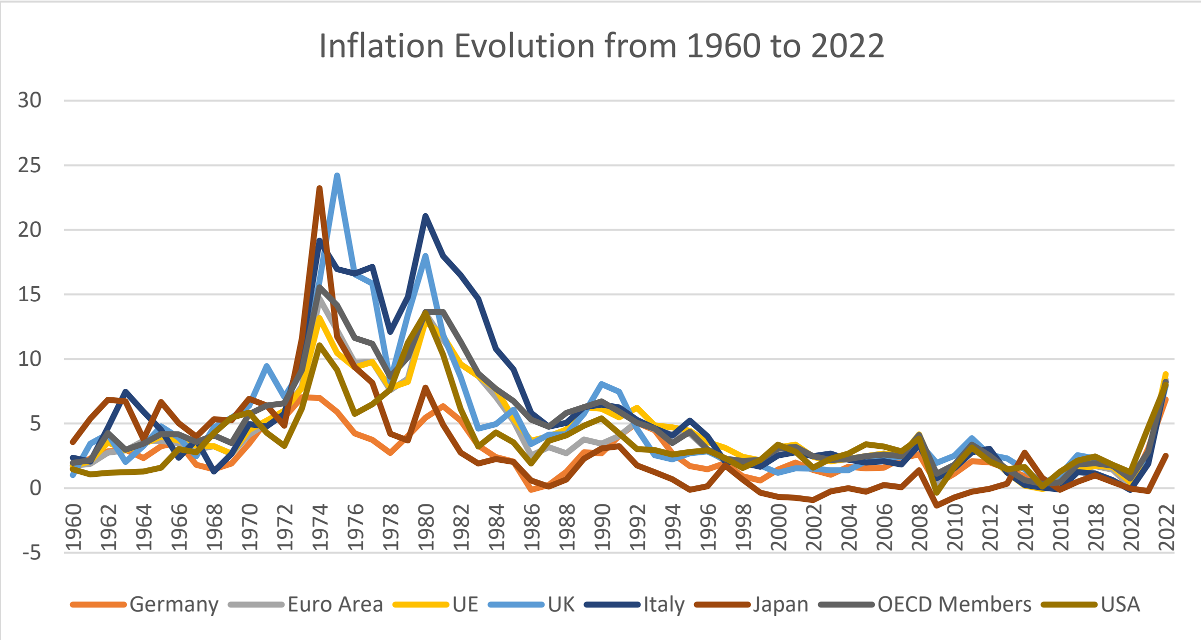
2.7. Zero Lower Bound

Inflation rates escalated significantly during the 1970s and 1980s, mainly attributed to the collapse of the Bretton Woods system and the occurrence of two major oil price shocks. However, after this period of sharp escalation, there was a significant and sustained decline in inflation. This downward trend can be seen in Figure 1[5], which illustrates how the inflation

[5] Source: OECD database

rate reached unprecedented levels in the 1970s and 1980s and then gradually declined. Consequently, these adjustments since the 1980s have

led to a simultaneous decline in real interest rates. The mid-1980s and the previous decade were characterised by a decline in the volatility of macroeconomic variables, particularly in the US, during a period known as the 'Great Moderation'. In particular, nominal and real rates fell



steadily. Following the financial crisis in the United States (2007-2008), which then spread worldwide, and the sovereign debt crisis in the European Union (2010-2013), the downward trend in interest rates strengthened. This was the result of extremely accommodative monetary policies instituted in response to the crisis and investors' demand for safe assets.

The nominal interest rate moved near to zero and eventually even fell below zero, reaching the zero lower bound (ZLB), therefore entering in a liquidity trap.

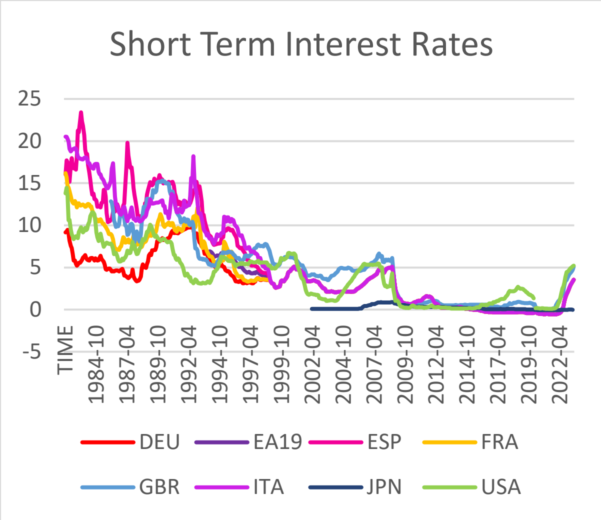


Figure 1: Short Term Interest Rates
Source: OECD

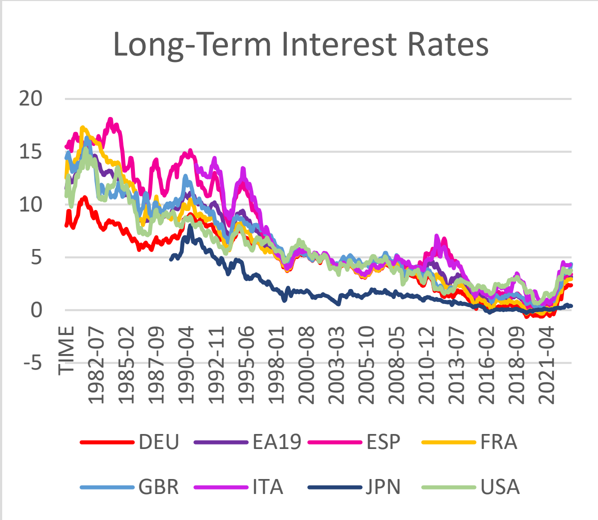


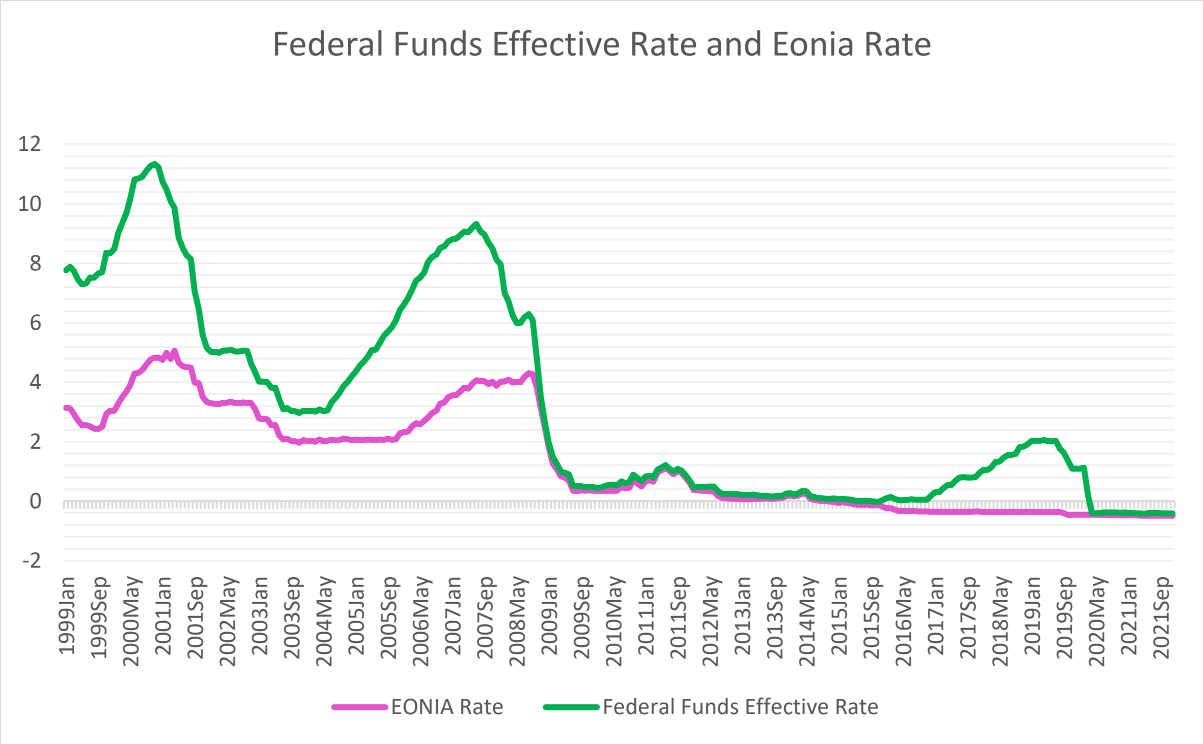
Figure 2: Long Term Interest Rates
Source: OECD

[6] [7]

The ZLB refers to the lower limit of nominal interest rates, at which point it becomes challenging or unfeasible for central banks to further stimulate the economy through conventional monetary policy. The zero lower bound was reached by the European Central Bank in 2013 and, according to many academics, is considered a significant problem for advanced economies (Bauer e Rudebusch 2016).

[6] Short-term interest rates are the rates at which short-term borrowings are effected between financial institutions or the rate at which short-term government paper is issued or traded in the market. Short-term interest rates are generally averages of daily rates, measured as a percentage. Short-term interest rates are based on three-month money market rates where available. Typical standardized names are "money market rate" and "treasury bill rate. (OECD)

[7] Long-term interest rates refer to government bonds maturing in ten years. Rates are mainly determined by the price charged by the lender, the risk from the borrower and the fall in the capital value. Long-term interest rates are generally averages of daily rates, measured as a percentage. These interest rates are implied by the prices at which the government bonds are traded on financial markets, not the interest rates at which the loans were issued. In all cases, they refer to bonds whose capital repayment is guaranteed by governments. Long-term interest rates are one of the determinants of business investment. Low long-term interest rates encourage investment in new equipment and high interest rates discourage it. Investment is, in turn, a major source of economic growth. (OECD)



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Bauer and Rudebusch (2016) argue that, when rates reach the ZLB, it is not possible to set negative rates without incurring to arbitrage opportunities. Indeed, investors may decide to hold currency rather than debt securities that offer negative interest rates. The zero lower bound, therefore, imposes a constraint on the monetary policy thereby restricting the autonomy of central banks in the pursuit of their objectives. However, some scholars, including Benoît Cœuré (2015), suggest that the effective lower bound is significantly below zero. According to Cœuré, several transactions can take place at negative interest rates without negatively affecting the economy (B. Cœuré 2014), so negative short-term rates do not represent a challenge for monetary policy. Furthermore, Cœuré describes various ways to further reduce interest rates in the long run, while remaining within the effective lower bound, which also takes into account the costs associated with holding cash. He argues that unconventional monetary policy, through asset purchases and forward guidance, may yield comparable outcomes to negative interest rates (B. Cœuré 2015).

[8] Source: Federal Reserve Economic Data, https://fred.stlouisfed.org for the Federal Funds Effective Rate
European Central Bank - Statistical Data Warehouse
https://sdw.ecb.europa.eu/quickview.do?SERIES_KEY=194.RTD.M.S0.N.C_EONIA.E&periodSortOrder=ASC, for the EONIA rate.

2.8. Shadow rates

Several academics hold the view that monetary aggregates can serve as a tool for comprehending the developments in monetary policy. In order to achieve feasibility, it is necessary that the aggregate considered exhibits stability and predictability over a considerable period of time, as stated by Bernanke (2006). As per the findings of Bernanke and Blinder's (1992) study, certain policy makers argue that the federal funds rate can function as a reliable measure for anticipating upcoming variations in real macroeconomic variables.

Another aspect that has been highlighted is the comprehension of the mechanism through which monetary policy impacts actual economic activity. In order to address this issue, many researchers have employed a VAR (vector autoregressive) framework to isolate and identify monetary shock. However, this particular framework experiences challenges when faced with a scenario close to the zero lower bound. This phenomenon can be attributed to the tendency of consumers to withhold available funds during liquidity traps, resulting in the ineffectiveness of monetary policy. As a result, academics are presently trying to develop a metric that can effectively evaluate and comprehend the effects of these policies.

The challenge lies in developing a comprehensible and straightforward monetary policy metric that effectively assesses non-interest rate policy measures during periods of zero lower bound (ZLB). Economists seek to create a comprehensible measure that will continue to operate even in the scenario of the non-existence of the zero lower bound (Lombardi e Zhu 2014).

The authors Chen et al. (2012) proposed a solution that involved utilising term and corporate spreads within the United States as a proxy for Federal Reserve (FED) monetary policy. The study uses this methodology to investigate the impacts of quantitative easing. The research conducted by the authors reveals important cross border effects. Nevertheless, the present research employs non-monetary instruments that display substantial fluctuations in contrast to monetary aggregates and this present greater challenges in relation to predictability. Hence, it is improbable that these indicators would function as long-lasting measures of monetary policy.

Meaning and Zhu (2011) proposed an alternative methodology to evaluate the impact of non-conventional monetary policies. This approach involves extracting data from the Federal Reserve's balance sheet, specifically the volume and maturity of Treasury securities and private securities. However, these statistics are insufficient in effectively capturing the substantial monetary growth that the Federal Reserve has reached.

The first who introduced the concept of shadow rates was Black (1995). In his influential paper, he proposed a sophisticated method for computing shadow rates.

The concept of shadow rates was formulated with the aim of evaluating the actual monetary policy stance and comprehending the impacts of unconventional monetary policies. Black demonstrated the similarity in structure between shadow rates and short-term interest rates, while highlighting the absence of lower bound constraints for shadow rates. This was achieved through the creation of an interest rate option pricing model, which relied on the assumption of normal distribution of interest rates. This model was introduced as the Shadow Rate Term Structure Model (SRTSM).

Subsequent to Black's elaboration, over time three main methodologies were developed to construct shadow rates.

Krippner (2015) employed a two-factor affine term structure model. The two-factor model is a theoretical approach used to understand the structure of interest rates. Krippner (2013) proposes a framework that is based on the use of Gaussian affine term structure models (GATSM), while imposing the zero lower bound on nominal interest rates. The GATSM is a two-factor model that is used to describe the dynamics of bond yields. In particular, these models allow the identification of short-term interest rate expectations and maturity risk premia. The author relies on Black's (1995) model, which assumes that individuals have an alternative, namely cash, so that nominal returns cannot be negative. Krippner then develops a CAB-GATSM (currency adjusted bond) which is an extension of Black's model that also includes the zero lower bound. This model is based on the use of short-term shadow rates, which are precisely an estimate of what nominal rates could be if they could go above zero. The CAB-GATSM framework allows shadow rates to be used to correctly discount future bond cash flows, taking into account the effect of the zero lower bound. This allows for closed-form expressions of forward rates, which provide an accurate estimate of long-term interest rates corrected for the lower bound context.

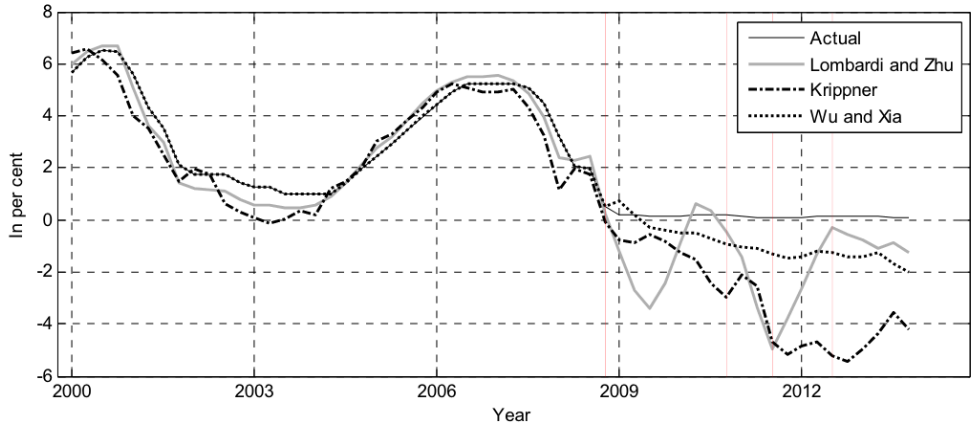
An alternative approach is presented by Wu and Xia (2016) who employ a three-factor model. Also, this one utilised Black's SRTSM shadow rate to construct a new metric for assessing the monetary policy stance in situations where the effective interest rate is constrained to 0. A factorial vector autoregression (FAVAR) model with three factors was employed to investigate the impact of unconventional monetary policy on the real economy. The third factor considered is the level of long-term interest rates.

Finally, a third modality is that employed by Lombardi and Zhu (2014)[9]: the dynamic factor model. The researchers assembled a comprehensive dataset containing variables that represented the majority of monetary policy actions. Subsequently, the data is summarised using a dynamic factor model. A dynamic factor model assumes that latent factors exhibit temporal variation and, in consideration of the substantial variability in the dataset, aims to reduce its dimensionality. The model is applied until the federal funds rate reaches the zero lower bound (ZLB), and the optimal specification is selected. Subsequently, the researchers obtain a set of shadow rates by employing the dynamic factor model, assuming the absence of the federal funds rate. Hence, the shadow rate employed by the researchers serves as an indicator of non-traditional monetary policy measures and takes into account the monetary and financial elements used to construct the dataset.

According to empirical evidence (Anderl e Caporale 2022) it is believed that two-factor models provide a more accurate estimation of the shadow rate with respect to the federal funds rate, especially during times of zero lower bound. Shadow rates derived from yield curve parameters

[9] Lombardi, M. J. & Zhu, F., 2014. A shadow policy rate to calibrate US monetary policy at the zero lower bound.

often exhibit substantial noise due to their dependence on market interest rate expectations, which may be influenced by non-monetary policy factors.

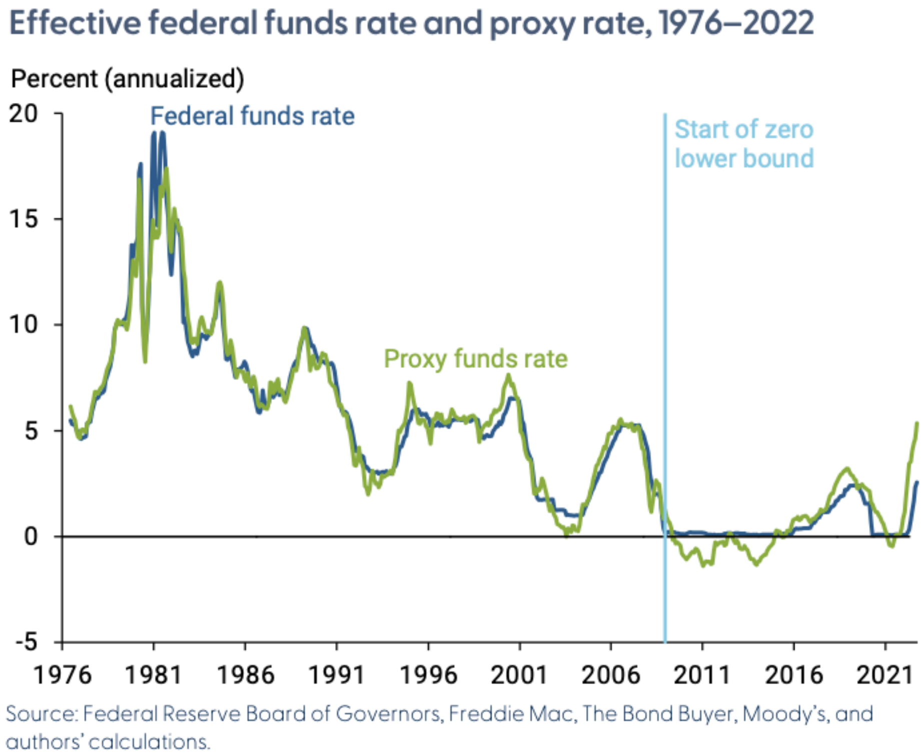


[1] The red vertical lines correspond to the dates of introduction of the major asset purchase programmes implemented by the Federal Reserve: LSAP1 (November 2008), LSAP2 (November 2010), MEP (September 2011) and LSAP3 (September 2012).

Source: See Footnote 9

In contrast, the dynamic factor model employed by Lombardi and Zhu is deemed extremely reliable, particularly in times of non-traditional monetary policy, due to its ability to extract data from the central bank's balance sheet.

Another approach to consider is that of Choi et al. (2022). Utilizing the statistical technique of principal component analysis, they produced a proxy for the federal funds rate. They extracted the common fluctuations between the 12 financial indicators they considered, such as treasury rates, mortgage rates and borrowing spreads. They then mapped the components with the federal funds rate to infer pre-2008 relationships, which were subsequently utilized to develop the new shadow rate.



This concept was then adopted by Volpi (2023), who produced a shadow rate for the European union utilizing a database of EU financial and market variables, based on the technique developed by the San Francisco Fed.

Several academic papers have contributed to the advancement of new methods to compute shadow rates and their evaluation in relation to the implicit rates of monetary policies. Lombardi and Zhu (2014) conducted an analysis to estimate the shadow rate for the United States, utilising the aforementioned methodology. They subsequently evaluated the coherence between the shadow rate and the federal funds rate during both ZLB and non-ZLB periods, thereby verifying the efficacy of the shadow rate as an accurate indicator of monetary policy within the Taylor rule. Furthermore, the authors demonstrated that the inclusion of the shadow rate in a VAR model improves the precision of shock estimation compared to relying on the reference rate.

Wu and Zhang (2019) have proposed a novel New Keynesian model that incorporates the shadow rate as a measure of both conventional monetary policy and liquidity trap scenarios. In the scenario where the zero lower bound is binding, the shadow rate will assume a negative value and will account for non-traditional monetary policies like credit easing and quantitative easing (QE). Similarly, their findings indicate that the shadow rate effectively captured the prevailing financial conditions and balance sheet of the Federal Reserve.

Ellington (2021) extended the aforementioned model, but with the assumption of a binding zero lower bound. A VAR model with variable coefficients was utilised to examine the efficacy of the shadow rate in signalling monetary policy. The results indicate that the shadow rate is an effective indicator. The study revealed that the reference rate exhibits greater sensitivity to shocks in GDP and inflation compared to the shadow rate.

Bauer and Rudebusch (2013) used a dynamic term structure model to derive the shadow rate and they observed their similarity to policy rates used in the Taylor rule. They stated, however, that shadow rates should not be used to evaluate the stance of monetary policy due to their reliance on the model and the limited information provided by the short end of the term structure. In their study, Bernanke et al. (2019) employed the shadow rate as a means of assessing ten distinct monetary rules in the context of the United States economy. The study revealed that the incorporation of shadow rates in the rule exhibits superior performance compared to the classical Taylor's rules.

After delving into the concepts of the Zero Lower Bound and unconventional monetary policies, I want to focus on the specific landscape of the European Central Bank (ECB) and its responses to these challenges. In the next chapter I will elaborate on the role of the ECB and its approach in the context of the Zero Lower Bound, looking at unconventional policies and their influence on the ECB's strategy. This exploration will provide a more comprehensive perspective on the implementation and measurement of monetary policies in the euro area during a period of near-zero interest rates, emphasising the importance of the shadow rate measurements and methodologies discussed in this chapter.

3. The European Central Bank

3.1 Overview

The European Central Bank (ECB) follows a single mandate, where the goal is maintaining price stability in the area (ECB 2023). It aims to target a 2 percent inflation rate over the medium term, with a symmetrical approach (BIS 2022).

The ECB operates as part of the Eurosystem, working alongside the national central banks of the euro area countries. According to the Article 105.2 of the Treaty of Maastricht, also known as Treaty of European Union (TEU) (1992), the European System of Central Banks (ESCB) is responsible for several tasks. These include: (i) defining and implementing the monetary policy of the Community, (ii) conducting foreign exchange operations, (iii) managing the official foreign reserves of the Member States, and (iv) promoting and regulating the operations of payment systems.

The ECB relies on a two-pillar approach to make monetary policy decisions (Holm-Hadulla et al., 2021). The first pillar pertains to the economic analysis, which looks at factors such as the economic growth, the employment, the interest rates, and the international trade. The goal is to understand the evolution of the euro area economy over the medium term and to identify possible price pressures. This analysis enables the Board of Governors to make appropriate monetary policy decisions to maintain price stability in the euro area in the long run.

The second pillar concerns the monetary analysis, which focuses on the assessment of the volume monetary aggregate M3 and its impact on inflation. This entails the examination of long-term interest rates, as well as the expansion or contraction of the monetary base. The aim is to identify any inflationary pressures arising from the growth of the money supply, and to take appropriate monetary policy measures to control inflation.

These two pillars represent a complementary approach to assessing the economic outlook and the inflation trends in the euro area. The central bank periodically releases regular economic reports and forecasts to explain its policies, guide inflation expectations, facilitating the achievement of the price stability objective over the medium term.

3.2 Unconventional Monetary Policies

Central banks employ various instruments to implement monetary policy, with policy rates being a key tool. The ECB relies on three key rates: the main refinancing operations rate (MRO), which is the interest rate at which the central bank provides loans to commercial banks for one week; the deposit rate, and the marginal lending rate (ECB, 2023). The ECB uses the MRO rate as a primary instrument to achieve its two main goals: maintaining price stability and promoting sustainable economic growth in the euro area. Indeed, in response to rising inflation, which reached 8.1 % in May 2023, the highest level since 1985, the European Central Bank (ECB) decided to raise the three main interest rates. As of September 20th 2023, the new rates are 4.50% for the main refinancing rate, 4.75% for the marginal lending rate and 4% for the deposit rate.

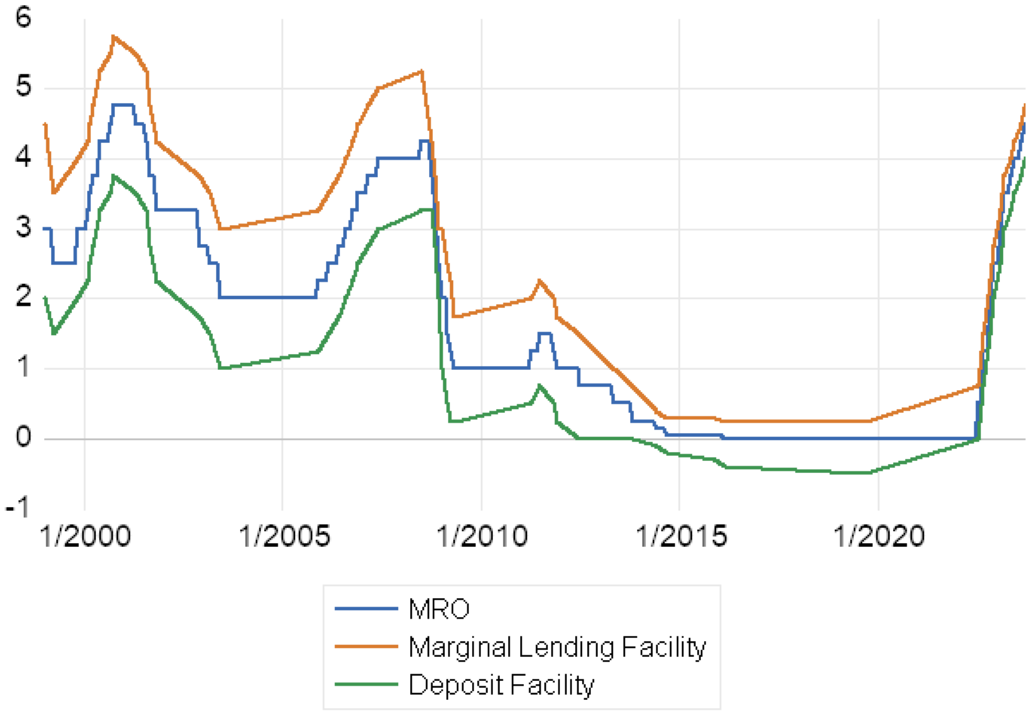


Figure 3
Data source: ECB Portal
Key interest rates for the ECB

Specifically, the central bank targets these rates to influence the Euro Overnight Index Average (EONIA), the benchmark interbank interest rate for overnight lending transactions in euros, although the ECB has recently elaborated the Euro Short-Term Rate (€STR), which measures the wholesale unsecured overnight borrowing costs of a sample of euro-area banks (Banca D'Italia 2021).

If the main tool to influence the monetary stance, as we said, is the use of the key interest rate, during economic downturns conventional monetary policies may prove insufficient. For instance, at the onset of the 2007 financial crisis, the ECB was forced to adopt unconventional measures. The central bank took steps such as providing the necessary liquidity on an overnight basis. In addition, it conducted fine-tuning operations (FTO) at fixed overnight rates and granted multiple long-term loans (LTROs) to banks in dire straits, while also working with the Federal Reserve (Fed) to provide European counterparties with US dollar liquidity through a swap arrangement (ECB, European Central Bank | Eurosystem 2018). This was followed by an aggressive policy of reducing the ECB's reference rate to 1% to facilitate the uniform transmission of monetary policy impulses between the euro area member states (Hartmann and Smets, 2018). With Mario Draghi’s leadership as ECB president, other unconventional measures were taken, again with the aim of tackling deflation risks and bringing inflation back to levels close to 2%.

We can distinguish three main types of unconventional monetary policies:

- **Balance sheet policies**: this category includes measures that focus on the size of the central bank's balance sheet. In particular, within this group we distinguish new landing facilities and large-scale asset purchases (LSAPs). The former includes facilities that have similar characteristics to discount window and deposit facilities but with different maturities and new accepted collaterals. These facilities often directly target markets or financial instruments, in this case they are called credit policies. The latter involve large scale asset purchases (LSAPs), commonly referred to as quantitative easing (QE). In this case, the central bank acquires large quantities of assets, including government bonds, thereby increasing reserves. Such purchases are called outright purchases, and they aim to lower long-term yields, ease financing conditions for households and businesses, and thus stimulate the economy and help bring inflation rates back in line with the central bank's price stability objective.
- **Forward guidance**: this entails providing explicit information on the expected policy rate trajectory to directly influence long-term rates.
- **Negative or zero interest rate policies**: these policies involve setting interest rates at or below zero percent.

As the US financial system is based on capital markets, large scale asset purchases, and thus QE, played a dominant role from the beginning. In contrast, the ECB's financial system is mainly bank-based, so the first measures taken were liquidity provisions such as LTROs, later followed by LSAPs.

If we take a broader view of the action taken by the ECB, it becomes evident that their initial unconventional monetary policy step occurred in 2009 with the introduction of the Covered Bond Programmes (CBPP). This measure was implemented to help support the mortgage market and was then followed by the Security Market Programmes (SMP) in 2010. However, the first outright transactions were done in 2014, with the implementation of the Asset-Backed

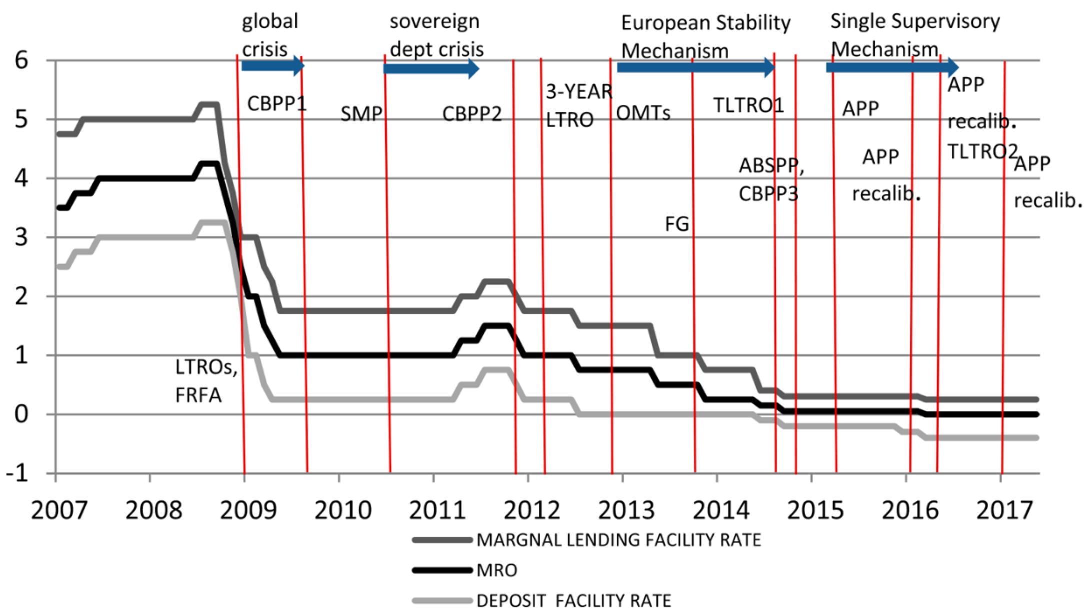


Figure 4

Data source: Ouerk, S., Boucher, C., & Lubochinsky, C. (2020). Unconventional monetary policy in the Euro Area: Shadow rate and light effects. Journal of Macroeconomics, 65, 103219.

ECB's unconventional monetary policies at the zero lower bound

Securities Market Program (ABSPP) and the third round of CBPP. The former, which consisted of the purchases of debt securities backed by consumer and real estate loans, was then followed by the Public Sector Purchase Programme (PSPP) which involves purchasing of public sector securities through the secondary market, and was supposed to be continued until at least September 2016, but was then extended until 2019. In the graphic above, we can see the implementation of the UMP and their effect on the key interest rates.

With the Asset Purchase Programme, the ECB joined the other major central banks in using large-scale purchases to provide monetary policy accommodation near the effective lower bound of interest rates. In June 2016, the ECB started with the fourth APP, the corporate sector purchase programme, i.e., the purchase of corporate sector bonds aimed at improving the financing conditions of the real economy. Despite the ECB committed itself to purchase EUR 60 billions of public and private sector bonds per month from March 2015 until at least September 2016, subsequent rounds of the APP brought the final portfolio size to around EUR 2.6 trn by the end of net purchases in December 2018, equivalent to around 25 per cent of euro area GDP as can be seen in the graphics below.

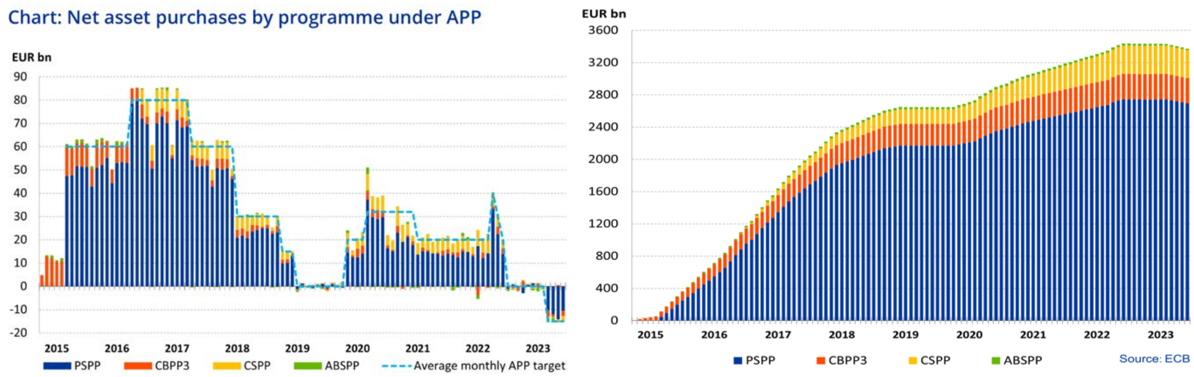


Figure 5
Data source: ECB Report
On the left the net purchases by programme under APP: the stock of Eurosystem APP bonds stood at €3373 billion at the end of June 2023. On the right the cumulative net purchases.

3.3 Measuring unconventional monetary policies

As mentioned, the unconventional measures were all implemented with the goal of reducing financing costs for businesses and households, stimulate inflation, and support economic growth. Many economists, however, have attempted to understand and especially quantify the effect of the unconventional monetary policies[10].

Several approaches have been utilized to achieve this objective, primarily by examining the changes in the size of balance sheet. However, this method failed to account for the impact of

[10] See Eser, F., & Schwaab, B. (2016). Evaluating the impact of unconventional monetary policy measures: Empirical evidence from the ECB′ s Securities Markets Programme. *Journal of Financial Economics*, *119*(1), Baumeister, C., & Benati, L. (2010). Unconventional monetary policy and the great recession-Estimating the impact of a compression in the yield spread at the zero lower bound, or Ouerk, S., Boucher, C., & Lubochinsky, C. (2020). Unconventional monetary policy in the Euro Area: Shadow rate and light effects. *Journal of Macroeconomics*, *65*, 103219.

official announcements. This is due to the fact that when a central bank announces the implementation of certain measures, the market anticipates these changes and responds beforehand. This phenomenon was observed in 2015 with the introduction of the QE. Notably, there was a gradual decline in the spread between 10-year treasuries and 3-month treasuries prior to the announcement as can be seen in the figure below.

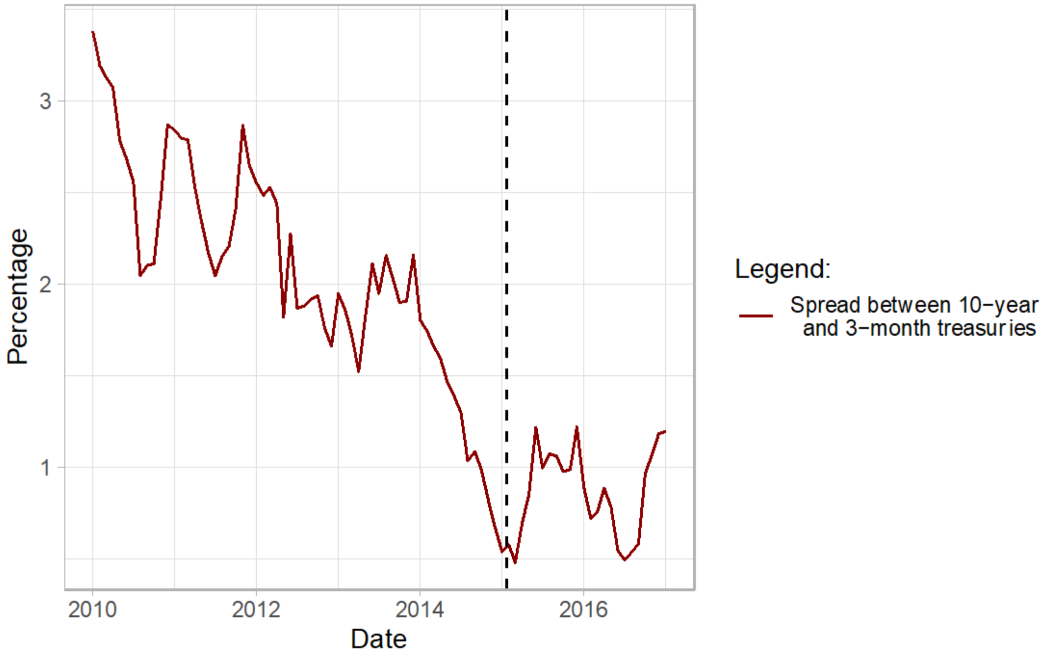


Figure 6
Data source: ECB Portal
The vertical line represents the announcement of the QE.

Another study conducted with this goal is the analysis of the impact of the Security Market Programme's by Eser and Schwaas (2016). Their analysis focuses on the effect of the SMP on bond yields, employing a VAR model and a difference in difference (DID) approach. They focus on five European countries (Greece, Ireland, Italy, Portugal, and Spain) where the SMP was strongly implemented and later replaced by the OMTs program in 2012. Their findings indicate that the unconventional measures had a significant impact on the sovereign bonds yield, reducing them by an average of 3 basis points for every 100 basis points of sovereign bonds purchased by the ECB. Additionally, the SMP also succeed to reduce bond yield volatility and tail risk. However, the complexity of the measure implemented doesn't allow to fully capture all the effect, making possible that the SMP had other aspect that were not properly analysed in the paper.

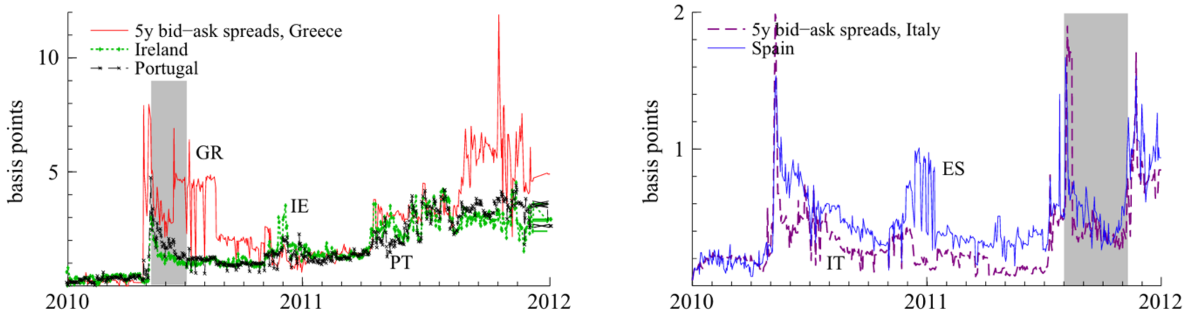


Figure 7
Data source: Eser, F., & Schwaab, B. (2016). Evaluating the impact of unconventional monetary policy measures: Empirical evidence from the ECB' s Securities Markets Programme. Journal of Financial Economics, 119(1), 147-167.
They represent the impact on the CDS-bond basis. They are at the five-year maturity for Greek, Irish, and Portuguese bonds (left panels) and for Italian and Spanish bonds (right panels) from 2010–12. The shaded areas mark frequent purchases in these periods. CDS data are from CMA via Thomson Reuters.

An alternative approach involves considering financial market prices, since they adapt promptly to changes and new information, making them suitable for capturing shocks, similar to the methodology employed by Baumeister and Benati (2013). In their approach, they substituted the MRO at the zero lower bound with the interest rate spread to capture the impact of the unconventional monetary policies on the long-term rates in a Bayesian time-varying parameter structural vector autoregressive model.

Many economists suggest that a valid alternative to these approaches involves the use of shadow rates. This concept reflects the ECB's additional stimulus through unconventional policies and can be incorporated in VAR models to replace the short-term interest rate at the zero lower bound. Utilizing the shadow rates enables the observation of the effects of quantitative easing (QE) and forward guidance as much as quantitative tightening on various aspect, including the yield curve, the unemployment rate, the GDP, or the consumer price inflation (Ouerk et al., 2020).

While the rationale behind this concept and its application were previously discussed in the preceding chapter, it is crucial to emphasize the practicality of this instrument. In particular, according to Andrea Volpi (2023), the shadow rate might offer a better representation of the monetary policy's effects, as it is constructed on financial indexes. Moreover, its utility is supported by research, such as studies like the one conducted by Choi et al (2022), which suggests that incorporating the shadow rate into a Taylor rule might give a more comprehensive perspective of the monetary stance.

Especially since the end of 2021, there has been an increasing trend of interest rates in both the United States and Europe, raising substantial concerns that monetary policy might be more restrictive than implied by the reference rates. Furthermore, with the implementation of quantitative tightening (QT) already in progress, which involves shrinking the central bank's balance sheet and withdrawing liquidity from the financial system, the challenge of accurately assessing the policy stance becomes even more significant. Therefore, it is imperative to question the effectiveness of a traditional Taylor rule in accurately describing the trajectory of monetary policy, especially in light of these developments.

In the next chapter, I will examine the effectiveness of different Taylor rules, including the one with Shadow Rates, to assess which approach performs best in the euro area context. This will allow us to better understand the role of Shadow Rates and unconventional policies in addressing economic and monetary challenges in the euro region.

4. Data

In this section I will present the data utilised in my analysis. The reference period of my analysis goes from 1999Q1 to 2023Q1. The in-sample-period spans from 1999Q1 to 2018Q1, consisting of 295 monthly observations for six key variables. All data series are real-time and derived from the ECB statistical Warehouse.

1) €STR (Euro Short-Term Rate)
2) HICP (Harmonised Index of Consumer Prices)
3) Real GDP growth
4) Shadow rate
5) Instrumental variables

4.1 Reference rate

As a measure for the nominal short-term rate, I used the Euro Short-Term Rate (€STR). As already presented in chapter 3.1, the €STR is the European Central Bank's benchmark interest rate introduced on 2nd October 2019 to replace the Eonia rate. The €STR acts as a complement to the benchmark indices produced by the private sector and their reserve (fall-back) rate and measures the cost of unsecured wholesale funding with one-day maturity of a sample of euro area banks (Banca D'Italia 2021). It is published on each business day of the TARGET2 system at 8 a.m. on the ECB's website. The data concerning the €STR were extracted from the Eurostat dataset but are available from 2019. Before that time, the official reference rate was the EONIA rate. Thus, for the period until the 30th of September 2019, the €str is calculated as the EONIA rate minus a spread of 8.5 basis points. Figure 4.1 illustrates the development of the euro short term rate.

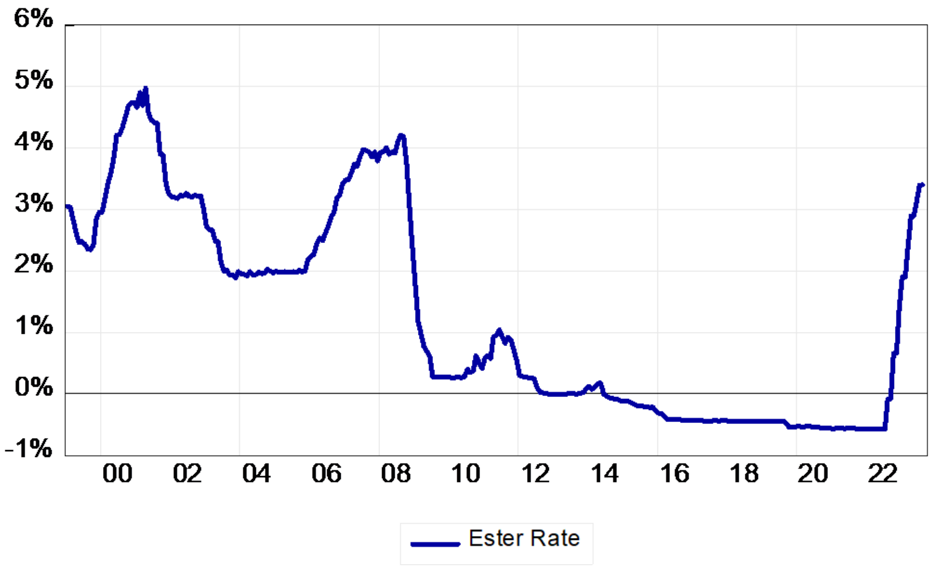


Figure 4.1
Data source: ECB Portal
in the picture, the trend of the €STR from 1999 to 2023

4.2 Shadow Rate

I utilized two distinct shadow interest rate measures: one is developed by Wu and Xia (Wu e Xia 2016), while the other one is created specifically for the European Union by Andrea Volpi in 2023 (Volpi 2023). The data pertaining to Wu and Xia's shadow rate were obtained from their website[11]. This rate is derived as follows: it is based on one-month forward interest rates commencing n years from the present, with *n* taking values of ¼, ½, 1, 2, 5, 7, and 10 years, estimated using the parameters of the Svensson and Nelson-Siegel models. In their paper "Measuring the macroeconomic impact of monetary policy at the zero lower bound" (2016), they introduce the shadow rate as a function of three unobservable factors following a VAR (1) process. Both the shadow rate and the latent factors are estimated using a Kalman filter, which offers a linear approximation due to the non-linear nature of these factors. The forward rate they compute can be approximated using equation 4.1:

$$f_{n,n+1,t}^{SRTSM} = \underline{r} + \sigma_n^{\mathbb{Q}} g\left(\frac{\alpha_n + b_n' X_t - \underline{r}}{\sigma_n^{\mathbb{Q}}}\right)$$

$$\text{where } (\sigma_n^{\mathbb{Q}})^2 \equiv \mathbb{V}ar_t^{\mathbb{Q}}(s_{t+1}) \quad (4.1)$$

Where the function $g(z) = z\Phi(z) + \varphi(z)$ is formed by a normal cumulative distribution function and a normal probability density function.
For further information on the computation of this shadow rate I refer to the paper written by Wu and Xia (2016)[12].

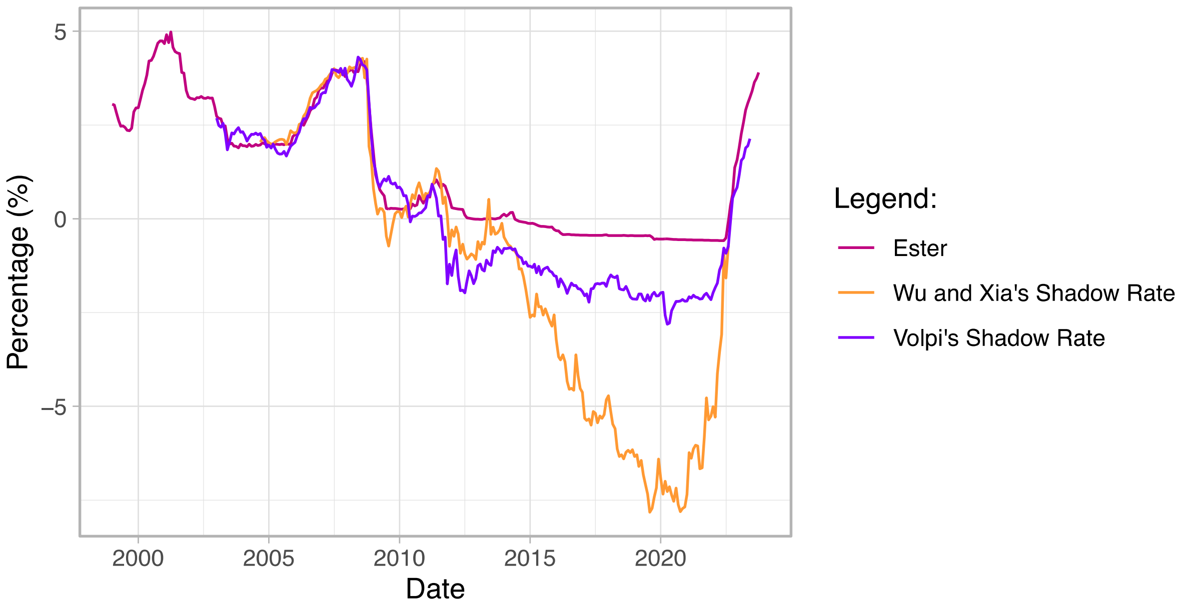
As for the second rate employed, Volpi (2023) developed a shadow index of monetary policy specifically for the Eurozone. The indicator, which aggregates a dataset of credit and financial market factors and is calibrated to overnight rates (€STR), was created following the Choi et al. (2022) methodology. He developed a comprehensive set of 29 interest rate metrics, which cover different aspects of monetary policy, including conditions in corporate and government bond markets, as well as credit conditions for businesses and households. To synthesize the available information and identify common patterns among different sets of information, he focused on the two main components that explain around 80% of the variability. Subsequently, a relationship is established between the vector of principal components and the €STR, calculated as previously outlined, until May 2009. Employing the derived coefficients, he

[11] https://sites.google.com/view/jingcynthiawu/shadow-rates
[12] Wu, J. C., & Xia, F. D. (2016). Measuring the macroeconomic impact of monetary policy at the zero lower bound. *Journal of Money, Credit and Banking*, *48*(2-3), 253-291.

conducted a regression analysis, involving the €STR as the dependent variable, and the two principal components, yielding the shadow effective policy rate[13].

As previously mentioned in the second chapter, there are alternative indicators already available to gauge the actual level of expansiveness of monetary policy. However, many of these indicators are designed in such a way that they align with the federal funds rate when the policy rate is above zero. One such example is the shadow index developed by Wu and Xia (2016). Volpi (2023), on the other hand, considers these indicators unsuitable for effectively tracking the progress of policy tightening in situations where official interest rates are increasing. In contrast, the index formulated by Choi et al. (2022), and consequently the one developed by Volpi (2023), draws upon a more comprehensive dataset, imposes no restrictions on sign, and does not suffer from theoretical arbitrage constraints. Hence, it is deemed to be a more suitable choice for application across diverse contexts (Volpi 2023).

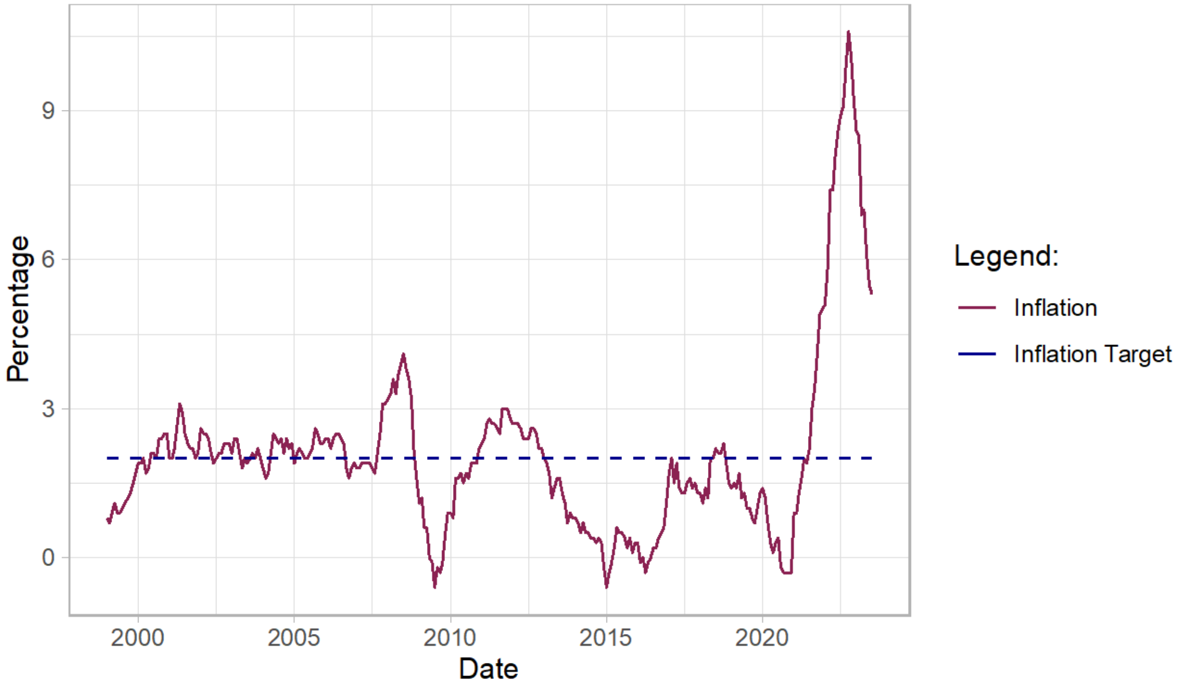
For completeness I included both of them in the analysis.



4.3 HICP

[13] Extracted by: Volpi, A. *Uno Shadow Rate per monitorare la restrizione monetaria.* Intesa Sanpaolo, 2023.

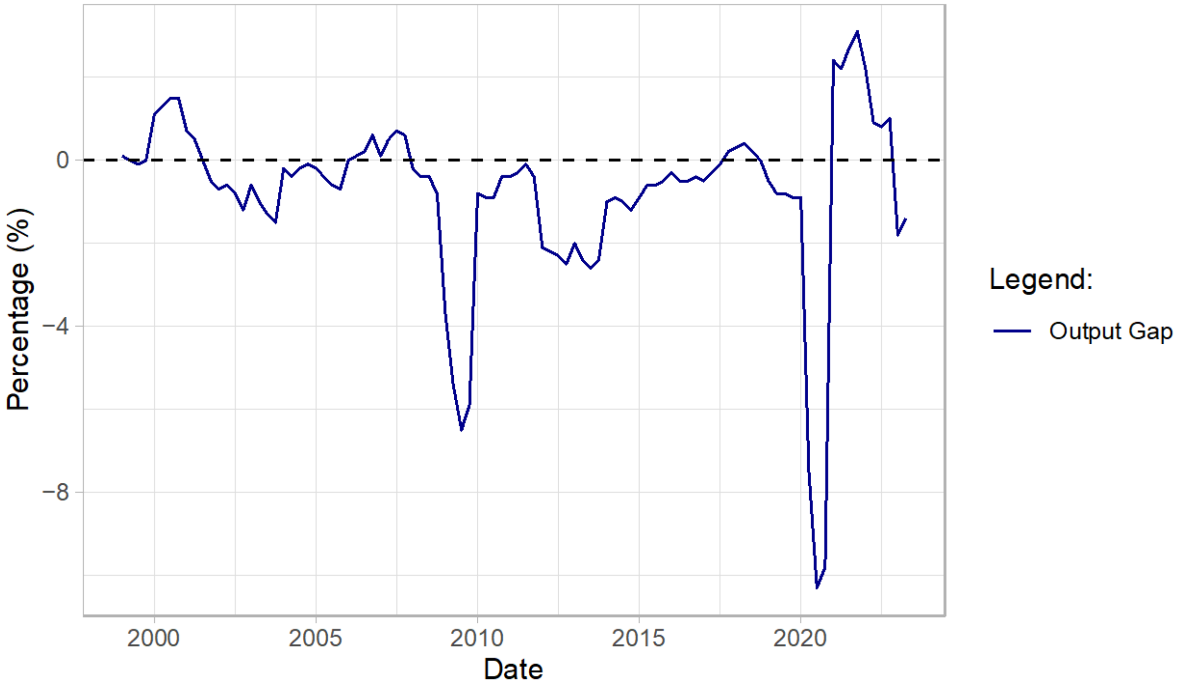
According to the European Central Bank definition, inflation is defined as the annual percentage change in the Harmonised Index of Consumer Prices (HICP) of the euro area. I will use a prospective measure, published quarterly. The prospective version is assumed to be the mid-point estimate of one-year expected inflation (HICP) (Passamani, et al., 2020). I chose to use 1-year expected inflation, since 5-year expectations are pegged to the medium- to long-term target of 2%. This constant characteristic for the entire sample would make this indicator unsuitable. As already mentioned in the literature review, the use of forward-looking inflation allows the ECB to also consider other aspects such as asset prices; moreover, compared to the backward-looking variable, it is less prone to inertia, considering that the transmission mechanism of monetary policy is subject to lags. However, a long-term perspective might have high estimation errors.



4.4 Real GDP growth

According to the standardized European System of national and regional Accounts 2010, real GDP growth is defined as the annual percentage change in real GDP (in volume) of the area. As with inflation, I will use a forward-looking measure. The forward-looking version is again the average point estimate of real GDP growth one year earlier. It is available in real time for the ECB.

I will take into account the deviation of GDP from an assumed long-term target of 2%, as suggested by Gali (2004), who proposed a growth range between 2 and 2.5%. Given the recent growth prospects for the eurozone, I have chosen to use the lower limit. In fact, according to the Survey of Professional forecaster, GDP growth expectations in the third quarter of 2023 stood at 0.6% for 2023, 1.1% for 2024 and 1.5% for 2025[14].

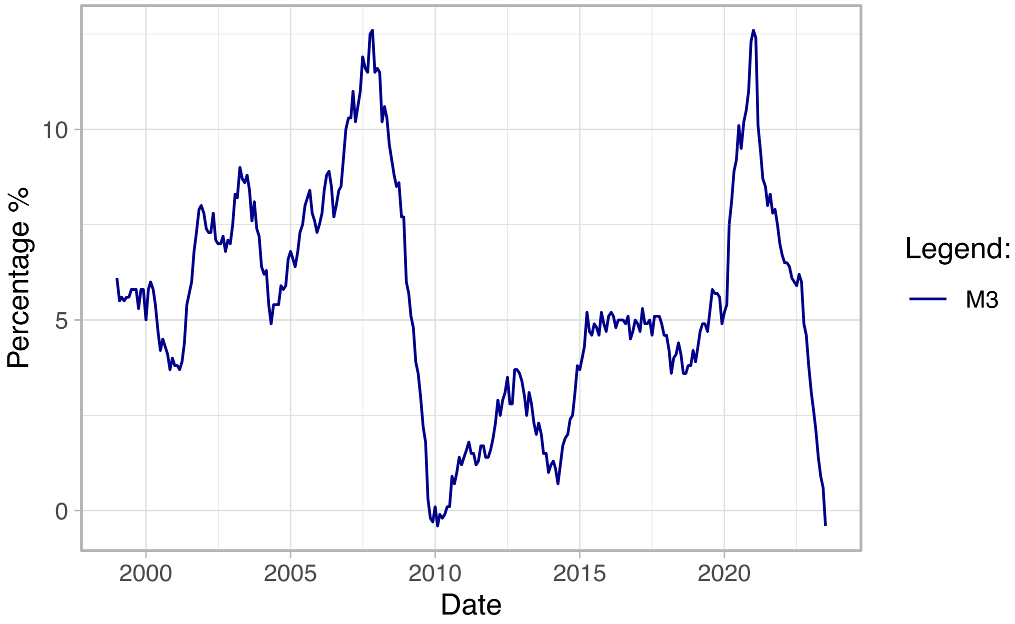


4.5 Instrumental variables

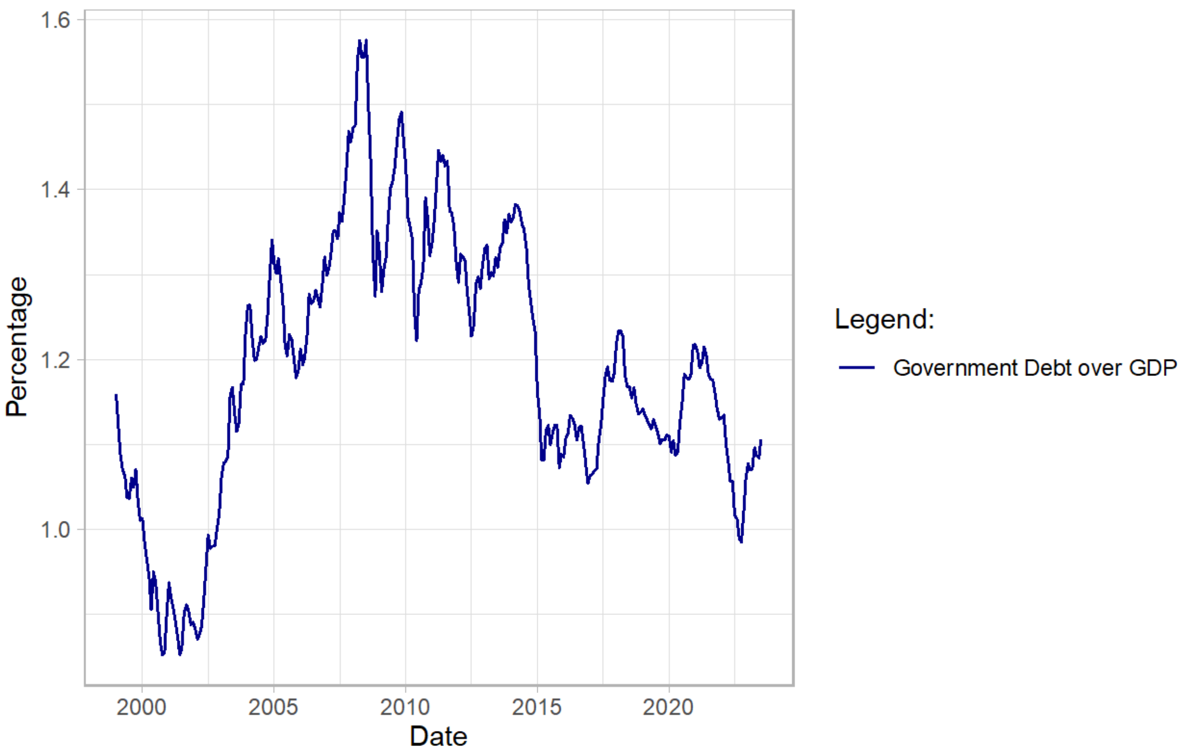
As a solution for the endogenous regressors, I opted to incorporate instrumental factors. This method enables me to have consistent parameters. Since the chosen model is derived from the framework of Clarida et al. (2000), I opted to utilize the instruments they presented, namely commodity price inflation, the spread between 10-year and 3-month treasuries and M3 growth. However, in order to build a model with more significant parameters, I chose to extend the information set, based on the work of Anda and Carron (2019). The goal is to approximate an information set that is as plausible as possible. Therefore, I decided to also utilize the house price inflation, the total public debt, and the EU/US exchange rate. All information was

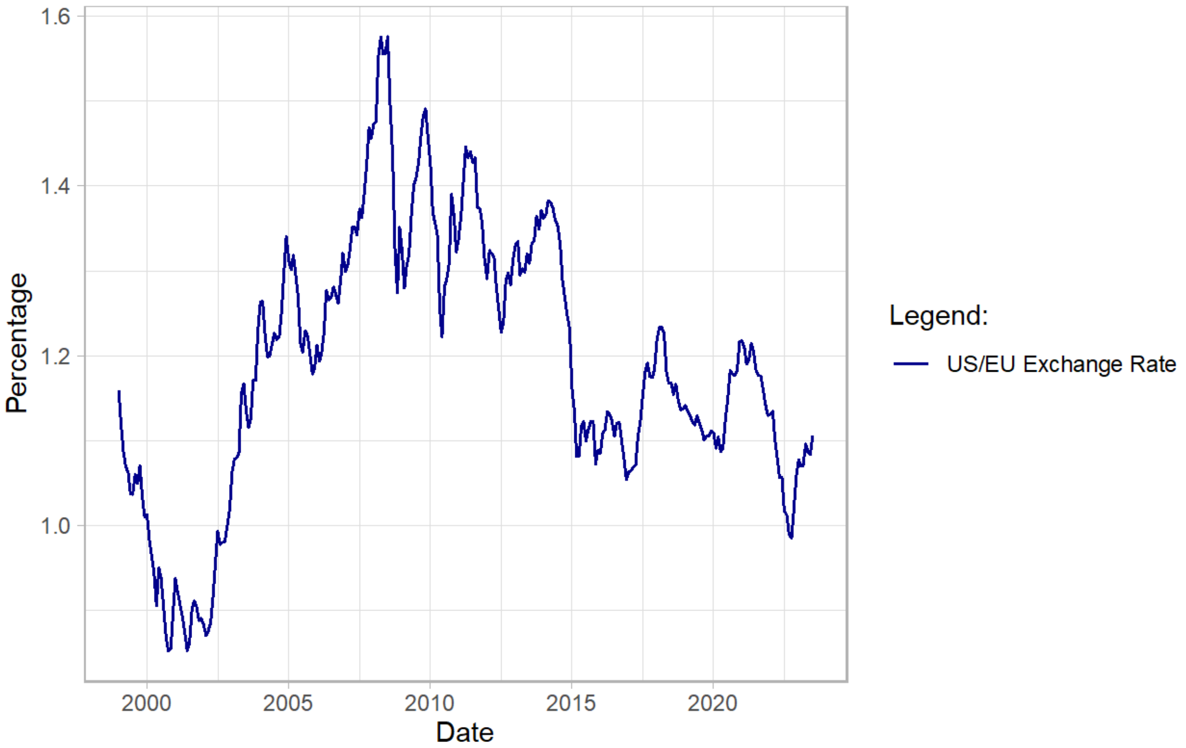
14 https://www.ecb.europa.eu/stats/ecb_surveys/survey_of_professional_forecasters/html/ecb.spf2023q3~7fb4e0b3a7.en.html#:~:text=Annex%20(chart%20data)-,Summary,2023%2C%20with%2054%20responses%20received.

obtained from Eurostat. Particularly, I added housing price inflation since it might provide light on future economic forecasts. In fact, as shown by Anundsen and Jansen (2013), this variable reveals the household's personal and macroeconomic expectations. Furthermore, it has demonstrated that the EU/US exchange rate affects market agents' expectations. The change in public debt also affects GDP per capita; hence, an increase will then have a negative impact on



markets.




1.6
1.4
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Percentage
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2015
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Date
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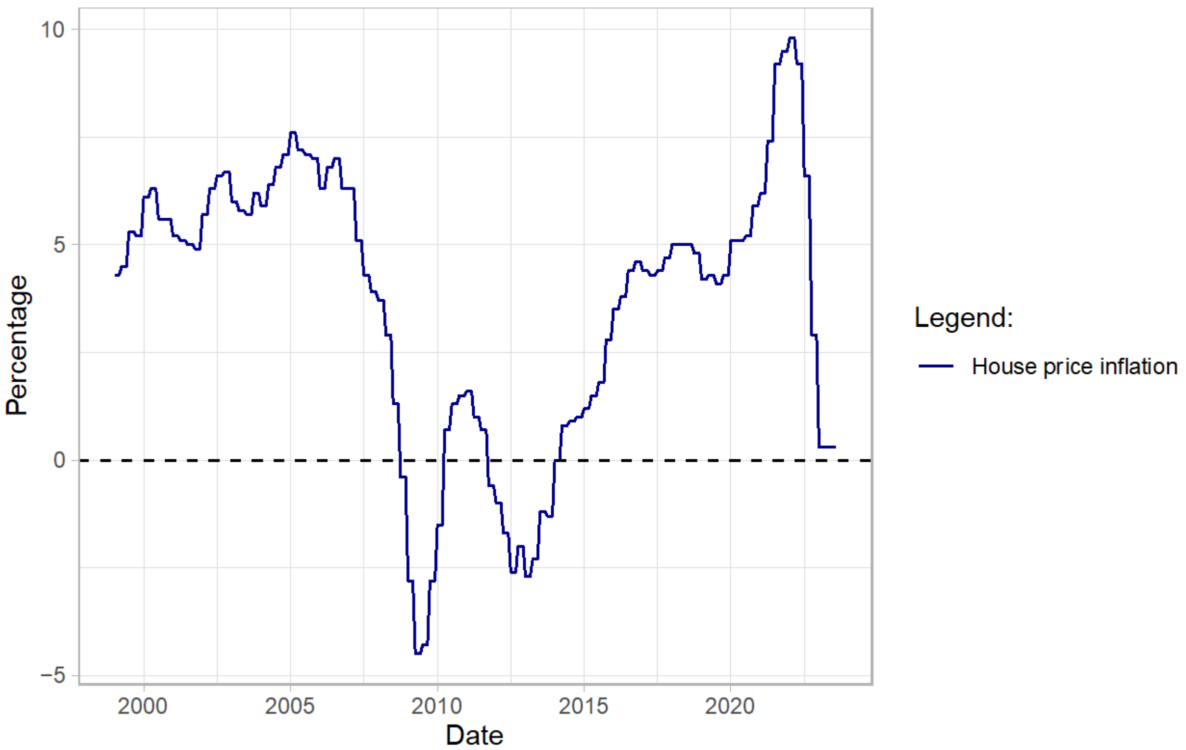
US/EU Exchange Rate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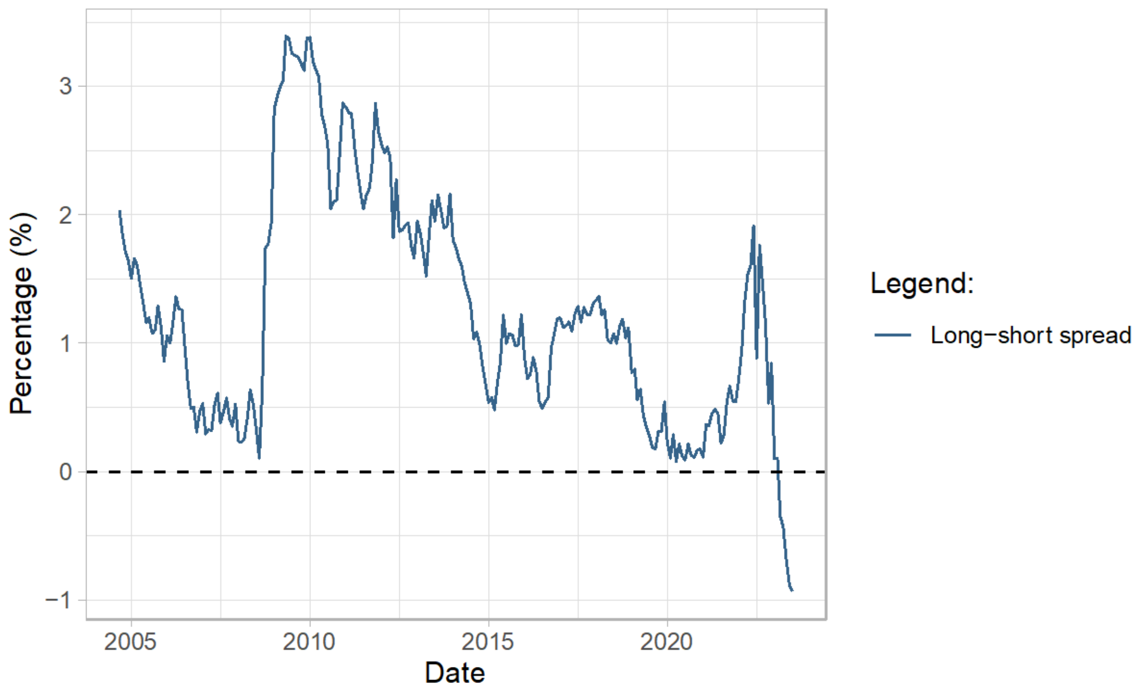
House price inflation



5. Methodology

As previously explained, the Taylor rule I am going to estimate is a policy rule that includes a smoothing parameter, which is not present in the Golden Taylor rule (J. Taylor 1993). According to Clarida et al. (2000), the introduction of this smoothing parameter together with the expected variables, improves the precision of the estimation. However, it should be noted that the use of a smoothing parameter makes the function no longer linear, which goes against the Ordinary Least Squares (OLS) assumptions to obtain a consistent and unbiased estimator. Furthermore, the use of forward-looking variables, which are correlated with the error at time t, requires an econometric model that considers endogeneity.

5.1 GMM

The method I chose to use, following the work of Clarida et al. (1998), is the Generalized Method of Moments (GMM). This method, using instrumental variables, is able to estimate non-linear functions that include endogenous explanatory variables. In addition, the GMM also takes into account heteroskedasticity and autocorrelations in the residuals.

An important basic assumption is that the GMM does not require complete knowledge of the data distribution (Hansen, 1982).

The general GMM model follows this equation:

$$y_t = h(X_t; \theta) + \epsilon_t, \quad t = 1, \dots, T \qquad (5.1)$$

Where y_t represents the vector of dependent variables, which is determined by the function h applied to the matrix of explanatory variables X_t and the matrix of parameters θ, with the addition of an error vector ϵ_t. In addition, an instrument matrix Z_t is introduced, which is correlated with X_t. Z_t is a vector of dimension $qx1$ consisting of instrumental variables, while θ is a vector of dimension $kx1$ of parameters, with $k < q$. Next, an orthogonality condition is established, which implies that instruments are uncorrelated with errors. In other words, this condition states that:

$$E[Z'_t \epsilon_t] = 0 \qquad (5.2)$$

Substituting ϵ_t into equation 12 we obtain:

$$E[Z'_t(y_t - h(X_t; \theta)] = 0 \qquad (5.3)$$

And so, defining the function f:

$$f(\theta, y_t, Z_t, X_t) = Z'_t\big(y_t - h(X_t; \theta)\big) \qquad (5.4)$$

We can write the orthogonality condition as

$$E[f(\theta, y_t, Z_t, X_t)] = 0 \qquad (5.5)$$

To apply this empirically we define:

$$g_T(\theta, y_t, Z_t, X_t) = \frac{1}{T}\sum_{t=1}^{T} f(\theta, y_t, Z_t, X_t) = \frac{1}{T}\sum_{t=1}^{T} Z'_t(y_t - h(X_t; \theta)) \qquad (5.6)$$

From the definition of the GMM estimator of a general model, one must therefore identify the estimator θ such that $g_T(\theta, y_t, Z_t, X_t)$ is as close as possible to the value of 0 of the theoretical counterpart $E[f(\theta, y_t, Z_t, X_t)]$. If and when the model is over identified, the function is minimized using the numerical optimization method.

5.2 GMM and Taylor rule

Taking up the work of Clarida et al., (1998, 2000), I will use the reaction function they developed. In period t, the reaction function will be:

$$i_t^* = i^* + \beta\left(E\left[\pi_{t,k}\middle|\Omega_t\right] - \pi^*\right) + \gamma E[x_{t,q}|\Omega_t] \qquad (5.7)$$

The model can be simplified by introducing a constant $\alpha = i^* - \beta\pi^*$. Thus, we obtain:

$$i_t^* = \alpha + \beta\left[\pi_{t,k}\middle|\Omega_t\right] + \gamma E[x_{t,q}|\Omega_t] \qquad (5.8)$$

Where β and γ determine the responsiveness of policy to changes in expected inflation and the GDP gap, respectively.

Considering, for the reasons stated above, also interest rate smoothing, the relationship will become:

$$i_t = (1-\rho)i_t^* + \rho i_{t-1} + v_{1t} \qquad (5.9)$$

Plugging it into function 5.8 yields

$$i_t = (1-\rho)(\alpha + \beta E[\pi_{t+4}|\Omega] + \gamma E[x_t|\Omega]) + \rho i_{t-1} + v_{1t} \qquad (5.10)$$

In addition, to eliminate unobserved forecast variables, an auxiliary variable ϵ_{1t}
is introduced.

$$\epsilon_{1t} = -(1-\rho)\left(\beta\left(\pi_{t,k} - E\left[\pi_{t,k}\middle|\Omega\right]\right) + \gamma(x_t - E[x_t|\Omega_t])\right) + v_{1t} \qquad (5.11)$$

The equation above is a combination of forecast error and exogenous error term, so it is orthogonal to the variables in the information set; subsequently, solving for v_{1t}:

$$v_{it} = -(1-\rho)\left(\beta\left(\pi_{t,k} - E\left[\pi_{t,k}\middle|\Omega_t\right]\right) + \gamma(x_t - E[x_t|\Omega_t])\right) + \epsilon_{1t} \qquad (5.12)$$

To remove the expected value, 5.12 is plugged into 5.10, obtaining:

$$i_t = (1-\rho)\left(\alpha + \beta E\left[\pi_{t,k}\middle|\Omega_t\right] + \gamma E\left[x_{t,q}\middle|\Omega_t\right]\right) + \rho i_{t-1} + (1-\rho)\left(\beta\left(\pi_{t,k} - E\left[\pi_{t,k}\middle|\Omega_t\right]\right) + \gamma\left(x_{t,q} - E\left[x_{t,q}\middle|\Omega_t\right]\right)\right) + \epsilon_{1t} \qquad (5.13)$$

So, rewriting the function in terms of realised instead of expected variables, we get:

$$i_t = (1-\rho)\left(\alpha + \beta\pi_{t,k} + \gamma x_{t,q}\right) + \rho i_{t-1} + \epsilon_{1t} \qquad (5.14)$$

We thus obtain the policy reaction function, however, to obtain the parameter vector for estimation, we use a set of instruments Z_t which is orthogonal to ϵ_{1t}. Within this matrix, both lagged variables and current variables that are uncorrelated with the error can be included. Thus, we impose as a condition $E[\epsilon_{1t}|Z_t] = 0$ which can be written as:

$$E\left[i_t - (1-\rho)\left[\alpha + \beta\pi_{t,k} + \gamma x_{t,q}\right] - \rho i_{t-1}\middle|Z_t\right] = 0 \qquad (5.15)$$

Thus, the vector of parameters we want to estimate is $[\rho, \alpha, \beta, \gamma]$.

To do this, therefore, I use a GMM model. In a first approach, I use the instrumental variables proposed by Clarida et al. (1998), i.e., 4 lags of the regressors, commodity inflation, M3 growth and long-short spread. In this case, the number of instrumental variables exceeds the number of regressors, and the model is therefore over-identified. Following the work of Anda e Carron (2019), to check whether the Shadow rate Taylor rule outperforms the standard Taylor rule, I apply the RMSE and MAE criteria. The RMSE estimates the root-mean-squared-error and gives a measure of the difference between the two values. Therefore, when comparing the two series with the out-of-sample period's Ester, the series that results in a lower RMSE will indicate a more precise estimation of the Ester. Since with the RMSE also the residuals are squared, therefore penalizing large errors, I will also introduce the mean absolute error (MAE) which measures the mean of the absolute differences between the estimates and the realized values.

5.3 Structural breaks

I also examine the presence of structural breaks in our sample. Identifying structural changes is of paramount importance since not detecting them correctly can lead to inaccurate forecasts and misleading conclusions. The test we employ is the Quandt Likelihood Ratio Test (QLR), which is built on the Chow test but doesn't need for picking a breaking point, since it computes the Chow test for all the available dates.

The Chow considers a linear model split in two samples at a predetermined break point such that:

$$y_t = x_t'\beta_1 + u_t \qquad for\ t \leq T_b$$

$$y_t = x_t'\beta_2 + u_t, \qquad for\ t > T_b$$

The Chow test for the null hypothesis $H_0': \beta_1 = \beta_2 | H_0''$ is:

$$CH_1: \left(\frac{RSS_T - RSS_{T1} - RSS_{T2}}{RSS_{T1} + RSS_{T2}}\right) \frac{(T-2k)}{k} \sim f(k, T-2k) \text{ where } T = T_1 + T_2$$

In case of instability, we expect $RSS_t > RSS_{T1} + RSS_{T2}$

The Chow test for the null hypothesis $H_0'': \sigma_1^2 = \sigma_2^2 | H_0''$

The test statistic is $CH_2 = \frac{\widehat{\sigma_2^2}}{\widehat{\sigma_1^2}} = \frac{RSS_{T2}}{RSS_{T1}} \left(\frac{T_1 - k}{T_2 - k}\right) \sim F(T_2 - k, T_1 - k)$

5.4 J-test

In order to verify that the estimates obtained are valid, it is necessary to check the absence of overidentification through the J-test. In particular with Hansen's J-statistics the null hypothesis of orthogonality is tested:

$$g_T(\theta, y_t, Z_t, X_t) = 0 \qquad (5.16)$$

Where the alternative hypothesis is:

$$g_T(\theta, y_t, Z_t, X_t) \neq 0 \qquad (5.17)$$

Rejecting the null hypothesis implies that the orthogonality condition is violated and that there are relevant omitted variables and that the model is therefore mis specified.

5.5 Robustness

In the GMM model, I employ the HAC matrix to face problems of autocorrelation and heteroskedasticity. Moreover, I use the N-step iterative approach for weight updating. In this way, as opposed to the 1-step iteration, the N-step re-calculates the covariance matrix repeatedly until it achieves estimates that converge. This process assigns various weights to different moment conditions: the observation with lower significance and higher variability receive a lower weight, while those with higher significance and lower variability receive a higher weight. The fact that the matrix is recalculated until it achieves numerical convergence implies that the estimations of $\hat{\beta}$ and $\hat{\Omega}$ converge numerically. This means that we obtain a more robust model with a more stable β-coefficient. However, for the extended analysis, due to limitations in iterations and convergence concerns, I employ the two-step iteration.

5.6 Stationarity

This estimation assumes that variables are stationary, where a stationary series does not contain unit roots. In particular, a variable is said to be stationary if $E(y_t) = \mu$, and thus statistical properties such as mean and variance are constant over time. The results of the ADF test, which is used to check the stationarity, can be found in the appendix **A.1.** In the ADF test, I also added lags to correct for potential serial correlation. In particular, we can reject the null hypothesis of unit root for the inflation, while the GDP gap, the reference rate and the shadow rate are not stationary. However, we can assume that those are stationary in the long run since they will converge with the long-run equilibrium rate and to the long run equilibrium GDP.

6. Results

The Taylor rule model chosen for the analysis is the one presented by Clarida et al. (1998). Following an initial application of Ordinary Least Squares (OLS) estimation, the model is further estimated using the Generalized Method of Moments (GMM). Subsequently, the analysis aims to assess whether the incorporation of the shadow rate improves accuracy, as measured by the Root Mean Squared Error (RMSE) and Mean Absolute Error (MAE) criteria. Additionally, attention will be given to identifying which specific shadow rate yields lower estimates.

6.1 Structural Break

During the initial analysis of the dataset, I conducted a test to check for structural break, considering that the sample spans from 1999M01 to 2023M08. Given the significant events that have occurred over this period, it was suspected that a significant shift had occurred in the data's underlying trends. Therefore, by employing the Quandt Likelihood Ratio Test the presence of a structural break occurring in October 2009 was confirmed. Consequently, the analysis is segmented into two periods: the first covering 1999 to October 2009, and the second spanning November 2009 to January 2018, marking the conclusion of our in-sample period. The coefficient estimated with the analysis of the second period will be employed for conducting out-of-sample estimations.

6.2 OLS Estimation

I begin my estimation by considering a linear Taylor rule with no smoothing parameter.
As we can see, the coefficients are all significative and consistent with the literature.

OLS PRE 2009M10

VARIABLES	(1) Baseline	(2) SR Volpi	(3) SR Wu and Xia
Hicp Gap	1.962***	1.644***	2.837***
	(0.230)	(0.234)	(0.283)
GDP Gap	0.836***	0.743***	0.751***
	(0.0884)	(0.0899)	(0.135)
Constant	3.238***	3.208***	3.117***
	(0.0762)	(0.0775)	(0.0937)
Observations	130	130	62
R-squared	0.583	0.503	0.793

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

However, in the second part of the sample, after October 2009, for the baseline estimation and the one with the shadow rate elaborated by Volpi, the coefficient is lower than 1, indicative an accommodative monetary policy.

OLS POST 2009M10

VARIABLES	(1) Baseline	(2) SR Volpi	(3) SR Wu and Xia
Hicp Gap	0.934***	1.064***	3.042***
	(0.129)	(0.341)	(0.726)
GDP Gap	0.0343	0.524**	-1.252**
	(0.0895)	(0.238)	(0.505)
Constant	0.542***	-0.119	-0.795
	(0.106)	(0.282)	(0.599)
Observations	99	99	99
R-squared	0.379	0.103	0.272

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

The inflation’s coefficient of the shadow rate elaborated by Wu and Xia is high, however, the GDP gap’s coefficient is negative, inconsistent with the literature. Nonetheless, it's crucial to exercise caution when interpreting these findings. Particularly in the second period of estimation, these coefficients may be subject to alteration due to unconventional monetary policies and the reaching of the effective lower bound.

6.3 Baseline Estimation

The actual model analysed is the one introduced by Clarida et al. (1998); in particular the baseline model is constructed with the same instruments that they employ in their paper: the lags of the regressors for four periods, the commodity inflation, the long short spread and the M3 growth rate. The model estimated spans from 1999M01 to 2017M12, and it is presented as follows:

$$i_t = (1-\rho)\left(\alpha + \beta E\left[\pi_{t,k}\right] + \gamma[x_t]\right) + \rho i_{t-1} + \epsilon_t$$

These are the results obtained by the GMM estimation:

Baseline PRE2009M10

VARIABLES	(1) Standard	(2) SR Volpi	(3) SR Wu and Xia
ρ	0.879***	0.944***	0.893 ***
	(0.0088)	(0.0155)	(0.0157)
α	-4.716***	-0.248	-2.513***
	(0.6869)	(1.763)	(0.805)
β	3.807***	1.687**	2.721***
	(0.3928)	(0.8383)	(.462)
γ	2.564***	0.244	1.189***
	(0.1519)	(0.4995)	(.1635)
J-stat	5.42057	12.65	8.115
P-value	0.9963	0.9204	0.964

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Estimated model: $i_t = (1-\rho)\left(\alpha + \beta E\left[\pi_{t,k}\right] + \gamma[x_t]\right) + \rho i_{t-1} + \epsilon_t$
Model (1) includes four lags of the €str, while model (2) and (3) include both four lags of the €str and four lags of the shadow rate. Both models include four lags of inflation, GDP gap, commodity inflation, M3 growth, the spread between 10-years and 3-months Treasury as instrumental variable.

Baseline POST2009M10			
VARIABLES	(1) Standard	(2) SR Volpi	(3) SR Wu and Xia
ρ	0.984***	0.935***	0.9397***
	(0.0036)	(0.0155)	(0.0157)
α	-3.259***	-3.018***	-22.646***
	(0.6070)	(1.763)	(0.805)
β	2.199***	0.779***	14.381***
	(0.4534)	(0.8383)	(0.462)
γ	0.899***	-1.314 ***	-4.866***
	(0.3245)	(0.4995)	(0.1635)
J-stat	4.435	4.792	4.941
P-value	1	0.999	0.999

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Estimated model: $i_t = (1-\rho)(\alpha + \beta E[\pi_{t,k}] + \gamma[x_t]) + \rho i_{t-1} + \epsilon_t$
Model (1) includes four lags of the €str, while model (2) and (3) include both four lags of the €str and four lags of the shadow rate. Both models include four lags of inflation, GDP gap, commodity inflation, M3 growth, the spread between 10-years and 3-months Treasury as instrumental variable.

For the *pre2009M10* period we can see that we have larger coefficient for inflation and for gdp gap. This is understandable since we introduced a smoothing parameter. Most of the coefficient are significant with the exception of model (2), where both the constant and the gdp coefficient are not significant. The J-statistic's p value is high for all three models.

Turning our attention to the second part of the sample, after the structural break, similar observations prevail. In this case as well, we note the presence of larger coefficients. However, it's worth mentioning that in model (2), the coefficient for inflation does not exhibit significance. An interesting point is the abnormally high coefficient of inflation in model (3), which could be attributed to a combination of the smoothing parameter and the observation that, based on the graphical representation, the shadow rate decreases significantly after reaching the zero lower bound.

6.4 Extended estimation

Building upon the study conducted by Anda and Carron (2019), and reapplying it to the European Union (EU), I have incorporated supplementary data. Specifically, following their model, I have included information on EU house price inflation, total public debt, and Euro to US dollar exchange rates. These indicators were added with the goal of representing the information encompassed by the financial market and therefore obtain an approximation of the true information set.

As can be seen in the figure below, with the only exception of the GDP gap, all coefficients are significant at 5% level. This finding aligns with the rationale that, in accordance with the ECB mandate, the central bank's primary objective is to uphold price stability. As we can see, the J statistic p-value is 1, meaning that all the instruments are significant, and the model is not overidentified.

Extended POST2009M10

VARIABLES	(1) Standard	(2) SR Volpi	(3) SR Wu and Xia
ρ	0.967 ***	0.9299 ***	0. .9885 ***
	(0.0068)	(0.0072)	(0.0032)
α	-2.615 ***	-2.078 ***	-21.9890***
	(0.3196)	(0.2858)	(5.3673)
β	2.425 ***	0.480**	10.1015***
	(0.2719)	(0.2214)	(3.387)
γ	1.869 ***	0.108	-1.5479
	(0.2958)	(0.2071)	(1.834)
J-stat	5.29403	5.29906	5.50327
P-value	1	1	1

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Estimated model: $i_t = (1-\rho)(\alpha + \beta E[\pi_{t,k}] + \gamma[x_t]) + \rho i_{t-1} + \epsilon_t$
Model (1) includes four lags of the €str, while model (2) and (3) include both four lags of the €str and four lags of the shadow rate. Both models include four lags of inflation, GDP gap, commodity inflation, the spread between 10-years and 3-months Treasury, house price inflation, exchange rate between euro and $dollars, government debt in proportion to GDP, M3 growth as instrumental variable.

From this point forward, the extended models incorporating shadow rates will serve as the focal point of our thesis. Similar to the baseline model, the coefficients are notably high, with the exception of model 2. Furthermore, the smoothing parameter exhibits a relatively high value, although this aligns with the established literature on GMM estimation for the European Union[15]. This implies that the ECB reacts gradually to shifts in inflation and output gap. However, it's important to note that gauging the short-term impact solely based on the smoothing parameter coefficients is insufficient; rather we should focus on the combination of different parameters. In order to gain a comprehensive understanding of the ECB's responses, we should deduct the value of the smoothing parameter. Then $(1 - \rho) * \pi$ equals the one period respond to a one percent change in inflation. Therefore, a change in inflation at time t is 0.08% for the regular model, 0.034% for the estimation with the shadow rate developed by Volpi and 0.1162% for the shadow rate developed by Wu and Xia.

Hence, the inclusion of the Volpi's shadow rate leads to a less responsive policy with respect to inflation targeting, while the inclusion of the Wu and Xia's shadow rate to a more responsive one.

Moreover, for a one percent increase in GDP gap, the response of the ECB is as follows: 0.06% following the standard model, 0.0008% following the model with Volpi's shadow rate and 0.0178% following the model with Wu and Xia's shadow rate. Once again, the shadow rates result in a lower response to changes in the GDP gaps. The coefficients are notably lower than the inflation's ones, this is coherent with the mandate of the ECB.

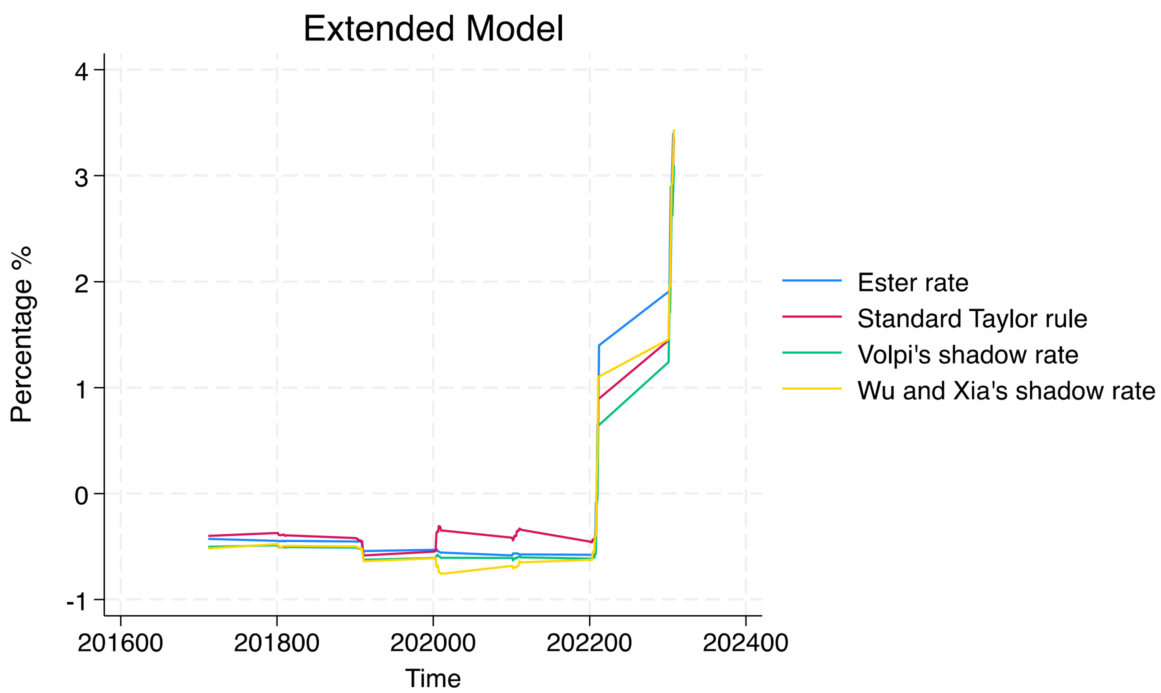
The estimated shadow coefficient is high, but coherent with the existing literature which assigns a value of 0.95 for the European Union[16].

6.4.1 Postestimation

To further compare the three rules, I have represented them graphically, also including the actual ester rate.

[15] See footnote 12

[16] For more information: Gerdesmeier, D., & Roffia, B. (2005). The relevance of real-time data in estimating reaction functions for the euro area. The North American Journal of Economics and Finance, 16(3), 293-307; Sauer, S., & Sturm, J. E. (2007). Using Taylor rules to understand European Central Bank monetary policy. German Economic Review, 8(3), 375-398; Fourçans, A., & Vranceanu, R. (2007). The ECB monetary policy: choices and challenges. Journal of policy Modeling, 29(2), 181-194.



As we can see the estimations yield similar results, however, from the figure, it appears that the shadow rate elaborated by Wu and Xia is closer to the ester rate. To confirm which estimate is more precise we employ the RMSE and MAE criteria.

RMSE and MAE

	(1) Baseline	(2) SR Volpi	(3) SR Wu and Xia
RMSE	0.752	0.683	0.592
MAE	0.506	0.485	0.437

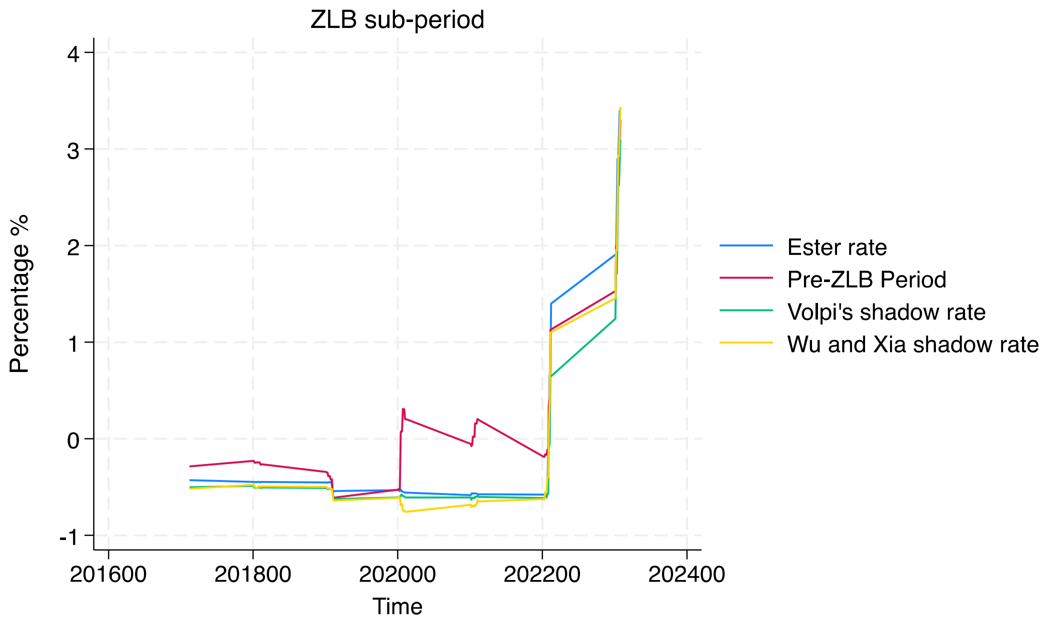
From the results we can see that our graphical findings were confirmed. Then a Taylor rule estimated with Wu and Xia's shadow rate is a better fir for the Ester rate, followed by the Taylor rule estimated with the shadow rate elaborated by Volpi. Indeed, the output from the model (3) shows 0.592 and 0.437 for RMSE and MAE respectively, which is substantially smaller than the results obtained with the standard estimation.

6.5 Robustness

The entire analysis is based on the assumptions presented throughout the thesis. However, additional hypotheses can be tested to corroborate the results. Building upon the methodology outlined in the work of Andas and Carron (2019), further exploration could involve adjusting the time horizon and conducting additional estimations with adjusted sample sets.

6.5.1 Different time horizons

Throughout this study, I stated that previous research primarily concentrated on understanding the impacts of monetary policy before the zero lower bound (ZLB) period. This suggests that when interpreting our results using an extended model, they should be compared to an estimation from before the ZLB period. In pursuit of this, I conducted an analysis employing a modified dataset that concludes in July 2012, coinciding with the commencement of the zero lower bound. In order to affirm the reliability of our results, we visually present the outcomes and draw a comparative analysis with our extended model that incorporates the notion of a shadow rate.



From the figure we see that our shadow rate model yields a more precise estimation, so we can confirm our findings are robust.

RMSE and MAE ZLB estimations

VARIABLES	(1) Baseline sub-period	(2) SR Volpi	(3) SR Wu and Xia
RMSE	2.76996	0.683	0.592
MAE	2.4151	0.485	0.437

6.5.2 Unemployment gap

To enhance the robustness of the analysis, an alternative variable, the unemployment gap, can be considered instead of the GDP gap. Despite the ECB's mandate not explicitly focusing on the GDP gap, it does take into account also other economic indicators, such as the unemployment gap, when making monetary decisions. This is because unemployment closely mirrors the real economic activity (Carlstrom e Zaman 2014).

I apply the same methodology as the previous estimation, using the dataset of the extended model, but replacing the GDP gap with the unemployment gap. The unemployment gap is computed by subtracting the NAIRU rate from the projected one-year-ahead unemployment rate.

Extended version with unemployment gap PRE2009M10			
VARIABLES	(1) Baseline	(2) SR Volpi	(3) SR Wu and Xia
ρ	0.982***	0.980***	0.983***
	(.004)	(.003)	(0.005)
α	-10.8798***	-6.906***	-19.915***
	(2.995)	(1.374)	(4.81)
β	6.824***	4.347***	11.485***
	(1.777)	(.782)	(2.673)
γ	0.254	-0.697 **	-0.657
	(0.274)	(.281)	(0.537)
J-stat	9.1023	10.4598	17.1702
P-value	0.9985	0.9994	0.9595

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

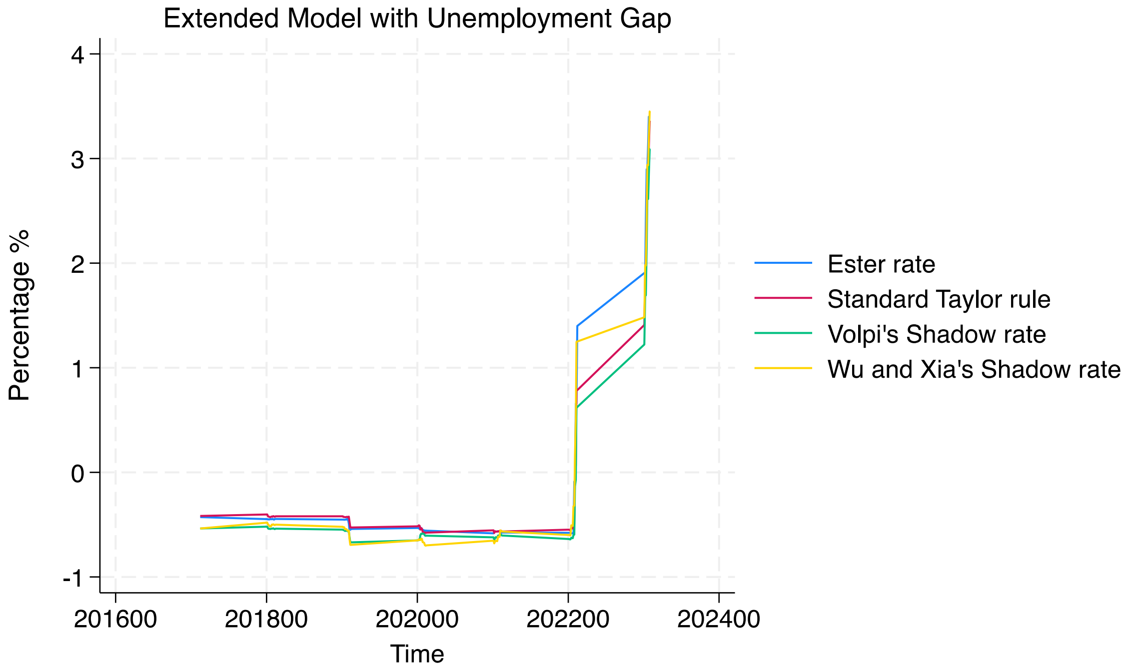
Figure 8
Estimated model: $i_t = (1-\rho)(\alpha + \beta E[\pi_{t,k}] + \gamma[x_t]) + \rho i_{t-1} + \epsilon_t$
Model (1) includes four lags of the €str, while model (2) and (3) include both four lags of the €str and four lags of the shadow rate. Both models include four lags of inflation, gdp gap, commodity inflation, the spread between 10-years and 3-months Treasury, M3 growth, house price growth, government debt over GDP as instrumental variable.

Extended version with unemployment gap POST2009M10			
VARIABLES	(1) Baseline	(2) SR Volpi	(3) SR Wu and Xia
ρ	0.973***	0.936***	0 .986***
	(0.007)	(0.006)	(0.003)
α	-1.301***	-2.876***	-24.158***
	(0.264)	(0.418)	(4.763)
β	1.052***	0.614**	11.815***
	(0.163)	(.269)	(2.608)
γ	-0.299***	0.299***	0.852
	(0.074)	(0.069)	(0.609)
J-stat	5.03535	4.97413	5.52717
P-value	1	1	1

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Figure 9
Estimated model: $i_t = (1-\rho)(\alpha + \beta E[\pi_{t,k}] + \gamma[x_t]) + \rho i_{t-1} + \epsilon_t$
Model (1) includes four lags of the €str, while model (2) and (3) include both four lags of the €str and four lags of the shadow rate. Both models include four lags of inflation, gdp gap, commodity inflation, the spread between 10-years and 3-months Treasury, M3 growth, house price growth, government debt over GDP as instrumental variable.

In the results, shown in figures 8 and 9, particularly for the post-break sample, the coefficients are generally not very high, except for the policy rule computed using the shadow rate developed by Wu and Xia, which exhibits a notably high value. Also, the smoothing parameter exhibits a high value, still aligned with the literature. Additionally, the p-value for the J statistics is high, indicating no overidentification. To compare the out-of-sample estimations, we use the same graph as before.



Looking at the graph, it seems that the model created using the Wu and Xia shadow rate provides the closest estimation to the Ester, especially after 2022.

RMSE and MAE

	(1) Baseline	(2) SR Volpi	(3) SR Wu and Xia
RMSE	0.38458571	.44795543	.48837864
MAE	.30049425	.32270351	.36367553

However, when we follow the standard practice of comparing using residual mean squared error and mean absolute error, the model based on the standard Taylor rule emerges as the closest estimate.

Thus, taking into account all the models estimated so far, we can deduce that the analysis conducted with the unemployment gap is inconclusive and the model incorporating the GDP gap, specifically the shadow rate model, is more accurate than the one incorporating the unemployment gap.

6.6 Shadow rate and the market

To further demonstrate the robustness of the model and the relevance of the shadow rate, it is necessary to focus on market expectations. Starting in 2021, the ECB conducted direct surveys of SMAs (survey monetary analysts) in order to understand the expectations about the future development of monetary policy instruments and initiatives already announced by the ECB, as well as about the conditions in financial markets and the economy in general. It is therefore of interest to understand whether or not the market had anticipated and taken the shadow rate into account when formulating its expectations.

Following the idea and approach of Bernardini and Lin (2023), I revisited their model to show that, after the exit from the zero lower bound, the market reverted to following Taylor's rule in forming its expectations. However, my main focus was to check whether or not the market had taken the shadow rate into account in the formulation process, and thus whether it had followed a standard Taylor rule or a shadow Taylor rule. To do this, I used the model presented so far, introducing some modifications; let us therefore take the standard Taylor rule:

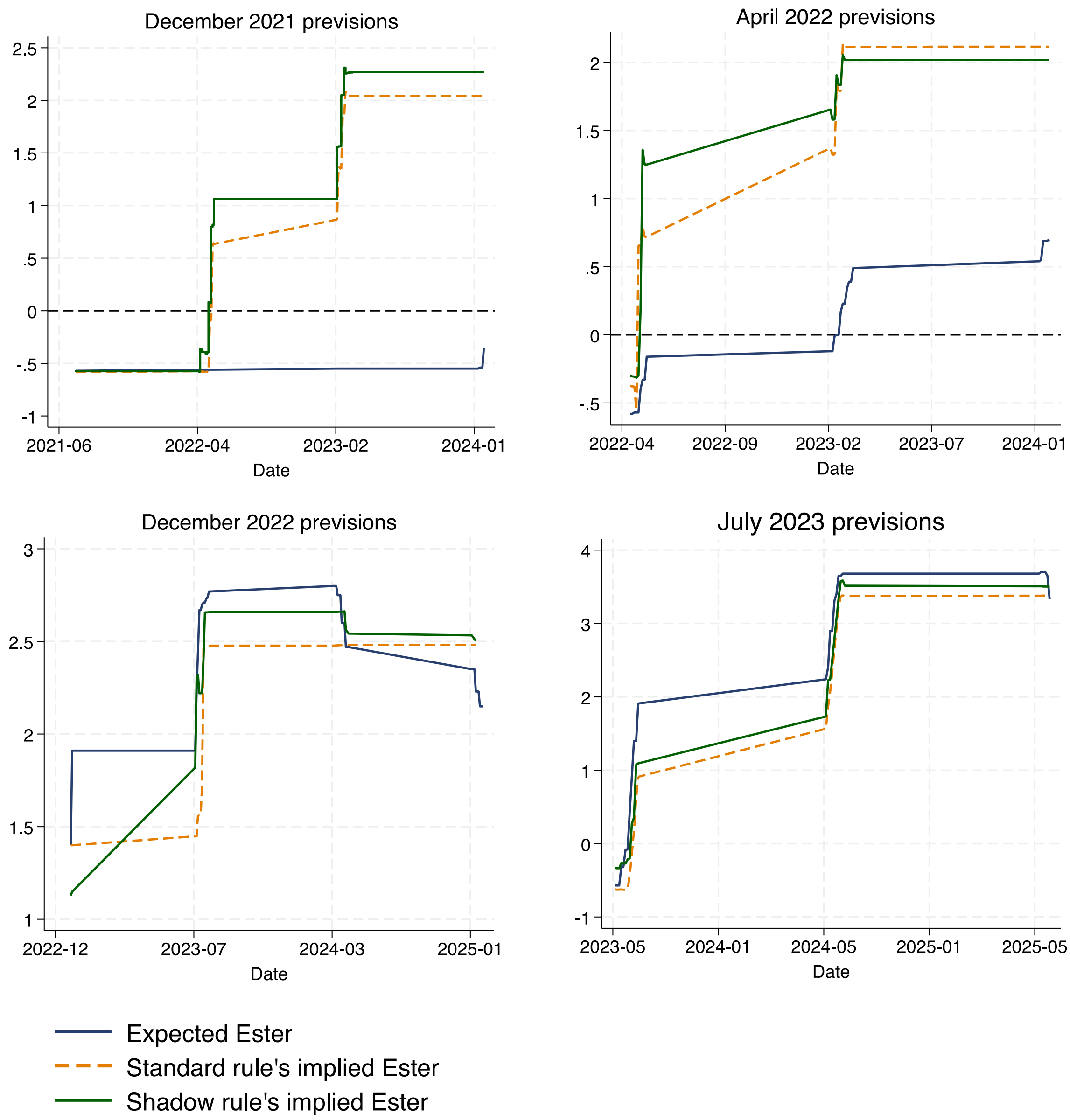
$$i_{t+h}^{R} = (1-\rho)\left(E_s i^* + \beta\pi_{t,k} + \gamma x_{t,q}\right) + \rho i_{t+h-1}^{R} + \epsilon_{1t}$$

$with\ h = 0, \dots, 8.$

Where:

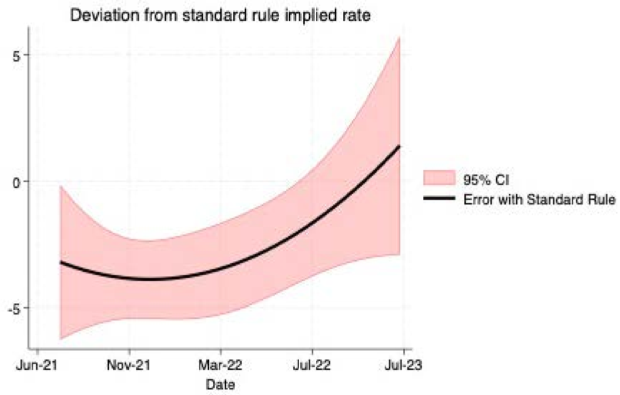
- $E_s i^*$ is the median long-run expectation from the ECB’s SMA
- $\pi_{t,k}$ is the median h-quarter ahead expectation from the ECB’s SMA of the euro-area core-HICP inflation;
- $x_{t,q}$ is the median h-quarter ahead expectation from the ECB’s SMA of the euro-area GDP gap;

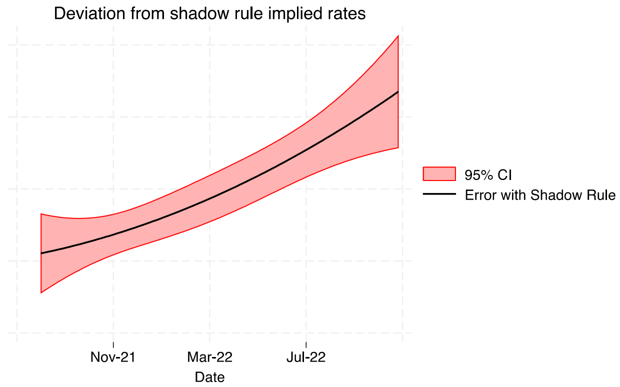
Leveraging a GMM estimation, using as instrumental variables four lags of the regressors, I then worked out the rule implied rate by projecting this model over four different survey rounds: December 2021, April 2022, December 2022, and July 2023, and compared the implied policy rate with the expected policy rate obtained from the survey. I then did the same utilizing the shadow rate elaborated by Volpi (2023).

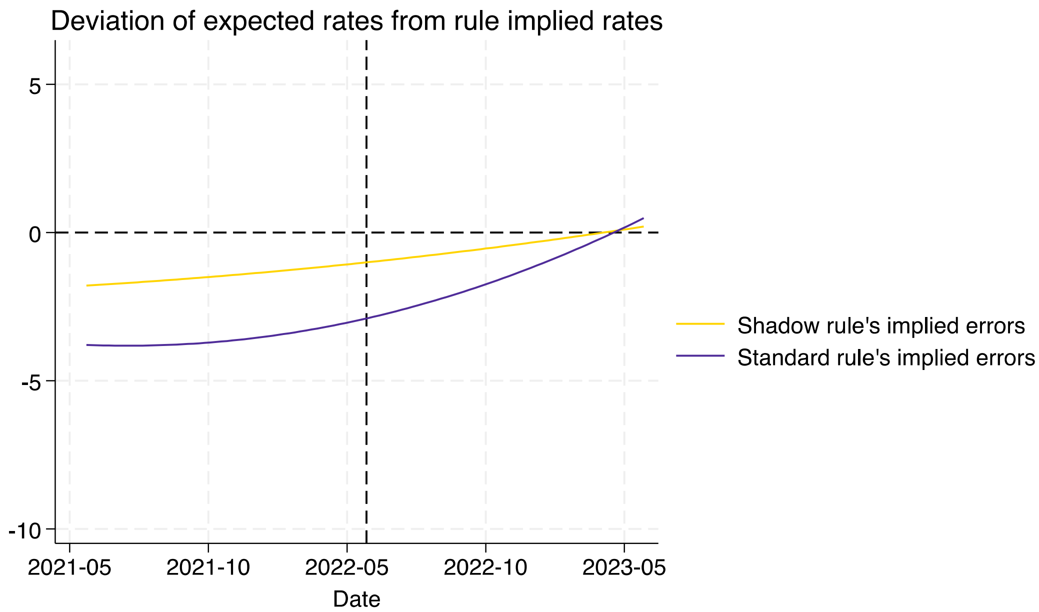


It is clear from the graphs above that the interest rate expectations diverged significantly from the interest rate suggested by the Taylor rules. Only as of June 2022 the expected rate came back into line with Taylor rule’s implied rate.

I then computed the average deviation for each survey between the expected and the rule implied rate by computing $Error = \sum(E_s i_{t+h} - E_s i^R_{t+h})$







The final graph shows that the expected interest rate deviated significantly from the rule implied rate, coming back into alignment only after October 2022. This phenomenon can be attributed to the ECB's policy: before June 2022, the ECB used forward guidance as its main instrument. In fact, although the Taylor rule indicated that the rate should have been increased, market expectations were anchored on the communication of the ECB, which was considered reliable and safe, assuring the market that the policy rate would remain unchanged at that level.

Rising inflation then led the central bank to raise rates abruptly and without unanimous agreement within the Governing Council, which hindered clear and reliable communication, especially regarding the intensity and timing of the increases. Therefore, with the return to interest rate policy as the main instrument of monetary policy, it is possible that monetary analysts went back to taking the Taylor rule into account when formulating their expectations. It is very interesting to note, however, that market expectations were more closely aligned with the Taylor rule based on the shadow rate, rather than with the standard Taylor rule, underlining how this rate might be already taken into account by the market and suggesting that it could be very useful to the central bank itself in formulating its policy.

Before drawing any firm conclusions, however, it is important to stress that the sample used for the analysis was limited from 2021 onwards and could lead to estimation errors. Nevertheless, it might be interesting to continue this analysis over time, taking advantage of the new data available.

6.7 Limitations

Before delving into our concluding remarks, it's imperative to undertake a critical evaluation and acknowledge potential limitations within our research. My findings are based on the assumptions of the validity of the shadow rate and its ability to explain the stance of monetary policy, as discussed in the previous sections. This means that the conclusions we draw are inherently tied to the reliability of the shadow rate. A significant limitation lies in the fact that both employed shadow rates are estimated, unlike the official interest rate (€str), which is directly observed. This creates concerns about the accuracy of the rate estimation and the standard errors obtained. Therefore, conducting further research to assess the efficacy of the shadow rate would yield more dependable results.

Moreover, our estimation was conducted with the Generalized Method of Moments, which is a model whose validity depend on the pertinency of its instrumental variable. Our selection of the variables is well-grounded and draws inspiration from existing literature. Furthermore, the validity of our model has been tested through orthogonality restrictions and overidentification tests. Nevertheless, the literature suggests that the instrumental variables could be individually weak, potentially undermining our model.

7. Conclusion

The goal of this analysis was to see whether the inclusion of the shadow rate would make the Taylor rule more accurate and indicative of the economic position of central banks. I therefore analysed the existing literature, focusing on the types of shadow rates and their construction. I then introduced and identified the role of the European Central Bank and the measures it took in the years following the 2007 financial crisis.
To perform the analysis, I used the Taylor rule presented by Clarida et al. (1998) and attempted to add instrumental variables that would better describe the financial market. I tested and verified the presence of a structural break in October 2009, and consequently divided the sample into two parts. I eventually used the period from 2018M1 to 2023M08 to test the accuracy of our model.

Analysing the results obtained, we saw that, using Volpi's shadow rate, the actual response to inflation is smaller than what would be obtained from a traditional Taylor rule, while using Wu and Xia's shadow rate, the response is more substantial. Therefore, the different ways used to compute the shadow rates greatly influence the outputs obtained and this makes our results less significative and uncertain. As far as the central bank's response to the GDP gap is concerned, using the shadow rate leads to a lower coefficient. The coefficients I obtained from the analyses were mostly significant at 5% and the overidentification test was rejected for all the estimates. I also verified the robustness of our analysis by changing the estimation period to the pre-ZLB, still obtaining the same results.
In order to test which of the two measures was more accurate, I used the RMSE and the MAE analysis, which showed that the shadow rate estimate was closer to reality. In line with the methodology proposed by Anda and Carron (2019), an alternative model was tested,

substituting the unemployment gap for the GDP gap. However, the obtained results proved inconclusive.

Lastly, I conducted an analysis to investigate whether the shadow rate, particularly Volpi's version, had been factored into the expectations of monetary analysts. I discovered that starting from June 2022, expectations shifted back to conforming to a Taylor rule, specifically one that incorporates a shadow rate.

We wanted to understand if incorporating the shadow rate into the Taylor rule could provide valuable insights for guiding monetary policy and give a more complete view of the macroeconomic situation.

With the current rise in interest rates, the shift from accommodative to restrictive monetary policy requires an accurate assessment of the impact on the economy. My findings show that the shadow rate is an indicator that could help providing a comprehensive perspective on the situation, helping policy makers to avoid overly restrictive decisions that could dampen economic activity. However, it is important to note that the analysis of the effect of including the shadow rate in the reaction functions, especially in a post zero lower bound period, requires further future research.

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Appendix

A1: Tables

A1.1: Augmented Dickey-Fuller test for unit root for the GDP gap

```
Augmented Dickey-Fuller test for unit root

Variable: gdpgap                           Number of obs  = 158
                                           Number of lags =   3

H0: Random walk without drift, d = 0

                                       Dickey-Fuller
                  Test      ———————— critical value ————————
             statistic           1%           5%          10%
─────────────────────────────────────────────────────────────
 Z(t)           -1.668       -3.491       -2.886       -2.576
─────────────────────────────────────────────────────────────
MacKinnon approximate p-value for Z(t) = 0.4475.
```

A1.2: Augmented Dickey-Fuller test for unit root for the inflation

```
Augmented Dickey-Fuller test for unit root

Variable: hicp                             Number of obs  = 158
                                           Number of lags =   3

H0: Random walk without drift, d = 0

                                       Dickey-Fuller
                  Test      ———————— critical value ————————
             statistic           1%           5%          10%
─────────────────────────────────────────────────────────────
 Z(t)           -3.333       -3.491       -2.886       -2.576
─────────────────────────────────────────────────────────────
MacKinnon approximate p-value for Z(t) = 0.0135.
```

A1.3: Augmented Dickey-Fuller test for unit root for the Ester

Augmented Dickey–Fuller test for unit root

Variable: **ester**

Number of obs = **158**
Number of lags = **3**

H0: Random walk without drift, d = 0

	Test statistic	Dickey–Fuller critical value 1%	5%	10%
Z(t)	**-1.722**	**-3.491**	**-2.886**	**-2.576**

MacKinnon approximate *p*-value for Z(t) = **0.4198.**

A1.4: Augmented Dickey-Fuller test for unit root for the shadow rate elaborated by Volpi (2023).

Augmented Dickey–Fuller test for unit root

Variable: **shadowester**

Number of obs = **158**
Number of lags = **3**

H0: Random walk without drift, d = 0

	Test statistic	Dickey–Fuller critical value 1%	5%	10%
Z(t)	**-0.336**	**-3.491**	**-2.886**	**-2.576**

MacKinnon approximate *p*-value for Z(t) = **0.9203.**

A1.5: Augmented Dickey-Fuller test for unit root for the shadow rate elaborated by Wu and Xia

Augmented Dickey–Fuller test for unit root

Variable: **shadow_wuxia** Number of obs = **158**
Number of lags = **3**

H0: Random walk without drift, d = 0

	Test statistic	Dickey–Fuller critical value 1%	5%	10%
Z(t)	**-1.281**	**-3.491**	**-2.886**	**-2.576**

MacKinnon approximate *p*-value for Z(t) = **0.6376.**