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Dissecting Time-Varying Risk Exposures in Cryptocurrency Markets*

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Abstract

In this paper we take an empirical asset pricing perspective and investigate the dominant view (possibly, an instinctive reflection of the media hype surrounding the surge of Bitcoin valuations) that cryptocurrencies represent a new asset class, spanning risks and payoffs sufficiently different from the traditional ones. Methodologically, we rely on a flexible dynamic econometric model that allows not only time-varying coefficients, but also allow that the entire forecasting model be changing over time. We estimate such model by looking at the time variation in the exposures of major cryptocurrencies to stock market risk factors (namely, the six Fama French factors), to precious metal commodity returns, and to cryptocurrency-specific risk-factors (namely, crypto-momentum, a sentiment index based on Google searches, and supply factors, i.e., electricity and computer power). The main empirical results suggest that cryptocurrencies are not systematically exposed to stock market factors, precious metal commodities or supply factors with the exception of some occasional spikes of the coefficients during our sample. On the contrary, crypto assets are characterized by a time-varying but significant exposure to a sentiment index and to crypto-momentum. Despite the lack of predictability compared to traditional asset classes, cryptocurrencies display considerable diversification power in a portfolio perspective and as such they can lead to a moderate improvement in the realized Sharpe ratios and certainty equivalent returns within the context of a typical portfolio problem.

Keywords: Cryptocurrencies, predictability, portfolio diversification, dynamic model averaging, time-varying parameter regressions.

JEL codes: E40, E52.

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1 Introduction

In the wake of the appearance of cryptocurrencies in the space of media-covered phenomena and, more specifically, in the set of investable assets, one question has loomed large in the minds of financial analysts and portfolio managers: are they a novel, innovative asset class, potentially *segmented* away—i.e., driven by alternative economic forces and factors—from other, traditional asset classes? As a matter of fact, the highly fragmented, multi-platform, decentralised and granular nature of cryptocurrency markets adds plausibility to the conjecture that crypto assets may indeed be separated from traditional, centralized asset market exchanges.[1]

The property of segmentation acts as a double-edged sword in empirical finance and asset pricing. On the one hand, when an asset class is segmented, often this occurs because we understand less precisely what are the factors and the structure of the pricing kernel that manages to price the asset class, both in the time series and in the cross-sectional dimensions. This is of course a source of a difficulty that should advise investors to be cautious when they decide whether to allocate any portion of their wealth to the new asset. On the other hand, exactly because an asset class is segmented away from the rest of the asset menus, it tends to offer large and persistent diversification opportunities. An asset class driven by forces and factors that are not common to other assets may offer a considerable hedge, especially during market downturns.[2] Therefore, there is considerable debate as to whether and how cryptocurrencies may be segmented from traditional asset classes. Our paper contributes to the empirics of this debate with reference to the statistical and portfolio value properties of the major and most liquid cryptocurrencies.

In particular, in this paper we are driven by the belief that a particular appealing way to understand what cryptocurrencies are is to investigate whether their returns behave similarly to other asset classes in a *forecasting framework*. In other words, we assess how investors and markets value current and future prospects of cryptocurrencies. We use standard tools of empirical asset pricing to comprehensively analyze cryptocurrency risks and returns. Specifically, we study whether major cryp-

[1] For instance, Makarov and Schoar (2020) study the efficiency and price formation of Bitcoin and other cryptocurrencies. They show empirically that there are large and persistent arbitrage opportunities in cryptocurrency trading relative to fiat currencies across different exchanges. Similarly, Bianchi and Dickerson (2019) show that market activity is primarily driven by heterogeneous investors' trading that speculate on private, superior, information; consistent with the idea that trading in virtual currencies inherently generates heterogeneous beliefs due the opaqueness of the information flow.

[2] Segmentation is at best only sufficient for this implication to follow. Asset classes priced by the same pricing kernel may be uncorrelated or even negatively correlated when their respective loadings on the same factors that enter the pricing kernel are sufficiently different (e.g., when the loadings carry different signs).

tocurrencies *dynamically* co-move with stocks, currencies, commodities, macroeconomic variables, as well as with cryptocurrency market specific factors in ways that can be exploited to forecast their returns and—eventually, using a portfolio diversification approach—to generate positive economic value to investors. We tackle such an empirical endeavour by using a flexible time-varying econometric approach that eschews the perils of standard linear regression analysis.

It is widely acknowledged that recursive, regression methods suffer from three problems. First, the sensitivity of a the (expected) return to a change in a given risk factor may not necessarily be time invariant in the data generating process. There is a large literature in macroeconomics and finance which documents structural breaks and other sorts of parameter change in many time series (see, among many others, Stock and Watson, 1996; Bianchi et al., 2017). Recursive regression methods are poorly designed to capture such parameter changes: to build models designed to capture it often turns out to be a dominant strategy. Second, the number of potentially relevant predictors can be large and not known a priori, i.e., the nature and the number of risk factors that explain the dynamics of cryptocurrency returns is uncertain.[3] In light of this fact, an ever expanding literature has turned to Bayesian methods, either by performing Bayesian model averaging (BMA) or by automating the model selection process, e.g., see Maltritz and Molchanov (2013). Yet, even in these cases, computational demands can become daunting when the researcher is facing a number of models that grows with the power of the number of predictors, P, i.e., 2^P. Third, the model relevant to a forecasting application may potentially change over time (see, e.g., Pesaran and Timmermann, 2005). That means that not only there may be dynamics in a time-series sense, i.e., time-varying parameters, but also in a cross-sectional sense, i.e., the number of parameters that are significant at a given point in time can change throughout the sample. This issue further complicates an already daunting econometric exercise: if the researcher faces 2^P models and, at each point in time, a different predictive model applies, the number of combinations of models which must be estimated in order to forecast the target variable is $2^{T \times P}$. Even in relatively simple forecasting exercises, it can be computationally infeasible to forecast by simply going through such an enormous amount of model combinations.

In this paper, we follow a strategy first developed by Raftery et al. (2010) and then exploited in macroeconomic forecasting applications by Koop and Korobilis (2012), which is commonly referred to as dynamic model averaging (henceforth, DMA, see e.g., Wang et al., 2016). Their approach can

[3] If the set of models is defined by whether each of P potential predictors is included or excluded, then the researcher has 2^P models. This raises substantive statistical problems for model selection strategies.

also be used for dynamic model selection (DMS) in which a single (potentially varying) model is used to derive forecasts at each point in time. DMA and DMS seem ideally suited to yield maximum flexibility within the problem of predicting cryptocurrency returns as they allow for the predictive model to undergo structural change,at the same time, allowing for the coefficients in each model to evolve over time.[4]. Moreover, DMA and DMS only involve standard econometric methods routinely applied to state space models such as the Kalman filter but (via some empirically-sensible approximations) achieve vast gains in computational efficiency. DMS and DMA can be interpreted as applying shrinkage in a number of ways. In particular, DMS puts zero weight on all models other than the one best model, thus “shrinking” the contribution of all models except a single one towards zero. In fact, one cannot rule out a prior that this additional shrinkage may provide some additional predictive power over DMA, which just shrinks the model weights based on past predictive performance. Furthermore, in times of rapid change, DMS will tend to switch more quickly than DMA does because it can select an entirely new model as opposed to adjusting the weights within all the models. In any event, both DMA and DMS may allow for both gradual or abrupt changes in the role of a predictor. Standard time-varying parameter regressions by construction—especially when the coefficients are assumed to follow a random walk, as it is typical—fail to allow for abrupt changes and instead force smooth variation of the slope coefficients over time.

We systematically apply DMA and DMS techniques to the time-series returns of four major cryptocurrencies (Bitcoin, Ethereum, Litecoin, and Ripple) and five traditional asset classes (US stocks, worldwide developed country stocks ex-US, US investment grade corporate bonds, spot gold, and long-short positions equivalent to the trade-weighted US dollar exchange rate) over a 2011-2019 sample. The choice of these four cryptocurrencies is dictated by (1) the length of time time series available, (2) the considerable depth of market liquidity (being prominently traded cryptocurrencies), and (3) the fact that they have been regularly investigated in existing, related, research (see, e.g., Liu and Tsyvinski, 2018). The latter adds to the comparability of our results to the existing literature.

Empirically, we report four key results. First, cryptocurrencies are characterized by returns that are less predictable on average when compared to other asset classes, including gold and the trade value-weighted US dollar exchange rate. Both recursive, total mean-squared forecast error (MSFE) and relative out-of-sample (OOS) R-square measures show that while dynamic modeling strategies

[4]The issues caused by structural change in the risk-return trade-off characterizing cryptocurrency returns has also been investigated by Krueckeberg and Scholz (2019)

deliver relatively low MSFEs and positive (often high) OOS R-squares for traditional asset classes, this is not the case for cryptocurrencies. Second, crypto assets are characterized by returns that can be forecast according to patterns and with a measurable degree of time variation that differ from most other asset classes, including gold, which has been often indicated as the most closely related asset class (see Klein et al., 2018). For instance, consistently with the empirical results in Drobetz et al. (2019) and Li and Yi (2019), sentiment—here measured by the rate of growth of the Google searches concerning each cryptocurrency—proves to be a key prediction variable, which is clearly not the case with traditional asset classes. Third, in recursive asset allocation experiments, cryptocurrencies are able to generate considerable, realized OOS economic value (especially when measured in terms of ex-post Sharpe ratios) when they are added to otherwise traditional asset menus of cash, corporate bonds, US and international stocks, and long-short exchange rate positions. However, cryptocurrencies are unable to offer much advantage in terms of realized, risk-adjusted portfolio performances deriving from any predictability patterns characterizing them. Finally, the value of cryptocurrencies cannot be reduced only to the fact the investors may have access to long positions in Bitcoin, in the sense that also Ethereum, Litecoin, and Ripple appear to generate substantive OOS realized economic value when they are added to the asset menu in addition to Bitcoin, also because they offer diversification benefits vs. Bitcoin returns.

1.1 Related literature

This paper contributes to three main strands of research. First, this paper adds to a growing literature that aims at understanding the investment properties of cryptocurrencies. Yermack (2015) and Dyhrberg (2016) have investigated the diversification properties of Bitcoin within the context of a diversified portfolio and reached opposite conclusions. In particular, Yermack (2015) argues that Bitcoin is uncorrelated with the majority of fiat currencies and is much more volatile, therefore being of limited usefulness for risk management purposes and diversification. Bianchi (2019) uses a larger set of cryptocurrencies to find that, except for a mild correlation with the returns on precious metals, there is no significant relationship between returns on cryptocurrencies and global proxies of traditional asset classes. He also reports that standard macroeconomic factors such as in inflation expectations, the yield curve, and real exchange rates do not play a significant explanatory role, lending support to a segmentation of cryptocurrency markets with respect to more traditional asset classes. Borri (2019) has shown that the returns dynamics of major cryptocurrencies is exposed to tail risk possibly due to trading activity. Similarly to these papers, we find that adding cryptocurrencies—importantly not only Bitcoin—to a standard asset menu may generate considerable economic value, despite the scant predictability evidence.

The most closely related paper is Liu and Tsyvinski (2018) who have established that the risk-return trade-off of the same, major cryptocurrencies investigated in our paper is distinct from those of stocks, currencies, and precious metals. However, differently from their in-sample investigation simply based on multivariate linear regressions, we adopt a flexible dynamic modeling approach to study the predictability of cryptocurrency returns and resorts to OOS recursive asset allocation experiments to measure the ex-post realized economic value of both predictability and of cryptocurrency as an alternative asset class.

Secondly, we contribute to growing literature on the economics of cryptocurrency markets. Examples are Gandal et al. (2018), Makarov and Schoar (2020), Sockin and Xiong (2018), Foley et al., 2019; Bianchi and Dickerson (2019). For instance, Gandal et al. (2018) and Foley et al. (2019) show that the trading activity and the consequent price dynamics may not necessarily be driven by risk sharing purposes but rather by fraudulent behaviours. Similarly, Makarov and Schoar (2020) and Bianchi and Dickerson (2019) show empirically that there are large and persistent arbitrage opportunities in cryptocurrency trading relative to fiat currencies across different exchanges, and that the crypto returns dynamics is primarily driven by investors who "speculate" on private information, respectively. We complement and extend this literature by showing that the return dynamics of cryptocurrencies cannot be reconciled by standard risk factors. That is, cryptocurrency markets may be indeed segmented away from traditional asset classes.

Third, ontribute to a large literature on time-varying return predictability. Among many others, examples are Pastor and Stambaugh (2009), Rapach et al. (2010), van Bisbergen and Koijen (2010), Dangl and Halling (2012), Pettenuzzo et al. (2014), Johannes et al. (2011), and Zhu (2013). We extend this literature by looking at the predictive power of otherwise standard risk factors within the context of cryptocurrency markets. Note that the aim of our paper is not to overthrow existing results from the traditional returns predictability literature, but rather to draw a direct comparison with the existing research on other asset classes in order to better understand the economics of cryptocurrency markets. In this respect, due to the institutional differences, we view this paper as an out-of-sample test of existing evidence on time-varying predictability developed in more traditional financial markets.

The rest of the paper is organized as follows. Section 2 introduces the empirical methodology applied in the paper. Section 3 describes data sources and the main features of the series investigated. Section 4 reports detailed empirical results derived from the estimation of DMA and DMS models for each cryptocurrency under investigation. Section 5 seeks direct answers to our key question concerning the segmentation of cryptocurrencies, by comparing their forecasting features and their realized OOS predictive and portfolio performances with traditional asset classes. Section 6 concludes.

2 Methodology

Let's start from a standard, off-the-shelf time varying parameter (TVP) regression model specification for the return on some generic, jth asset or security (see, e.g., Cogley and Sargent, 2005, Dangl and Halling, 2012, Bianchi et al., 2017, Bianchi and McAlinn, 2018, and Guidolin et al., 2019 among others):

$$r^j_{t+1} = \theta^j_{0,t} + \sum_{p=1}^{P}(\theta^j_{p,t})' z^j_{p,t} + \epsilon^j_{t+1} = (\theta^j_t)'\mathbf{z}^j_t + \epsilon^j_{t+1} \qquad t = 1, ..., T,\ j = 1, ..., J \tag{1}$$

$$\theta^j_t = \theta^j_{t-1} + \eta_t \qquad t = 1, ..., T,\ j = 1, ..., J \tag{2}$$

where $\mathbf{z}^j_t$ is a jth asset-specific $(P+1)\times 1$ vector of predictors potentially specific to currency/asset j (always including a unit constant to absorb the intercept coefficient and possibly encompassing lags of r^j_{t+1}), θ^j_t is a $(P+1)\times 1$ of possibly time-varying coefficients (states), ϵ^j_t IID $N(0, h^j_t)$, η_t IID $N(\mathbf{0}, \mathbf{L}^j_t)$, and the errors ϵ^j_t and η_t are assumed to be mutually independent at all leads and lags and for all currencies or assets.

This model displays all the flexible power of TVP models, especially over and above constant coefficient models (including the case in which these are estimated recursively). Yet, in forecasting applications, (1)-(2) suffer from the drawback that the same set of explanatory variables is assumed to be relevant at all points in time. Additionally, when P is large, the potential for such a rich TVP regression to over fit the data appears to be substantial, which may rapidly damage the predictive power of the overall framework. Following Koop and Korobilis (2012), we overlay on this model a simple technique to let alternative models to hold at different points in time while their coefficients remain time-varying and average across them.

Suppose to have a set of Q models which are characterized by having different subsets of $\mathbf{z}_t^j$ as predictors. Denoting these by $\mathbf{z}_t^j(q)$ for $q = 1, 2, ..., Q$, our set of models can be written as (we have dropped the j index for clarity) of:

$$r_{t+1} = \theta_t{}'(q)\mathbf{z}_t(q) + \epsilon_{t+1}(q) \qquad t = 1, ..., T,\ q = 1, ..., Q \tag{3}$$

$$\theta_t(q) = \theta_{t-1}(q) + \eta_t(q) \qquad t = 1, ..., T,\ q = 1, ..., Q, \tag{4}$$

where $\epsilon_t(q)$ IID $N(0, h_t(q))$, $\eta_t(q)$ IID $N(\mathbf{0}, \mathbf{L}_t(q))$, and the errors are mutually independent at all leads and lags. Let $M_t \in \{1, 2, ..., Q\}$ denote which model applies at each time period t, which implies that we shall be letting different models hold at each point in time but dynamically average across them, i.e., when forecasting time $t + 1$ returns using information through time t:

- DMA involves calculating $Pr(M_t = q|r_t, r_{t-1}, ..., r_1)$ for $q = 1, 2, ..., Q$ and averaging forecasts across models using these probabilities;
- DMS involves selecting the single model with the highest value for $\Pr(M_t = q|r_t, r_{t-1}, ..., r_1)$ and using this to forecast.

However, both DMA and DMS are cursed by a inherent risk of over-parameterization while the computational burden which arises when Q is large remains a high hurdle. In particular, although the natural approach is to let the model selection to be driven by a latent Markov chain so that the how predictors enter/leave the model in real time is simply captured by a transition matrix, $\mathbf{T}$, with elements $\tau_{ij} = \Pr(M_{t+1} = j|M_t = i)$, see e.g., Guidolin (2011), inference in such models is theoretically straightforward, but computationally infeasible because $\mathbf{T}$ will be considerably large for interesting choices of the number of predictors, P.[5] However, in our implementation of DMA, standard Kalman filter methods to be ran only Q times can be used at the cost of skipping the exact specification of a transition matrix and therefore giving up on the exact structure of Markov switching predictive regressions. As we shall explain below (or see Raftery et al., 2010, for an exhaustive treatment), the econometric framework (3)-(4) implies that the entire state vector, t, breaks into blocks (with one block for each model) which are independent of one another (i.e. the predictive density depends on $\theta_t(q)$ only conditionally on $M_t = q$).

[5]For instance, in the case in which the models are defined according to whether each predictor is included or excluded, $Q = 2^P$ so that $\mathbf{T}$ will be a $2^P \times 2^P$ matrix and will contain (because of sum up constraints) 2^{2P-1} parameters to estimate. For instance, with $P = 17$, we shall be facing $2^{33} = 8,589,934,592$ free parameters to estimate, so that problem will become computationally unfeasible.

The approximation proposed by Koop and Korobilis (2012) depends on two parameters, λ and ψ, which we shall call forgetting factors and set to be slightly below one, to allow decay over time to occur. Their role is best explained in a standard Kalman filter iteration when model switching is (just for expositional purposes) ignored. For given values of h_t and $\mathbf{L}_t$, standard Kalman filtering starts with $\theta_t | r_t, r_{t-1}, ..., r_1 \sim N(\hat{\theta}_t, \mathbf{\Sigma}_{t|t})$, where $\mathbf{\Sigma}_{t|t}$ depends on h_t and $\mathbf{Q}_t$. Then filtering proceeds using:

$$\theta_{t+1} | r_t, r_{t-1}, ..., r_1 \sim N(\hat{\theta}_t, \mathbf{\Sigma}_{t+1|t}) \qquad \mathbf{\Sigma}_{t+1|t} = \mathbf{\Sigma}_{t|t} + \mathbf{L}_{t+1}. \tag{5}$$

Raftery et al. (2010) have noted that the computational burden simplifies substantially when the updating recursion is simplified to:

$$\mathbf{\Sigma}_{t+1|t} = \frac{1}{\lambda} \mathbf{\Sigma}_{t|t} = \mathbf{\Sigma}_{t|t} + \left(\frac{1}{\lambda} - 1 \right) \mathbf{\Sigma}_{t|t}, \tag{6}$$

or equivalently $\mathbf{L}_{t+1} = \left(1 - \frac{1}{\lambda}\right) \mathbf{\Sigma}_{t|t}$ where $0 < \lambda \leq 1$. Importantly, this formula does not depend on the estimate of $\mathbf{L}_{t+1}$. In line with the empirical literature, we set $\lambda = 0.99$.

Estimation in the one model case is then completed by the updating equations:

$$\begin{aligned}
\theta_{t+1} | r_{t+1}, r_t, r_{t-1}, ..., r_1 &\sim N(\hat{\theta}_{t+1}, \mathbf{\Sigma}_{t+1|t+1}) && (7)\\
\hat{\theta}_{t+1} &= \hat{\theta}_t + \mathbf{\Sigma}_{t+1|t} \mathbf{z}_t (h_{t+1} + \mathbf{z}_t' \mathbf{\Sigma}_{t+1|t+1} \mathbf{z}_t)^{-1} (r_{t+1} - \theta_t{}' \mathbf{z}_t) && (8)\\
\mathbf{\Sigma}_{t+1|t+1} &= \mathbf{\Sigma}_{t+1|t} - \mathbf{\Sigma}_{t+1|t} \mathbf{z}_t (h_{t+1} + \mathbf{z}_t' \mathbf{\Sigma}_{t+1|t} \mathbf{z}_t)^{-1} \mathbf{z}_t' \mathbf{\Sigma}_{t+1|t} && (9)\\
r_{t+1} | r_t, r_{t-1}, ..., r_1 &\sim N(\hat{\theta}_t' \mathbf{z}_t, h_t + \mathbf{z}_t' \mathbf{\Sigma}_{t+1|t} \mathbf{z}_t). && (10)
\end{aligned}$$

Conditional on h_t, these results are analytical and, thus, no Markov chain Monte Carlo (MCMC) algorithm is required. This greatly reduces the computational burden.

In the case of DMA, we use this approximation of the Kalman filter and an additional one. Call Θ_t the vector collecting all coefficients, $\Theta_t \equiv [\theta_t'(1) \; \theta_t'(2) \; ... \; \theta_t'(Q)]'$. First, the previous filtering equations for θ_{t+1} now become:

$$\begin{aligned}
\Theta_t | M_t &= q, r_t, r_{t-1}, ..., r_1 \sim N(\hat{\theta}_t(q), \mathbf{\Sigma}_{t|t}(q)) && (11)\\
\Theta_{t+1} | M_{t+1} &= q, r_t, r_{t-1}, ..., r_1 \sim N(\hat{\theta}_t(q), \mathbf{\Sigma}_{t+1|t}(q)) && (12)\\
\Theta_{t+1} | M_{t+1} &= q, r_{t+1}, r_t, r_{t-1}, ..., r_1 \sim N(\hat{\theta}_{t+1}(q), \mathbf{\Sigma}_{t+1|t+1}(q)), && (13)
\end{aligned}$$

where $\hat{\theta}_t(q)$, $\mathbf{\Sigma}_{t|t}(q)$, and $\mathbf{\Sigma}_{t+1|t}(q)$ are computed from (8), (9), and (6), respectively. However, conditional on $M_t = q$, the prediction and updating equations will only provide information on $\theta_t(q)$ and not the full vector Θ_t: one needs a method for unconditional prediction to obtain probability-weighting of forecasts across values for $\hat{\theta}_t(q)$. We follow Koop and Korobilis (2012) and introduce one additional, forgetting-like factor to be applied to the state equation across models, ψ, comparable to the forgetting factor used with the state equation for the parameters, λ. Note that by definition the probability transition equation is,

$$\begin{aligned} pdf(\Theta_t|r_t, r_{t-1}, ..., r_1) &= \sum_{q=1}^{Q} pdf(\theta_t|M_t = q, r_t, r_{t-1}, ..., r_1) \Pr(M_t = q, r_t, r_{t-1}, ..., r_1) \\ &= \sum_{q=1}^{Q} pdf(\theta_t|M_t = q, r_t, r_{t-1}, ..., r_1)\pi_{t|t,q}, \end{aligned} \tag{14}$$

where $pdf(\theta_t|M_t = q, r_t, r_{t-1}, ..., r_1)$ comes from $\theta_t(q)|r_t, r_{t-1}, ..., r_1 \sim N(\hat{\theta}_t(q), \mathbf{\Sigma}_{t|t}(q))$. In the case of a standard Markov switching model, when the elements of the transition matrix $\mathbf{T}$ are $\tau_{ij} = \Pr(M_{t+1} = j|M_t = i)$, (14) would be:

$$\pi_{t+1|t,q} = \sum_{l=1}^{Q} \pi_{t|t,l}\tau_{lq}, \tag{15}$$

but we approximate it by:

$$\pi_{t+1|t,q} = \frac{\pi_{t|t,q}^{\psi}}{\sum_{l=1}^{Q} \pi_{t|t,l}^{\psi}} \qquad 0 < \psi \leq 1 \tag{16}$$

which is a type of multi-parameter power steady model. To understand further how the forgetting factor can be interpreted, note that this specification implies that the weight used in DMA which is attached to model q at time t is updated according to:

$$\begin{aligned} \pi_{t+1|t,q} &= \frac{[\pi_{t|t-1,q} pdf(r_{t-1}|M_{t-1} = q, r_{t-2}, ..., r_1)]^{\psi}}{\sum_{l=1}^{Q} [\pi_{t|t-1,l} pdf(r_{t-1}|M_{t-1} = l, r_{t-2}, ..., r_1)]^{\psi}} \\ &\propto [\pi_{t-1|t-2,q} pdf(r_{t-2}|M_{t-2} = q, r_{t-3}, ..., r_1)]^{\psi^2} [pdf(r_{t-1}|M_{t-1} = q, r_{t-2}, ..., r_1)]^{\psi} \\ &\propto ... \propto \prod_{i=1}^{t-1} [pdf(r_{t-i}|M_{t-i} = q, r_{t-i-1}, ..., r_1)]^{\psi^i} \pi_{0|0,q}^{\psi^t}, \end{aligned} \tag{17}$$

so that model q receive more weight at time t if it has forecast well in the recent past (where forecast performance is measured by the predictive density, $pdf(r_{t-i}|M_{t-i} = q, r_{t-i-1},..., r_1)$). The interpretation of "recent past"is controlled by the forgetting factor, and we have the same exponential decay at the rate ψ^i for observations i periods ago. Clearly, it is not enough for one time performance in the past to highly disappointing, i.e., $pdf(r_{t-i}|M_{t-i} = q, r_{t-i-1},..., r_1) \simeq 0$, for $\pi_{t+1|t,q} \simeq 0$ because the expression in (17) fails to iterate over a scaling factor which may also get very close to zero when in the past all the moments have produced some performances characterized by very small predictive density scores.[6] In line with the empirical literature, and also for consistency with our earlier choice for λ, we set $\psi = 0.99$.

The benefit of using this approximation in the model prediction equation is that we do not require an MCMC algorithm to draw transitions between models nor a simulation algorithm over the space of models. The reason is that a simple, but effective updating equation is:

$$\pi_{t|t,q} = \frac{\pi_{t|t-1,q} \cdot \overbrace{pdf(r_t|M_t = q, r_{t-1}, ..., r_1)}^{\text{from}\sim N(\hat{\theta}'_{t-1}(q)\mathbf{z}_{t-1}, h_t + \mathbf{z}'_{t-1}\mathbf{\Sigma}_{t|t-1}(q)\mathbf{z}_{t-1})}}{\sum_{l=1}^{Q} \pi_{t|t-1,l} \cdot \underbrace{pdf(r_t|M_t = l, r_{t-1}, ..., r_1)}_{\text{from}\sim N(\hat{\theta}'_{t-1}(l)\mathbf{z}_{t-1}, h_t + \mathbf{z}'_{t-1}\mathbf{\Sigma}_{t|t-1}(l)\mathbf{z}_{t-1})}}. \tag{18}$$

At this point, recursive forecasting can be performed by averaging over predictive results for every model using $\pi_{t+1|t,q}$ to perform the weighing across models. Therefore, DMA point predictions are given by:

$$\hat{r}^{DMA}_{t+1|t} \equiv E[r_{t+1}|r_t, r_{t-1}, ..., r_1] = \sum_{l=1}^{Q} \pi_{t+1|t,l}\hat{\theta}'_t(l)\mathbf{z}_t = (\hat{\theta}^{DMA}_{t+1|t})'\mathbf{z}_t(l), \tag{19}$$

where $\hat{\theta}^{DMA}_{t+1|t} \equiv \sum_{l=1}^{Q} \pi_{t+1|t,l}\hat{\theta}_t(l)$ is the filtered, real-time forecast of the predictive regression coefficients applicable at time $t+1$ given the information available at time t. In any event, this framework implies that forecasting is performed in real time in the sense that, for all our variables, we use the value which would have been available to the forecaster at the time the forecast was being made with no hindsight bias possible.

DMS proceeds instead by selecting the single model with the highest value for $\pi_{t+1|t,q}$ for $q = 1, 2,..., Q$ at each point in time and simply using it for forecasting:

$$\hat{r}^{DMS}_{t+1|t} = \hat{\theta}'_t(\hat{k}_t)\mathbf{z}_t(\hat{k}_t) \qquad \hat{k}_t \equiv \arg\max_{k=1,...,Q} \pi_{t+1|t,k}. \tag{20}$$

Of course, also the DMS forecast is a real-time one. In this case, we can define $\hat{\theta}^{DMS}_{t+1|t} \equiv \hat{\theta}_t(\hat{k}_t)$ and write about a filtered, sup-type real time forecast of the predictive regression coefficients applicable

[6]Of course, if we were to set $\psi = 1$, then $\pi_{t+1|t,q}$ would be simply proportional to the marginal likelihood using data through time t, which is a standard Bayesian model averaging (BMA) approach. However, one formally has BMA only when $\lambda = \psi = 1$ which implies using conventional linear forecasting models with no time variation in coefficients.

at time $t+1$ given the information available at time t.

In summary, conditional on knowing or estimating h_t for $t = 1, 2, ..., T$, the DMA and DMS algorithms surveyed above only involve updating formulas that identical or approximations of typical Kalman filter iterations. All recursions are simply started out by selecting a prior for $\pi_{0|0,q}$ and $\hat{\theta}_0(q)$, $q = 1, 2, ..., Q$. In a very pragmatic way, we set $\pi_{0|0,q} = 1/Q$ and $\hat{\theta}_0(q) = \mathbf{0}$ for all values of the state q, to allow the data to truly express the existence of any predictability relation.[7] As for $h_t(q)$, we follow Raftery et al. (2010) and simply proceed to plug in place of $h_t(q)$ a consistent estimate that—in line with existing applied work using DMA and DMS methods—is simply based on an Exponentially Weighted Moving Average (EWMA) estimator,

$$\begin{aligned} \hat{h}_{t+1}(q) &= \sqrt{(1-\delta)\sum_{i=1}^{t+1}\delta^{i-1}(r_{t+1-i} - \hat{\theta}'_{t-i}(q)\mathbf{z}_{t-i}(q))^2} \\ &= \sqrt{\delta\hat{h}_t(q) + (1-\delta)(r_t - \hat{\theta}'_t(q)\mathbf{z}_t(q))^2} \end{aligned} \tag{21}$$

with $\hat{h}_0(q) = T^{-1}\sum_{i=1}^{T}(r_{t+1-i} - \hat{\theta}'_{t-i}(q)\mathbf{z}_{t-i}(q))^2$. Notice that, in view of our use of weekly data we set the RiskMetrics-style parameter to $\delta = 0.96$ to guarantee a not too slow decay.

3 The Data

We use weekly returns and growth rate series throughout. Cryptocurrency price data series are from the CoinDesk website. For Bitcoin, we use data from December 27, 2010 to January 27, 2019 because there was not much liquidity and trading in earlier years. The data series for Litecoin ranges from July 14, 2013 to January 27, 2019; the one for Ripple from April 5, 2015 to January 27, 2019; finally, the price series for Ethereum are the shortest, spanning a May 30, 2016 - January 27, 2019 sample. In all cases, samples are relatively short than what may be nominally available, because these minor cryptocurrencies saw trading and liquidity increase only between 2013 and 2015. We construct cryptocurrency return series using the corresponding price data.

The spot price of precious metals are from several sources. The gold and silver prices are from the London Bullion Market Association (LBMA). Platinum prices are from the London Platinum and

[7]Note that $\mathbf{\Sigma}_{1|1}(q) = \mathbf{\Sigma}_{1|0}(q) - \mathbf{\Sigma}_{1|0}(q)\mathbf{z}_0(h_1(q) + \mathbf{z}'_0\mathbf{\Sigma}_{1|0}(q)\mathbf{z}_0)^{-1}\mathbf{z}'_0\mathbf{\Sigma}_{1|0}(q)$ just requires knowledge of $\mathbf{\Sigma}_{1|0}(q) = \frac{1}{\lambda}\mathbf{\Sigma}_{0|0}(q)$. We set $\mathbf{\Sigma}_{0|0}(q) = 100\hat{h}_0(q)\mathbf{I}_{P+1}$, where the last estimate is specified in the main text. This prior is of course, very diffuse and reflects considerable, initial uncertainty on the financial nature of crytpocurrencies as an asset class.

Palladium Market (LPPM). Also in this case, we compute return series from prices without including any value of carry. Even though spot precious metals are not directly or easily tradable, we have checked that the returns from 1-month futures contracts (obtained from Reuters Datastream) all give correlations between 0.97 and 0.99 with the spot returns obtained from spot prices.

Aggregate and individual stock return data are from CRSP. We obtain the Fama French 3-factor, Carhart 4-factor, and Fama French 5-factor models dataseries from Kenneth French's website. The data on corporate bond returns are from the ICE BofAML US Corporate Master Index, which tracks the performance of US dollar denominated investment grade rated corporate debt publicly issued in the US domestic market.[8]

Because earlier literature (see, e.g., Drobetz et al., 2019 and Li and Yi, 2019) has shown the importance of sentiment in explaining cryptocurrency returns, Google search data series are downloaded from Google.[9] To proxy for supply factors, we get back to basics and acknowledge that mining a cryptocurrency requires two inputs:[10] electricity and computer power. For electricity, we consider two proxies, i.e., the value-weighted stock returns of U.S.-listed electricity firms and the value-weighted stock returns of the China-listed electricity companies. The reason why we include the China proxies is because electricity supply is location specific and because China is considered to have the largest coin mining operation among all countries (see, e.g., Li et al., 2019). For proxies of computer power, we consider the stock returns of the companies that are major manufacturers of either GPU mining chips (Nvidia Corporation and Advanced Micro Devices, Inc, AMD for short) or ASIC mining chips (Taiwan Semiconductor Manufacturing Company, Limited and Advanced Semiconductor Engineering, Inc, TSMC fort short), see, e.g., Liu and Tsyvinski (2018).

Table 1 describes the series under analysis. While the four cryptocurrencies span partly by construction, partly by choice different sample periods—while always providing sufficiently long sample

[8]To qualify for inclusion in the index, securities must have an investment grade rating (based on an average of Moody's, S&P, and Fitch) and an investment grade rated country of risk (based on an average of Moody's, S&P, and Fitch foreign currency long term sovereign debt ratings). Each security must have greater than 1 year of remaining maturity, a fixed coupon schedule, and a minimum amount outstanding of $250 million.

[9]Da et al. (2011) uses Google searches to proxy for investor attention.

[10]Mining is the process of adding transaction records to a cryptocurrency public ledger of past transactions; this ledger of past transactions is called the block chain as it is a chain of blocks. The primary purpose of mining is to set the history of transactions in a way that is computationally impractical to modify by any one entity. The blockchain serves to confirm transactions to the rest of the network as having taken place. Cryptocurrency nodes use the blockchain to distinguish legitimate transactions from attempts to re-spend coins that have already been spent elsewhere. Mining is intentionally designed to be resource-intensive and difficult so that the number of blocks found each day by miners remains under control. Mining is also the mechanism used to introduce new crypto coins into the system: Miners are paid any transaction fees as well as a "subsidy" of newly created coins. This both serves the purpose of disseminating new coins in a decentralized manner as well as motivating people to provide security for the system.

periods of investigation, between 139 and 422 weekly observations—we report summary statistics for the remaining portfolio and asset returns as well for the non-financial predictors with reference to the longest, 2002-2019 sample. The first panel reports summary statistics on cryptocurrency returns. A few stylized facts well known from the literature (see, e.g., Liu and Tsyvinski, 2018) emerge. All cryptocurrencies carry very high mean weekly returns, in the order of 150% (for Ethereum) - 230% (for Ripple) in annualized terms. This is due the rapid growth in the value of cryptocurrencies experimented between 2014 and 2017. However such high mean returns are countered by two features. First, all cryptocurrencies are characterized by astonishingly high realized weekly volatilities, ranging between 18 and 32 percent (which amounts to a volatility of 125 - 210 percent in annualized terms). Second, on top of such high volatilities, all cryptocurrencies (but Ethereum, for which the recorded series are however shorter than for the rest of our data), total massive excess kurtosis coefficients, between 17 and 57. This means that, even after discounting their high variability as measured by their second moment, crypto returns are plagued by massive tails, which reflects that hugely negative and positive returns are always possible. One last feature of cryptocurrency returns is interesting: their weekly median returns are systematically much lower (even negative in the case of Litecoin and Ripple) vs. weekly mean returns. Even though an analysis of the empirical distributions of cryptocurrency returns reveals that multiple modes are possible, medians that are systematically lower than means are indicative (this is sufficient in the case of uni-modal distributions) of positive skewness, that indeed appears to be considerable, between 0.82 (Ethereum) and 5.94 (Litecoin). As already commented by Liu and Tsyvinski (2018), cryptocurrencies are assets representing "miracles" (as well as "disasters", as revealed by the massive excess kurtosis) or, equivalently, are a kind of securitized lottery tickets. In spite of their large variability, all cryptocurrencies are characterized by very attractive weekly Sharpe ratios, between 0.11 for Litecoin (that however has an extremely appealing right-skewness) to a stunning 0.19 for Bitcoin, which has probably contributed to its fame and to a recent drive towards introducing derivatives trading having Bitcoin as the underlying asset.[11]

The empirical facts about the other asset/portfolio returns are generally better known, with a few surprises. On the one hand, the value-weighted market excess returns yield an annualized mean return of approximately 8%, annualized volatility of just less than 14%, negative skewness (which makes aggregate stock market returns structurally different from cryptocurrency returns) and some non-negligible excess kurtosis that is however much lower vs. the case of crypto. The Sharpe ratio

[11] See, e.g., A. Osipovic, "Another Exchange Jumps on Bitcoin Bandwagon", the *Wall Street Journal*, 12 March 2018.

is between half and two-thirds compared to cryptocurrencies, which is largely known. The surprises come from most of the five Fama-French factor-mimicking long-short portfolio: SMB (representing size), HML (value), and CMA (investment) portfolio returns are all negative, either on average or in median terms. It is well known that the so-called "smart beta" factor portfolios are smart because they generate positive risk premia. However, such a requirement is to be intended to apply on average, unconditionally and over long periods of time: this explains why on our 10-year data sample, it occurs that many such factors fail to return a positive premium. In any event, the RMW (the quality) and Carhart's momentum factors do return large premia and strong Sharpe ratios. While gold, platinum and silver appear to have declined on average and therefore to have yielded performances very different from the cryptocurrencies, the three individual stock return series (especially NVIDIA and AMD, the producers of GPU mining chips) display properties that tend to be consistent with the general summary statistics for the cryptocurrencies.

Finally, the Google searches for the world "bitcoin" tend to grow on average but this happens through sudden jumps, as revealed by the fact that their median growth is in fact negative, the skewness is positive (4.33) and large and excess kurtosis is massive (29.2), similarly to the returns on cryptocurrencies.

4 Recursive Empirical Estimates

Using the methodologies of Section 2, we proceed to the recursive estimation of (3)-(4) for each of the cryptocurrency returns series, using in each of the four runs of the DMA and DMS algorithms all the available data. The selection of the initialization ("prior") parameters ($\pi^j_{0|0,q}$, $\hat{\theta}^j_0(q^j)$, $\mathbf{\Sigma}^j_{0|0}(q^j)$, $q^j = 1, 2, ..., Q^j$, j = Bitcoin, Litecoin, Ripple, and Ethereum) and of the forgetting factors (λ and ψ) are the ones reported in Section 2. In our exercises, we use $P = 17$ for all currencies which implies that Q^j = 131,072, which is a rather enormous number of models to deal with![12] To ease the interpretation of the estimated coefficients and of the resulting plots, the predictors in $\mathbf{z}_t$ are standardized by dividing their values by their recursively estimated standard deviation, as in Koop and Korobilis (2012).

[12]Instead a constant intercept is always included in all models.

4.1 Bitcoin

With reference to Bitcoin data, in Figure 1, we report 15 out of the available 18 plots for the recursive estimates of the vector $\hat{\theta}^{DMA}_{t+1|t}$ between January 2011 and January 2019. We include the estimated constant $\hat{\theta}^{DMA}_{0,t+1|t}$ but we exclude—because of space constraints and just to keep the resulting plots sufficiently readable—the graphs concerning the predictive slope coefficients for AMD and TSMS stock returns and for the returns on a value-weighted portfolio of U.S.-listed electricity firms. The plots remain available upon request but in what follows we express a few simple comments on their shape and dynamics. In Figure 1, for each estimated coefficient, we report the same values on a double scale. On left-hand axis, we adopt a scale homogeneous across different plots that ranges between -2 and +2. Therefore, on the left-hand scale, we can interpret the elements of $\hat{\theta}^{DMA}_{t+1|t}$ to measure the cryptocurrency return reaction to a one-standard deviation shock to the predictor on the right-hand side, when all other predictors are held constant. Clearly, coefficients exceeding ± 1 are predictors that cause a more than proportional effect on cryptocurrency returns, while coefficients close to zero indicate a small or no reaction of a cryptocurrency to a predictor. On the right-hand side, we report the estimated coefficients on a scale that varies instead across different predictors, for better readability of the variation of the coefficients over time. However, it must be stressed that in both cases, the estimates that are plotted are identical, just represented on different scales to allow for more meaningful interpretation and commentary.

The thicker lines, to be compared with the left-scale in Figure 1 reveal that—with minor exceptions concerning 2011-2012 for most coefficients, NVDIA stock returns between 2013 and 2015, and mostly importantly the rate of growth of Google searches concerning the word "Bitcoin" (consistently with the role of sentiment in Drobetz et al., 2019 and Li and Yi, 2019)—there is very limited overall predictability, across model averages, of Bitcoin returns: most of the time and for most predictors (apart from the Google search variable), the estimated coefficients carry values close to zero. The thinner curves (see the right scale) show that even if the slopes are modest, they display considerable variation: also because of the small of observations the DMAs are based on, they oscillate considerably early on in the sample, and many of them tend to spike up and increase between 2013 and 2015, when Bitcoin returns seemed to turn more predictable than on the average of the full 2011-2019 sample. The estimated constant plays a key role, taking a positive and large values that fits the initial rapid growth of Bitcoin prices, during the first half of 2011, when $\hat{\theta}^{DMA}_{0,t+1|t}$ declines from more than 3% to less than 0.3% per week and then fluctuates between 0 and 0.4% per week.

The top panel of Table 2 presents statistics summarizing the information in Figure 1. The means and medians of the recursively estimated DMA slopes are generally modest and very far off the ± 1 one-to-one reactivity. The only partial exception is $\hat{\theta}^{DMA}_{p,t+1|t}$ for $p =$ Google searches of the word "Bitcoin", characterized by a mean of 0.12 and by a median of 0.09. Moreover, the coefficient oscillates in sign but it seems to be above one in absolute value continuously between 2011 and 2017, with rare interruptions: Bitcoin is an asset dominated by the "wisdom of the crowds", dubbing the famous title by Chen et al. (2014), and is therefore a sentimental asset. Sensibly enough, as the "buzz" concerning Bitcoin spreads, its returns are predicted to be higher, although the marginal impact is rather modest (on average, 12 basis points per each standard deviation increase in the weekly growth of the searches). Interestingly, after the Google searches are taken into account, per se Bitcoin momentum or even a simple AR(1) term turn out to imply on average modest coefficient estimates. More generally—as shown by the boldfacing in the last two columns of the table—for 5 predictors out of 17, we have evidence of their empirical (probability-weighted across models) 90% confidence band failing to include a zero coefficient: the VW excess market return (a vestige of CAPM, albeit with modest coefficients), the returns on the investment factor-mimicking portfolio, spot silver returns, and especially the rate of growth of Google searches, as already emphasized. In fact, we have also computed the unconditional average number of predictors included by the DMA algorithm using the straightforward formula

$$Size^{DMA}_{j} \equiv \sum_{t=1}^{T^j-1} \sum_{l=1}^{Q^j} \pi_{t+1|t,l} Size(\hat{\theta}^{j}_{t}(l)) \qquad j = \text{Bitcoin, Litecoin, Ripple, and Ethereum}, \tag{22}$$

where $Size(\hat{\theta}^{j}_{t}(l))$ returns the number of predictors included in model l at time t. It turns out that under DMA, $Size^{DMA}_{Bitcoin} = 4.66$, which appears to be rather modest. Inspection of unreported plots of $Size(\hat{\theta}^{Bitcoin}_{t}(l))$ shows that the average size of the models favored by the recursively updated probabilities increases sensibly (to exceed 6) between 2014 and 2015 and in the final part of the sample, after 2017. However, there are long spells (2012-2013 and 2016) in which $Size(\hat{\theta}^{Bitcoin}_{t}(l))$ falls below two: basically nothing but Google searches can forecast Bitcoin returns.

Figure 2 has a structure similar to Figure 1, but instead of showing $\hat{\theta}^{DMA}_{t+1|t}$, it plots the time t probability of each predictor to be included across the $Q^{Bitcoin} = 131{,}072$ models entertained in the paper, $\Pr(z^j_{p,t}$ included) $\equiv \sum_{l=1}^{Q} \pi_{t+1|t,l}\, I\left\{\hat{\theta}_{p,t}(l) \neq \varnothing\right\}$, where $I\left\{\hat{\theta}_t(l) \neq \varnothing\right\}$ indicates that predictor $p = 1, 2, ..., P$ is included in the vector $\mathbf{z}_t(l)$. Although this does not apply in general, the plots indicate that the reason for the low estimated $\hat{\theta}^{DMA}_{t+1|t}$ vectors in Figure 1 and in the summaries in Table 2 does not really derive from negligible estimated probability of inclusion, but from generally modest estimates of the regression coefficients. For instance, Bitcoin has a low DMA "beta" relative to the market portfolio not because the latter is a predictor carrying a low probability of inclusion (as this exceeds on average 0.35 and 0.50 between 2013 and 2016), but because the estimated slope is persistently small. However, in the case of Google searches and of NVIDIA stock returns, Figure 2 gives evidence of a persistently strong probability of inclusion of these two predictors, that also tend to represent the lower bound of inclusion commented earlier on. Bitcoin returns are almost always predicted by these two variable, with peaks during 2012-2015 and then again after 2017, when $\Pr(z^j_{p,t}$ included) comes to exceed 0.6, with peaks in excess of 0.9. Finally, we note that after a peak in late 2013, the probability of inclusion for most predictors tend to drift up starting in mid-2017, i.e., Bitcoin return seem to have recently become more predictable, even though it is harder for us to judge whether this corresponds to a temporary, regime shifting-like increase or not.

Figure 3 reports 15 out of the available 18 plots for the recursive estimates of the vector $\hat{\theta}^{DMS}_{t+1|t}$ for the recursive out-of-sample January 2011 - January 2019. We include the estimated constant $\hat{\theta}^{DMS}_{0,t+1|t}$ but we exclude—because of space constraints—the graphs concerning the predictive slope coefficients for AMD and TSMS stock returns and for the returns on a value-weighted portfolio of U.S.-listed electricity firms.[13] Visibly, all plots flat-line at zero, indicating that the corresponding predictor is never selected in the optimal mix with three exceptions, in order of importance and magnitude: the rate of weekly growth of Google searches of the term "Bitcoin", which enters the best prediction model over 2013-2015 and 2018, giving a rather sizeable $\hat{\theta}^{DMS}_{Google,t+1|t}$ estimate; NVIDIA returns which are included between mid-2011 and 2015, and then sporadically in 2017; the market portfolio excess returns, that matter—although with a rather model CAPM-style beta—after 2014. In fact, an overview of Figure 3 reveals that over the initial part of the sample, especially 2011 and 2012, Bitcoin returns are predicted by very simple models, occasionally just featuring a constant, with the exception of short sub-intervals in which own-momentum, the investment factor portfolio or the US electrical stock index (unreported) do play some role. However, in spite of the fact that

[13] The predictor $r_{AMD,|t}$ has a blip in correspondence of late 2011 and it is otherwise never selected by the DMS algorithm.

depending on past market and NVIDIA returns may make some sense on rational grounds, it remains the case that Bitcoin is largely a sentimental asset, essentially and uniquely predicted by Google searches. In fact, $Size^{DMS}_{Bitcoin} = 1.81$ only and for approximately 40% of our 8-year sample, only the best prediction model $\hat{k}_t$ just includes Google search growth and a constant. Moreover, the bottom panel of Table 2 gives a less rosy picture (we leave as a blank the rows corresponding to predictors that never entered the optimal model set): the means and medians of the standardized predictors are generally small, never exceed 0.20 in absolute value, while their (across time series and cross-sections of models) empirical 90 percent confidence interval always includes zero. In other words, even though Google search momentum comes close to display *some* predictive power for Bitcoin returns, this is generally muted in a DMS algorithmic perspective.[14]

4.2 Litecoin

Figure 4 replicates Figure 1 in the case of Litecoin returns, while the top panel of Table 3 reports summary statistics for $\hat{\theta}^{DMA}_{t+1|t}$ in this case.[15] The table shows at a glance—especially when we focus on whether the empirical 90% confidence bands for the estimated coefficients happen to include zero or not—that there is a bit more predictability in the case of Litecoin vs. Bitcoin. However, also in the light of Figure 4, the source of such additional predictability is quickly established: lagged Bitcoin returns forecast Litecoin returns with a DMA coefficient that has a mean of 0.28 and a median of 0.32. Although these values are below one and imply a relatively muted reaction to past Bitcoin return shocks, such mean estimated coefficients are relatively large in light of the evidence in Table 2 and Figure 1, concerning Bitcoin. In fact, Figure 4 shows that the slope coefficient projecting Litecoin returns on past Bitcoin returns grows over the 2013-2019 sample, starting in fact from small but negative values and then strongly drifting up, especially between 2015 and 2017. This implies that Litecoin—even at rather low frequency weekly intervals—may be priced off recent Bitcoin returns. Also in this case, the rate of growth of the number of Google searches for "Litecoin" is a stable and strong forecasting variable, with a mean (median) DMA coefficient of 0.43 (0.31), which also characterizes this cryptocurrency as an a high attention-beta asset.[16] The empirical evidence also

[14]The set of predicted model/state probabilities under DMA and DMS are identical by construction because DMS is a probability-based refinement of DMA. Therefore the same analysis of Figure 2 performed above applies here.

[15]In Figure 4, to save space, we omit to report the DMA time series for the value-weighted Chinese and US electricity stock portfolio returns. However, as also shown in Table 3, the associated coefficients are generally small and hardly economically significant.

[16]However, Figure 4 shows that the DMA coefficient on Google searches has steadily declined in the case of Litecoin, going from levels close to 1 in late 2013 to neglible or negative values by the end of our sample.

emphasizes that a few other predictors tend to be effective most of the time: the returns on spot gold and silver (especially after 2016 but with heterogeneous signs, i.e., gold and Litecoin would be complements but silver and Litecoin would instead be substitutes); the individual share returns on AMD and TSMC, yet also in this case with estimated coefficients characterized by positive means and medians (albeit rather small and characterized by positive spikes against a background of negative, lagged dependence, see Figure 4).

These properties are magnified by the DMS algorithm, for which summary statistics are reported in the bottom panel of Table 3. There are only two predictors that turn out to be selected and therefore display non-zero means and medians and these are the expected ones, i.e., Bitcoin lagged returns and the rate of growth in the Google searches of the word Litecoin. Litecoin momentum and the returns on NVIDIA and AMD stocks enter the predictive regressions but only sporadically. Indeed while under DMA we have that $Size^{DMA}_{Bitcoin} = 5.38$ (which is in fact higher vs. what we had estimated for Bitcoin, consistently with Table 3), under DMS $Size^{DMS}_{Bitcoin} = 1.62$ which represents a stark contrast but also delivers a very powerful conclusion: Litecoin returns are only predicted by the growth in the crowds' attention and by past returns on Bitcoin; but because Bitcoin itself is mostly explained by the rate of growth in the Google searches of the word Bitcoin, it seems that in overall terms, also Litecoin is eventually an almost entirely attention-driven asset. Interestingly, there is (especially under a DMS algorithm) little or no evidence of Litecoin being predicted by standard equity finance factors.

Figure 5 shows the time series of probabilities of each predictor to be included across the $Q^{Litecoin} =$ 131,072 models entertained in the paper. The plots confirm the intuition on $\hat{\theta}^{DMA}_{t+1|t}$ expressed with reference to Figure 5: although their dynamics over the 2013-2019 sample is heterogeneous, the probabilities of models including lagged Bitcoin returns and the Litecoin Google search growth rate cumulate very high probabilities, generally exceeding 0.70; however, in the case of lagged Bitcoin returns, such probability is generally increasing over time, starting out at around 0.3 and reaching 1 by late 2016, while in the case of Google searches, these describe a "V"-shaped evolution, starting out at 1, declining by late 2016 to almost zero, and then bouncing back to basically 1 by the end of 2017. All the other probabilities of inclusion are generally modest and oscillate between 0.20 and 0.40, although many of them tend to climb up after 2017, which may be taken as evidence that Litecoin returns tended to become increasingly predictable.

Figure 6 shows times series plots for 15 out of the available 18 graphs for the recursive estimates of $\hat{\theta}_{t+1|t}^{DMS}$ for the Litecoin out-of-sample period July 2013 - January 2019. We include the estimated constant $\hat{\theta}_{0,t+1|t}^{DMS}$ but we exclude—because of space constraints—the graphs concerning the predictive slope coefficients for the value-weighted portfolio returns of electrical companies in China and the US. In any event, these predictors are never selected by the DMS algorithm. As one would expect, the patterns in Figure 6 closely mimic the probability ones in Figure 5 for the two predictors that enter most often the selected models, i.e., lagged Bitcoin returns and the rate of growth of Google searches of the word Litecoin.

4.3 Ripple

Figure 7 extends Figures 1 and 4 to the case of Ripple returns in showing the time series of $\hat{\theta}_{t+1|t}^{DMA}$ for the April 2015-January 2019 sample, while the top panel of Table 4 reports summary statistics. The table shows that although its structure may be different, Ripple is characterized by the same level of predictability vs. Litecoin. In fact, as in the case of Litecoin, also Ripple returns are forecast by lagged Bitcoin returns, even though the associated coefficients are relatively small. Even though the related coefficients all tend to be small at least in mean and median terms, in a few cases the empirical 90% confidence band fails to include zero: this happens for the returns on the size and quality factor mimicking portfolios (with sensible, positive coefficients), platinum spot returns (which turn out to be a substitute for Ripple coins), the returns on a value-weighted portfolio of Chinese electric power firms, and TSMC individual stock returns. In this case, investors' attention in the form of Google searches do not appear to predict Ripple returns, even though the delayed dependence of Ripple returns on past Bitcoin returns does imply an indirect, persistent dependence on Google searches. Yet Figure 7 shows that the slope coefficients are all close to zero and relatively flat throughout our sample. The only visible exceptions are 12-month own-momentum and the rate of growth of internet searches of the word "Ripple". Likewise, also because the corresponding values for the $\hat{\theta}_{t+1|t}^{DMA}$ coefficients are generally small, it is difficult to provide interpretation or account for stories underlying the dynamics of the recursively estimated coefficients in Figure 7. Not surprisingly in the light of Table 4, $Size_{Bitcoin}^{DMA} = 4.38$, which establish an unconditional level of model complexity similar to Litecoin.[17]

[17] The DMA probabilities of inclusion of each predictor over time are tabulated in an Appendix available upon request from the authors.

These results—also because of their weakness—are largely over-turned by the findings in the bottom panel of Table 4 and by Figure 8 concerning the DMS analysis: none of the predictors consistently enters the best selected model so to end up delivering mean and median coefficient estimates that are essentially zero. In fact, we find that $Size^{DMS}_{Bitcoin} = 0.30$, which means that on average the best selected model just includes the constant, i.e., no predictability exists. The results of DMS and DMA are consistent in the light of the recursively estimated coefficients that are essentially zero in Table 4.

4.4 Ethereum

Although on a rather short sample (2016-2019, for which we found sufficient liquidity and hence reliability in the corresponding market), the empirical evidence concerning Ethereum is similar to what we have unconvered in Section 4.3 with reference to Ripple returns. First, from Table 5, it emerges that under DMA, even though the empirical 90 percent bands show some sign of resilient predictability, this occurs with means and medians of the recursively estimated DMA coefficients are generally small, with the only partial exception of the Google search predictor, for which the mean (median) coefficient is 0.135 (0.158).[18] Therefore, there is some predictability—Figure 9 shows that this particularly visible in the first part of the sample, 2016-2017—for this crypto asset, but this seems to mostly come from its media attention exposure, as already reported above. Our calculations indeed reveal that $Size^{DMA}_{Ethereum} = 4.58$. This is consistent with Figure 10, concerning the recursive probabilities of inclusion of variables across DMA iterations, in which we note that—with the exception of the two predictors mentioned above and, at least between 2016 and 2017, for the momentum and SMB returns variables—all predictors imply probabilities that oscillate between 0.1 and 0.4 and that fail to display any interpretable trends over out sample. Second, as shown in the bottom panel of Table 5, when a DMS algorithm is applied, little of the predictability of cryptocurrency returns uncovered under DMA survives: as usual, only lagged Bitcoin returns and the rate of growth of Google search sentiment remain estimated with robust and non-zero coefficients (on average, these are 0.087 and 0.135, respectively, i.e., they remain rather modest, which is consistent with Drobetz et al., 2019 and Li and Yi, 2019); differently from previous cases (especially Ripple returns), Ethereum returns are also predicted by silver spot and NVIDIA stock returns. More interestingly, Table 5 shows considerable

[18] Interestingly, Ethereum returns display a resilient, negative predictive loading on past returns on the factor-mimicking portfolio representing size. A parallelism between the small-size cryptocurrency status of Ethereum and small-size stocks is of course suggestive (Ethereum would be a substitute for small cap firms).

variation over time of the models selected as the best one for forecasting purposes, even though—as already emphasized—past Bitcoin returns and the rate of growth of Google searches tend to play a prominent role. Consistently with this remark, we find $Size^{DMS}_{Ethereum} = 1.26$.

4.5 The dynamics of residual variance

For the sake of completeness in the presentation of our recursive OOS results, for each cryptocurrency return series, Figure 11 displays the recursively estimated time series of the DMA residual variances, $\left\{\hat{h}^{j,DMA}_{t}\right\}_{t=1}^{T}$,[19]

$$\hat{h}^{j,DMA}_{t+1} \equiv \sum_{l=1}^{Q^j} \pi_{t+1|t,l}\hat{h}^{j,DMA}_{t+1}(l) \qquad j = \text{Bitcoin, Litecoin, Ripple, and Ethereum,} \tag{23}$$

for $t = 0, 1, ..., T-1$. For the portions of their respective estimation samples that the different cryptocurrencies share, the series of residual EWMA variances are qualitatively similar: for instance, between November 2013 and January 2019, $\hat{h}^{Bitcoin,DMA}_{t+1}$ and $\hat{h}^{Litecoin,DMA}_{t+1}$ turn out to be rather close, even though the former declines from about 2.5 to 0.2 percent per week, while the for the latter cryptocurrency this takes place from a higher peak of 4 percent per week. Generally, Bitcoin and Litecoin display declining residual variances after initial peaks, why Ripple and Ethereum both features double initial peaks and then tame off to 1 percent per week by the end of the sample. Certainly, the early periods were characterized by rather unusual, unpredictable risks not explained away by any of the prediction variables between 2011 and 2014, then declining to more normal levels of 0.5-1 percent per week. As it has been frequently discussed in the literature (see Chaim and Laurini, 2018)), crypto assets are certainly "special" in their elevated and jump-like volatility. Our analysis confirms these more informal accounts but qualifies such risk as idiosyncratic within a predictive DMA framework and is therefore novel.

4.6 What is the impact of DMA/DMS methods on what learn from the data?

Because the net contribution of our paper to the literature, especially compared to Liu and Tsyvinski (2018), consists of the application of flexible averaging/selection forecasting methods to a subset of the predictors for cryptocurrencies isolated by earlier research, we close this section by presenting some

[19] We have also computed DMS time series of residual variance that however lead to largely similar conclusions.

evidence of the considerable impact of adopting simple recursive OLS instead of the DMA/DMS methods employed so far in our paper. Figure 12, plots recursive OLS coefficient estimates with reference to Bitcoin returns, using the same left scales as Figure 1 did. Even a cursory comparisons of Figures 1, 3, and 12 reveals that in the latter case, variation over time and even drifts in estimated coefficients appear that were completely missing in Figures 1 and 3 in which, in fact, most the flexible DMA and DMS recursive estimates flat-line at zero for the largest part of our sample, especially after late 2011. For instance, while in Figures 1 and 3, the estimated coefficients associated to the lagged spot gold return never significantly stray away from zero, Figure 12 shows (using the same left-scale as Figures 1 and 3) sustained variation of the OLS estimates through early 2013, followed by a smooth drift of the coefficients from approximately -0.4 towards -0.2; interestingly, even by the end of our sample, the recursive OLS fails to reach zero, which is instead typical of DMA and the only value featured by the DMS forecast algorithm. As we shall comment in Section 5, such differences vs. standard, recursive OLS deliver realized OOS and economic value performances that substantially differ from those of DMA/DMS.

To provide further corroborating evidence, with reference to Litecoin returns (arguably, the most predictable among the cryptocurrencies), in Figure 13 we perform a direct, side-to-side comparison of a few selected DMA coefficient estimates with recursive OLS estimates. Although there are cases in which the patterns are homogeneous (e.g., lagged bitcoin returns and SMB), in most plots the recursive OLS estimates are characterized by higher variance and by a dynamics over time that is rather different and of course will lead to differing forecasts and portfolio decisions. In Figure 13, the most glaring cases are offered by the coefficients associated to Gold returns and the Investment factors, which display patterns of time variation under OLS that are largely disconnected from those obtained under DMA.

5 Asset Pricing Implications

In this section we tackle the key research question of this paper, i.e., whether the empirical evidence of predictability (or lack thereof) in the returns of cryptocurrencies may justify a claim that these are segmented from more traditional asset classes. We articulate this argument along three main sub-arguments. First, we examine whether and how crypto currencies imply a different recursive dynamics of predictability relationships vs. traditional asset classes. Note that a different pattern of time variation may mean generally higher, lower, or simply different strength of any predictability relationships. Therefore, the second test that we perform concerns whether DMA and DMS models imply an overall amount of OOS predictability for cryptocurrencies that is significantly different from that recorded for traditional asset classes. However, such tests are always performed under standard loss functions, such as the squared loss that leads us to rank alternative models and to quantify OOS predictability using mean squared forecast errors criteria. Therefore, as a third step, we embrace more economically grounded loss functions and perform a simple and yet robust recursive mean-variance asset allocation exercise to test whether the economic value generated from the predictability in cryptocurrency returns may compare to that recorded for other, traditional asset classes.

5.1 Comparing the recursive dynamics of exposures to traditional asset classes

We have applied the same DMA and DMS techniques illustrated above also to data concerning returns on US investment grade corporate bonds, US value-weighed equities, a MSCI value-weighted equity index for developed markets (ex-US), the effective US dollar nominal exchange rate vis-à-vis a basket of other currencies, and gold spot prices. For each this large portfolios/asset classes, we have obtained empirical results similar in structure to those reported for cryptocurrency returns in Tables 2-5 and Figures 1-10.[20] Because the amount of results to be reported for five additional portfolios appears overwhelming and a number of the predictive factors have been selected from earlier literature on cryptocurrencies but it does appear to be inspired by equity market research, in this section we have opted to report results for US investment grade corporate bonds, for which the predictive power of the factors may result to be more in line with what one would expect of cryptocurrency returns. Figures 14 and 15 (as well as A3 in the Appendix) and Table 6 report results for US corporate bonds in standard format, showing that the very variables that have been staked out to forecast cryptocurrencies and, at least to some extent, stocks in past research, also carry some precision in the prediction of investment-grade corporate bond returns. For instance, in Table 6, under DMA, it turns out that *all* time-varying coefficients are characterized by empirical 90 confidence intervals that fail to include zero, which may be taken as an indication of precise estimation. Moreover, for all of them, the sample mean and median of the estimated coefficients carry identical sign, which is an

[20]Of course, momentum returns refer to the asset class under consideration, while the Google search variable measures Bitcoin searches, because Bitcoin is the most popular (and capitalized) among the cryptocurrencies. To ensure full consistency, we have also tried to use variables constructed after own Google searches for each asset class (like Guidolin, Orlov, and Pedio, 2018) but found largely similar results.

indication of them being well-behaved. In Tables 2-5, for cryptocurrencies, to find precise estimates of predictive regression coefficients had been almost the exception and not the rule. Under DMS, Table 6 shows that for seven predictors, the algorithm leads to their selection over time, while selection had represented more the exception than the rule in the case of cryptocurrencies (with the only exception of Litecoin).

Figure 14 shows much more variation of the DMA coefficients over our sample vs. the typical plots in Figures 1-10: especially the thicker lines in the plots are expressed on comparable, adjusted scales and while especially Figures 6, 7, and 9 had been dominated by flat lines, this is hardly the case in Figure 14. Figure 15 shows probability of DMA algorithm-driven inclusion of predictors that systematically exceeds 0.5 in the case of the US electricity stock portfolio index up to early 2014 and of lagged platinum returns after the end of 2015. The comparable Figures 2, 5, and 10 (and others available in an on-line appendix) do show occasional patterns of inclusion in excess of 0.5, but only when cryptocurrency specific variables are used to predict subsequent crypto returns; moreover, while in these figures the majority of the plots show average probabilities of inclusion that often fall towards zero, this is never the case for US corporate bonds. All in all, even though most of the differences that we have discussed appear to be of a qualitative type and are hardly formalized, it seems that even though our empirical exercise was hardly rigged in favor of traditional asset classes and against finding predictability in cryptocurrency returns, the data reveal that the same variables appear to have a easier time forecasting other types of returns, as the next section proceeds to formally quantify. Importantly, these results can be extended to both US and international stock returns as well as to gold and Figures 14-15 and Table 6 should be taken as examples. Although the plots and complete, tabulated results for gold have been dropped to save space but are available upon request, the finding that cryptocurrencies are less predictable than gold is of some interest. Often, at least in a fraction of the public commentaries on the nature of cryptocurrencies (see, e.g., Dyhrberg, 2016a; Wu et al., 2019), these have been compared to gold and other precious metals; our results show that such suggestions may be void of empirical content, at least as far as the predictability space goes, similarly to the key conclusion by Klein et al. (2018).

5.2 Are Cryptocurrencies just another currency?

An important, additional test is based on assessing whether cryptocurrencies are eventually just an alternative means of payment that differ from typical asset classes, such as stocks and bonds. Therefore, the specialty of cryptocurrency and possibly their segmentation from remaining asset classes can be best established by a comparison with the predictability patterns revealed by the US dollar exchange rate. Therefore we obtain data on the US dollar effective exchange rate (i.e., the trade-weighted exchange rate of the US dollar against a subset of major currencies that include the Euro, Canada, Japan, United Kingdom, Switzerland, Australia, and Sweden) for a Jan. 2011 - Jan. 2019 sample, similarly to Bitcoin data. The logic is that in the same way in which cryptocurrency returns measure relative percentage changes in the value of a cryptocurrency vis-a-vis the US dollar, the log changes in the effective nominal exchange rate of the US dollar measure the relative changes of its value vis-a-vis foreign currencies. Although in Table 1 we may notice that the 0.071 Sharpe ratio characterizing a long-short position in US dollars vs. a basket of other currencies is inferior to that typical of cryptocurrencies, Table 7 and Figure 16 (as well as Figures A4 and A5 in the Appendix) show that—even using predictor variables that are hardly optimized to be consistent with the exchange rate literature—there is considerably more predictability in US dollar log-changes vs. what we have reported for cryptocurrencies. In Table 7, all predictors but bitcoin momentum imply empirical 90% confidence interval for the DMA coefficients that fail to contain zero; in the case of DMS estimates, 11 coefficients give signs of accurate and persistent inclusion in the predictive relationship. In Figure 16, there is visible evidence, especially in comparison to Figures 1, 7, and 9 (Litecoin provides some intermediate grounds), of some forecasting power of our selected predictors for the trade-weighted value of the US dollar. In particular, SMB, silver, and electricity stock returns are characterized by time-varying coefficients that seem to capture pockets of predictability.[21]

5.3 Comparing the OOS realized predictive performance to traditional asset classes

In this section, we systematically compute and record the OOS realized forecasting performance of alternative models for different asset classes, with the differential performance across cryptocurrencies and other asset classes in the spot light. Table 8 provides the crucial piece of evidence in this paper

[21] In Figure A4, we obtain ex-post DMA evidence of high (exceeding 0.6) and time-varying probability of inclusion for the SMB, HML, stock momentum, platinum, silver, and especially technology stock returns.

and it is organized as follows. The upper panel concerns crypto returns and, for comparison, the lower panel the other asset classes investigated so far, including international and US stocks and gold. In this regard, we caution against the heterogeneity of the samples available, even though all recursive OOS forecasts for the benchmarks have been obtained on a sub-sample identical to that for Bitcoin, July 2011 - January 2019. This implies that strictly speaking, the results are perfectly comparable only when Bitcoin is involved.[22] All models are initialized on a 6-month weekly sample of observations. Table 8 has four blocks of results concerning four alternative prediction strategies: naive, recursive OLS; recursive OLS based on the full-sample best individual predictor, that is also listed in the table; DMA; DMS.[23] For each model, we report two measures of OOS predictive accuracy, i.e., the root mean-squared forecast error (RMSFE) defined in the conventional way and the OOS R-square, defined as:

$$R^2_{OOS}(j,\mathcal{M}) \equiv 1 - \frac{\sum_{t=P_j+1}^{T}(r_{j,t+1} - \hat{r}^{\mathcal{M}}_{j,t+1|t})^2}{\sum_{t=P_j+1}^{T}(r_{j,t+1} - \bar{r}_{j,t+1|t})^2} = 1 - \frac{MSFE(j,\mathcal{M})}{MSFE(j,mean)}, \tag{24}$$

where j refers to the asset class or cryptocurrency under consideration, $\mathcal{M}$ is one of the four predictive frameworks under consideration, and P_j denotes the end of the estimation sample for each potential j. Clearly, $R^2_{OOS}(j,\mathcal{M}) \lesseqqgtr$ according to whether $MSFE(j,\mathcal{M}) \gtreqqless MSFE(j,mean)$ so that a negative $R^2_{OOS}(j,\mathcal{M})$ is not only possible, but also highly meaningful: $R^2_{OOS}(j,\mathcal{M}) < 0$ occurs when a given forecasting model cannot manage to outperform the sample mean based on the naive expanding sample forecast

$$\bar{r}_{j,t+1|t} = \frac{1}{t}\sum_{\tau=1}^{t} r_{j,\tau}. \tag{25}$$

Of course, compared to the (square root of the) MSFE for a model, $R^2_{OOS}(j,\mathcal{M})$ can be more informative by providing a signed, relative measure. Finally, in Table 8, for each currency or asset class, we have emphasized in boldfaced the model with the highest $R^2_{OOS}(j,\mathcal{M})$ and the lowest RMSFE, although we must immediately caution that in a few cases, such a $R^2_{OOS}(j,\mathcal{M})$ turns out to be negative, indicating that none among the four predictive models under analysis manages to outperform a

[22] In the case of Ripple and Etheurem, the OOS back testing occurs on relatively short-subsamples (starting in Oct. 2015 and Jan. 2017, respectively), and applying them to Bitcoin and Litecoin returns would have implied a massive loss of information that we deemed to be unacceptable.

[23] Recursive OLS forecasts based on the full-sample best individual predictor are not true OOS forecasts because they could not have been implemented in real time, as they are based on a selection of predictor that becomes available only as of January 2019. We have purposefully specified this unfeasible predictive model to provide a quantitative indication of what is the best predictive power of simple OLS with our data. Unsuprisingly, in the case of three of the four cryptocurrencies, the best OLS predictor is the rate of growth in the crypto-specific Google searches.

simple, recursively updated historical average.

The message of Table 8 is stark: with the exception of Litecoin that keeps offering a bit of a puzzle, all other cryptocurrencies—and markedly, Bitcoin among them—are considerably less predictable than other, more traditional asset classes are. As already commented in the Introduction, while on the one hand this suggests great caution when cryptocurrency investments are approached, on the other hand it provides considerable evidence of their segmentation, of their being different from all other asset classes under consideration. Leaving Litecoin aside, the best R^2_{OOS} for a cryptocurrency is 1.1 percent (Ethereum, from a DMS framework), while the two other top R^2_{OOS} are in fact negative; on the opposite, all remaining asset classes are characterized by R^2_{OOS}s that range between 0.3 (the US exchange rate) and a rather large 6.2 percent (investment grade US corporate bonds). Moreover, with only two exceptions, out of nine currencies/asset classes covered by Table 8, in seven cases, it is DMS that provides the most accurate OOS forecasting performance, probably as one would expect given the parsimonious nature of the algorithm; in the other two cases, the best R^2_{OOS} is provided by the unfeasible recursive OLS model (for Ripple returns, but DMS ranks second with a negative R^2_{OOS} of -6.9%) and by DMA (for gold returns).

5.4 Comparing realized OOS portfolio performance with and without cryptocurrencies

As it is well known, assessing the predictability of asset returns under classical, statistical loss functions may turn out to be excessively remote when compared to the typical usage of these very forecasts in financial decisions, such as trading, portfolio allocation, and risk management, see e.g., Leitch and Tanner (1991). In fact, a number of papers (see, e.g., Cenesizoglu and Timmermann, 2012 and Dal Pra et al., 2018) have emphasized that often the typical statistical loss functions used in much research on predictability may reveal weak forecasting power that is able to generate substantial OOS, realized economic value under commonly used trading and asset allocation strategies based on the maximization of performance criteria and expected utility functions that are equally widespread in financial economics. With *economic value*, we refer to the fact that enriched asset menus (for instance, to include alternative asset classes) or more sophistical asset allocation methods ought to lead to superior OOS performances, for instance in terms of realized certainty equivalent returns, Sharpe ratios, and better higher-order properties of portfolio returns (skewness and kurtosis). The potential of a divergent informative content of statistical vs. economically grounded loss functions clearly represents an issue for our research design that has been so far based on statistical loss functions only. As a result, in this section we proceed to extend our earlier empirical evidence to a simple, representative portfolio problem to test whether exploiting the scant predictability in cryptocurrency returns may generate any economic value. If cryptocurrencies were segmented from all other asset classes—hence not (or less) predictable using standard variables and methodologies—we expect to hypothesis not to be rejected in our empirical experiments:

1. Accounting for any predictability in recursive, OOS portfolio strategies (as opposed to not exploiting any forecasting power from standard predictors) ought to generate small or even non-positve economic value in asset menus that include cryptocurrencies, besides cash, bond, stocks, and gold.
2. However, just because cryptocurrencies are segmented from all other asset classes, even (or especially) ignoring any predictability, their inclusion in the asset menu in addition to traditional assets classes ought to create substantial and, in any event, positive economic value.

Our portfolio allocation design is rather typical of the literature, see e.g., Barberis (2000) and Guidolin and Timmermann (2007). We model a US investor who, starting from unit wealth and in the presence of no-short sale constraints, maximizes expected power utility by selecting at weekly frequency the portfolio weights to be assigned to the N assets in the asset menu she faces:

$$\max_{\omega_t} E_t^{\mathcal{M}}\left[\frac{W_{t+1}^{1-\gamma}}{1-\gamma}\right] \quad (\gamma \neq 1) \qquad \text{s.t. } W_{t+1} = \sum_{j=1}^{N} \omega_{j,t}(r_{j,t+1} - r_{f,t+1}) + (1 + r_{f,t+1}), \quad \omega_{n,t} \in [0,1]. \quad (26)$$

In problem (26), $r_{f,t+1}$ is the riskless, cash rate known at time t and the conditional expectation of the one-week ahead utility is computed by an IID bootstrap of the empirical distribution of the data up to time t, under a given prediction model $\mathcal{M}$.[24] For instance, under DMA, we shall have $W_{t+1}(\omega_t) \sim \mathcal{D}\left(\widehat{\mu}_{t+1|t}(\omega_t), \omega_t' Cov(\mathbf{r}_{t+1} - \hat{\mathbf{r}}_{t+1|t}^{DMA})\omega_t\right)$, where $\mathcal{D}$ is the empirical distribution of the available data obtained by a simple IID bootstrap from the available returns up to time t with mean $\widehat{\mu}_{t+1|t}(\omega_t) = \sum_{j=1}^{N} \omega_{j,t}(\hat{r}_{j,t+1|t}^{DMA} - r_{f,t+1}) + (1 + r_{f,t+1}) = \sum_{j=1}^{N} \omega_{j,t}(\sum_{l=1}^{Q} \pi_{j,t+1|t,l} \hat{\theta}_{j,t}'(l)\mathbf{z}_{j,t} - r_{f,t+1}) + (1 + r_{f,t+1})$ and variance $\omega_t' Cov(\mathbf{r}_{t+1} - \hat{\mathbf{r}}_{t+1|t}^{DMA})\omega_t$, where $\omega_t' Cov(\mathbf{r}_{t+1} - \hat{\mathbf{r}}_{t+1|t}^{DMA})\omega_t$ is the $N \times N$ covariance matrix of the

[24] Because we impose on the shape of the empirical distribution of the data, means, variance and covariances that are affected by the selected model $\mathcal{M}$, this IID bootstrap scheme to optimize portfolio weights is also known as a filtered historical simulation approach. In our view, in the presence of the massive deviation from normality characterizing the returns on cryptocurrencies, any other parametric approach to optimal asset allocation would be of dubious applicability.

forecast errors from the DMA model.[25]

We recursively solve (26) using standard constrained optimization methods in Matlab over the same OOS period used already in Section 5.2. With reference to such OOS period, we compute and report two measures of realized, risk-adjusted portfolio performance, the certainty equivalent return (CER) associated to the recursive solution of (26),

$$CER(\mathcal{M}, N) \equiv \left\{ \frac{1}{T-P_N} \sum_{t=P_N+1}^{T} \left[\sum_{j=1}^{N} \hat{\omega}_{j,t}^{\mathcal{M}} (r_{j,t+1} - r_{f,t+1}) + (1 + r_{f,t+1}) \right]^{1-\gamma} \right\}^{\frac{1}{1-\gamma}} - 1, \tag{27}$$

where the OOS for the asset menu characterized by N assets starts at $P_N + 1$ and the notation $CER(\mathcal{M}, N)$ emphasizes that this the CER computed under the forecasts from model $\mathcal{M}$ for asset menu N, and the standard Sharpe ratio,

$$SR(\mathcal{M}, N) \equiv \frac{\frac{1}{T-P_N} \sum_{t=P_N+1}^{T} \sum_{j=1}^{N} \hat{\omega}_{j,t}^{\mathcal{M}} (r_{j,t+1} - r_{f,t+1})}{\sqrt{\frac{1}{T-P_N} \sum_{t=P_N+1}^{T} \left[\sum_{j=1}^{N} \hat{\omega}_{j,t}^{\mathcal{M}} (r_{j,t+1} - r_{f,t+1}) - \frac{1}{T-P_N} \sum_{t=P_N+1}^{T} \sum_{j=1}^{N} \hat{\omega}_{j,t}^{\mathcal{M}} (r_{j,t+1} - r_{f,t+1}) \right]^2}}. \tag{28}$$

Of course, the standard caution that only the CER is really meaningful when portfolio weights have been optimized under (26) also applies in this case, when asset returns are highly non-normal. However, reporting and commenting Sharpe ratios seems to be a common practice, especially in the industry, and so we also follow this custom here. In practice, we consider three alternative values of the coefficient of constant relative risk aversion γ (i.e., 3, 8, and 15) and three alternative asset menus, which correspond to six distinct experiments:

1. A first menu characterized by $N_1 = 5$ risky assets, i.e., US stocks, World developed markets stocks (ex-US), US investment grade corporate bonds, gold, and long-short position in US dollars vis-a-vis a trade-weighted basket of other major currencies.[26] Of course these are the five classical assets that we have considered in Section 5.1, when performing comparisons between the predictability patterns of cryptocurrencies and other asset classes. The portfolio calculations are performed under DMA, DMS, and when there is no predictability and returns are simply predicted by their historical sample means.

[25] Because we are considering the conditional, one-step ahead distribution of wealth, this is the relevant definition of covariance for our problem, see Campbell et al. (2004).

[26] Because we assume that the short position is 100 percent covered by depositing cash, we impute on the long-short exchange rate the same cost in terms of committment of available wealth as other assets/strategies.

2. A second menu identical to the first, but expanded to include Bitcoin, so that $N_2 = 6$; even though this second asset menu just includes one cryptocurrency only, this is the most famous and by far the most actively traded (to the point of acting as a medium of exchange and currency conversion for other, minor crypto assets) that therefore allows us to apply our OOS portfolio tests to a relatively long July 2011 - January 2019 sample. The portfolio calculations are performed under DMA, DMS, and when there is no predictability and returns are simply predicted by their historical sample means. A comparison of the CERs and Sharpe ratios of this asset menu vs. $N_1 = 5$ delivers an estimate of the economic value of adding Bitcoin to an otherwise traditional asset menu; a comparison of CERs and Sharpe ratios for $N_2 = 6$ provides instead an estimate of the economic value of capturing predictability when Bitcoin belongs to the asset menu.
3. A third asset menu, that further expands the second to include all cryptocurrencies, i.e., $N_3 = 9$; on the one hand, in the light of the goals of our paper, this expansion of the asset menu seems natural; on the other hand, this implies a remarkable cost in terms of length of the feasible OOS period, that indeed shrinks to 108 weeks only, January 2017 - January 2019, which is forced upon us by the limited data availability of Ethereum returns. A comparison of the CERs and Sharpe ratios of this asset menu vs. $N_1 = 5$ delivers an estimate of the economic value of adding all cryptocurrencies to an otherwise traditional asset menu; a comparison of CERs and Sharpe ratios for $N_3 = 9$ provides instead an estimate of the economic value of capturing predictability when all cryptocurrencies jointly belong to the asset menu; finally a comparison of the CERs and Sharpe ratios of this asset menu vs. $N_2 = 6$ delivers an estimate of the economic value of adding Ethereum, Litecoin, and Ripple to an asset menu that includes only Bitcoin, which is another interesting question.

With reference to the case of $\gamma = 3$, Table 9 provides preliminary evidence on our dynamic portfolio exercises by reporting the customary set of summary statistics concerning optimal portfolio weights for the three asset menus described above.[27] The table is organized in three panels, devoted to results obtained under DMA, DMS, and using historical sample means, respectively. When no cryptocurrencies are available, under DMA and DMS, wealth is on average invested across all asset classes, with a slight prevalence of cash and investment-grade corporate bonds, even though also US

[27] Tables A1 and A2 in the Appendix describe the empirical results for the cases of $\gamma = 8$ and 15 but are qualitatively similar, apart from the obvious finding that the optimal asset allocations in this cases are tilted away from risky assets and towards cash.

and global equities are demanded on average (about 15%), even though their demand is weaker, as shown by median statistics. However, when predictability is ignored, only corporate bonds enter the optimal portfolio. When bitcoin is added to the choice menu, the effects turn out to be considerably dependent on whether predictability or not is accounted for. Under DMA and DMS, Bitcoin enters with a weight between 12 and 21 percent, when simple historical means, variances, and covariances are considered, then the demand for bitcoin jumps to 62% on average (the median is 73%) and the rest is invested in US corporate bonds. This asset allocation is clearly unrealistic and yet its OOS economic value remains to be assessed. Finally, when all cryptocurrencies are made available to our representative investor—although only for the shorter 2017-2019 sample—we find that all allocations become heavily tilted towards crypto assets (in particular Bitcoin and Litecoin under DMA and DMS, Ripple when only historical moments are taken into account) and away from cash that appears to be completely "crowded out" by block-chain based payment methods; rather awkwardly, most (all, under historical moments) demands of stocks and bonds disappear, leading to less realistic and well-diversified portfolio allocations.

Just because the resulting average and median asset allocations appear to be quite unbalanced when predictability is ignored, it is imperative to check their resulting OOS realized portfolio performances, which is what Table 10 performs for $\gamma = 3$, 8, and 15. In the first case, which directly matches the results in Table 9, we observe that while ignoring predictability leads to the highest—albeit rather puny (0.041% per week)—CER, it is predictability, as flexibly captured by DMA, that leads to the highest economic value, with CERs between 0.091 and 0.207% per week, which are far from negligible. Moreover, the highest economic value of historical mean estimates comes from its superior higher-order moment properties, in the sense that it yields to systematically lower kurtosis (and essentially zero skewness) vs. DMA and DMS, while the two latter models guarantee higher realized mean returns and Sharpe ratios in each experiment/asset menu. For each asset menu, the last column of Table 10 estimates the value of predictability by comparing the CER when predictability is ignored vs. the highest CER between DMA and DMS. For a standard asset menu that ignores predictability, and consistently with earlier literature (see Goyal and Welch, 2007, Rapach et al., 2010 and the review by Rapach and Zhou, 2013), the value of such predictability turns out to be negative, -0.081% per week, i.e., an investor ought to be ready to pay to avoid accounting for the (modest) forecasting power discussed in Section 4. However, when cryptocurrencies are brought into the asset menu, the resulting economic value turns positive, although rather low, 0.014% when only Bitcoin is inverstable and 0.039% when all cryptocurrencies are available. These are incremental CERs of between 0.6 and 2% per year that appear to be close (or even below) plausible measures of transaction costs that may be involved in our trading strategy and that we have ignored for simplicity. Moreover, finding that the predictability of cryptocurrencies was elusive (absent) under standard, statistical loss functions such as MSFE and modest under economic loss functions is generally expected, as the two types of loss functions are generally imperfectly aligned in many financial applications.

Table 10 also allows to compute an estimate to the question that is central to this paper, i.e., what is the value of a seemingly segmented asset—cryptocurrencies, at least in a predictive framework—in an asset management perspective. By taking the difference between the highest CER when crypto assets are included in the asset menu ($N_2 = 6$ and $N_3 = 9$) and the highest CER under the baseline menu ($N_1 = 5$), we find an increase in CER of 0.166% per week, i.e., a rather hefty 8.3% per year. Yet, as we would expect on the basis of earlier results, if one takes the difference under the case of no predictability between the highest CER when crypto assets are included in the asset menu and the highest CER under the baseline menu, we find an increase of CER due to the inclusion of cryptocurrencies in the asset menu that equals 0.091% per weak, i.e., more than half of the 8.3% does not depend on the application of DMA and DMS models.[28] Although we have computed our recursive optimal portfolios by maximization of expected power utility, Table 10 also shows that the inclusion of cryptocurrencies into the asset menu systematically increased the realized, OOS Sharpe ratios vs. the baseline $N_1 = 5$: the best achievable weekly SR index climbs from 0.169 to 0.186 when Bitcoin is added to the asset menu, and then to 0.265 when all cryptocurrencies are considered (even though on a shorter sample, so that comparability is limited).[29]

Finally, the intermediate and top panel of Table 10, report results for the cases of $\gamma = 8$ and 15. Results are qualitatively similar, even though in the case of $\gamma = 8$ results are starker in the sense that ignoring the predictability of cryptocurrencies simply increases CER vs. DMA and DMS. Once more, this caused by the reduced kurtosis of strategies simply based on historical moments that imply excessively large portfolio returns only in rare occasions. In fact, while in overall terms, cryptocurrencies carry a rather modest economic value (0.065% per week), when predictability is

[28] All of the overall economic value of cryptocurrencies derives from the ability to hold Bitcoin, but when predictability is not taken into account, the value of Litecoin—being the most forecastable asset—is important and as such the increase in CER deriving from Bitcoin holdings only is a meager 0.036% per week.

[29] These weekly SR are rather considerable when annualized by simply multiplying by $\sqrt{52}$, ranging between 1.2 and 1.9. However, such an operation is of dubious nature because the data display features that starkly contrast with their independent and identical distribution over time.

ignored, this increases to 0.253% per week, which is almost 13% per year. Under $\gamma = 15$, we find that predictability has positive economic value (up to 0.212% per week) but that once more cryptocurrencies are more valuable when predictability is ignored (0.227% per week) than when it is taken into account (0.124%). Interestingly, one can easily check that in the case of $\gamma = 8$ and 15 the incremental CER recorded for crypto assets derives in a predominat portion from the ability to hold Bitcoin. Therefore, for aggressive, relatively non-risk averse investors, predictability is less important vs. the case of more risk-averse investors and the former care more than the latter to get access to more volatile, speculative cryptocurrencies such as Litecoin and Ripple, that are in fact also more easily predictable.[30]

6 Conclusion

In this paper we have investigated whether and how cryptocurrencies may represent a new asset classes segmented—i.e., with different statistical properties in a linear space—from traditional asset classes. We have done that using an exquisitely forecasting, flexible approach in which—given a set of plausible predictive variables drawn from earlier literature on the asset pricing of cryptocurrencies—we ask whether the patterns, the strength, and the economic value of any predictability characterizing cryptocurrency returns may differ from that typical of traditional asset classes. Our approach is flexible because it is based on the dynamic model averaging and dynamic model selection approach by Raftery et al. (2010) and popularized in macroeconomics by Koop and Korobilis (2012): instead of simply performing recursive OLS estimation of linear predictive models with fixed predictors, we allow the data—through the use of Bayes' formula and a few carefully selected approximations—to either recursively re-weight the forecasting variables to used in the model or to select which variables ought to be dynamically included in the model. Finally, we measure economic value through standard recursive asset allocation exercises in which a US investor maximized expected power utility across alternative asset menus—with and without cryptocurrencies—and predictability models, including a sample historical mean benchmark that does not feature any predictability of asset returns. As argued in the paper, the goal of such exercises—although potentially appealing by itself to Readers coming from the field of applied portfolio management—is ensure adequate robustness to the selection of

[30] Although the nature of the estimation is questionable in the light of the features of the asset allocation problem we have tackled, also for $\gamma = 8$ and 15, we find that the maximum SR monotonically increases as additional cryptocurrencies are added to the asset menu.

specific statistical loss functions, that remains a tricky choice in all research designs based on relative forecasting power.

We find evidence that cryptocurrencies do represent a new asset classes, substantially segmented from traditional asset classes. Cryptocurrency appear to be: (1) characterized by returns that are less predictable on average when compared to other asset classes, including gold and the external, trade value-weighted US dollar;[31] (2) characterized by returns that can be forecasted according to patterns and with a measurable degree of time variation that differ from most other asset classes, including gold that has been often indicated as the most closely related asset class; (3) able to generate considerable, realized OOS economic value (especially when measured in terms of ex-post Sharpe ratios) when they are added to otherwise traditional asset menus of cash, corporate bonds, US and international stocks, and long-short exchange rate positions; (4) unable to offer much advantage in terms of realized, risk-adjusted portfolio performances deriving from any predictability patterns characterizing them; (5) not reducible only to Bitcoin, in the sense that also Ethereum, Litecoin, and Ripple appear to generate substantive OOS realized economic value when they are added to Bitcoin, also because they offer diversification benefits vs. Bitcoin returns; because diversification benefits play a role of increasing importance as investors become more risk-averse, we find stronger evidence of the value of cryptocurrencies not being subsumed by Bitcoin when $\gamma = 15$, the highest risk aversion coefficients entertained in our exercise.[32]

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[31] Arguably, Litecoin returns provide an exception to this finding.

[32] However, this empirical implication is derived using a relatively short 2017-2019 data set while the empirical results concerning the economic value of Bitcoin are obtained from a longer, 2011-2019 sample.

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Table 1

Summary Statistics

The statistics are based on weekly data, expressed in US dollars. All commodity and foreign stock returns are expressed in US dollars. The interquartile range is defined as the difference between the 75th and the 25rh sample percentiles. Excess kurtosis is defined as sample kurtosis minus 3. The Sharpe ratio is computed with reference to the weekly yields of US 30-day T-bills.

	Sample	Number of weeks	Mean	Median	St. Dev.	Interquartile range	Skewness	Excess Kurtosis	Sharpe ratio
	Cryptocurrency Returns								
Bitcoin returns	Jan. 2011 - Jan. 2019	422	3.782	1.626	19.540	13.242	2.595	13.137	0.193
Litecoin returns	July 2013 - Jan. 2019	289	2.801	-0.203	25.135	12.077	5.938	57.288	0.111
Ripple returns	April 2015 - Jan. 2019	199	4.520	-0.770	31.608	13.363	3.548	16.932	0.143
Ehereum returns	June 2016 - Jan. 2019	139	2.989	0.469	18.016	20.614	0.823	0.782	0.165
	Other Asset/Portfolio Returns								
1-month T-bill	Jan. 2011 - Jan. 2019	422	0.008	0.001	0.012	0.009	1.741	1.795	___
Inv. Grade US Corporate bond returns	Jan. 2011 - Jan. 2020	422	0.036	0.034	0.211	0.221	-0.100	1.555	0.133
VW CRSP Mkt excess returns	Jan. 2011 - Jan. 2021	422	0.158	0.240	1.892	1.830	-0.607	2.784	0.079
US Dollar Trade-Weighted Exchange Rate	Jan. 2011 - Jan. 2021	422	0.053	0.042	0.747	0.974	0.212	3.184	0.071
SMB portfolio returns	Jan. 2011 - Jan. 2022	422	-0.022	0.000	0.647	0.893	-0.077	-0.270	-0.034
HML portfolio returns	Jan. 2011 - Jan. 2023	422	-0.024	-0.055	0.754	0.885	0.401	1.068	-0.032
RMW portfolio returns	Jan. 2011 - Jan. 2024	422	0.067	0.070	0.512	0.658	-0.130	0.488	0.130
CMA portfolio returns	Jan. 2011 - Jan. 2025	422	0.001	-0.020	0.474	0.598	0.094	0.598	0.001
Momentum portfolio returns	Jan. 2011 - Jan. 2026	422	0.096	0.155	1.188	1.270	-0.986	3.176	0.081
Bitcoin Momentum returns	Jan. 2011 - Jan. 2026	422	2.536	1.604	6.419	6.176	1.255	2.951	0.395
Litecoin Momentum returns	July 2013 - Jan. 2019	289	1.050	-0.563	6.794	5.753	1.568	2.201	0.155
Ripple Momentum returns	April 2015 - Jan. 2019	199	2.058	-0.021	9.992	4.719	1.849	3.344	0.206
Ethereum Momentum returns	June 2016 - Jan. 2019	139	2.015	0.305	7.268	8.870	0.931	0.123	0.277
Gold spot returns	Jan. 2011 - Jan. 2027	422	0.010	0.012	2.179	2.671	-0.125	1.287	0.001
Platinum spot returns	Jan. 2011 - Jan. 2028	422	-0.137	-0.209	2.653	3.473	0.092	0.172	-0.055
Silver spot returns	Jan. 2011 - Jan. 2029	422	-0.071	-0.109	3.932	4.155	-0.958	8.799	-0.020
NVIDIA individual stock returns	Jan. 2011 - Jan. 2030	422	0.690	0.519	5.698	6.309	0.729	3.524	0.120
AMD individual stock returns	Jan. 2011 - Jan. 2031	422	0.595	0.120	8.299	8.964	0.700	3.789	0.071
TSMC individual stock returns	Jan. 2011 - Jan. 2032	422	0.315	0.295	3.316	4.190	-0.101	0.211	0.093
	Other Predictors								
Growth of Google Bitcoin searches	Jan. 2011 - Jan. 2019	422	7.243	-3.206	44.606	25.459	4.331	29.188	___
Growth of Google Litecoin searches	July 2013 - Jan. 2019	289	4.951	-5.455	53.526	27.609	4.795	31.723	___
Growth of Google Ripple searches	April 2015 - Jan. 2019	199	4.076	-0.599	48.005	16.982	4.817	34.303	___
Growth of Google Ethereum searches	June 2016 - Jan. 2019	139	2.058	-0.021	9.992	4.719	1.849	3.344	0.206
VW returns on Chinese electricity stocks	Jan. 2011 - Jan. 2019	422	0.060	0.037	3.429	4.173	0.257	1.243	0.015
VW returns on US electricity stocks	Jan. 2011 - Jan. 2019	422	0.139	0.249	1.818	2.305	-0.275	0.046	0.072

Table 2

Summary Statistics for Recursively Estimated DMA and DMS Predictive Regression Coefficients: Bitcoin Returns

The statistics are based on weekly data, expressed in US dollars. The percentile statistics are across time and probability-weighted within time periods. We have boldfaced pairs of 5th-95th empirical percentile statistics that fail to include zero, which implies that in overall terms (under the assumption of independence over time), the series of recursively estimated slopes are statistically significant at a 10% size.

	Mean	Median	St. Dev.	Min	Max	5th percentile	95th percetile
	Dynamic Model Average Coefficients						
Bitcoin Lagged returns	-0.0068	-0.0054	0.0103	-0.0409	0.0507	-0.0111	0.0002
Bitcoin Momentum	-0.0175	0.0033	0.1523	-1.4194	0.0654	-0.7991	0.8058
VW CRSP Mkt excess returns	0.0714	0.0435	0.0888	-0.3042	0.6480	**0.0415**	**0.0456**
SMB portfolio returns	0.0343	0.0080	0.0764	-0.0579	0.4407	-0.0272	0.0432
HML portfolio returns	0.0039	-0.0005	0.0233	-0.0752	0.1864	-0.0180	0.0170
RMW portfolio returns	0.0782	0.0076	0.1962	-0.0009	1.3601	**0.0065**	**0.0087**
CMA portfolio returns	0.0005	0.0025	0.0781	-0.5621	0.1599	-0.4252	0.4302
Momentum portfolio returns	-0.0215	-0.0019	0.0728	-0.5110	0.0159	-0.2665	0.2626
Gold spot returns	0.0086	0.0073	0.0216	-0.0769	0.3510	-0.0474	0.0619
Platinum spot returns	0.0153	0.0068	0.0379	-0.0225	0.4620	-0.0034	0.0169
Silver spot returns	0.0266	0.0208	0.0359	-0.2007	0.1201	**0.0153**	**0.0263**
Growth of Google Bitcoin searches	0.1210	0.0935	0.1215	-0.0502	0.3742	**0.0590**	**0.1279**
VW returns on Chinese electricity stocks	-0.0042	0.0044	0.0735	-0.7438	0.0599	-0.3584	0.3673
VW returns on US electricity stocks	-0.0190	-0.0023	0.0810	-0.7298	0.0409	-0.1868	0.1822
NVIDIA individual stock returns	0.1043	0.0964	0.0697	0.0100	0.2091	**0.0897**	**0.1030**
AMD individual stock returns	0.0082	0.0006	0.0609	-0.0616	0.3665	-0.0097	0.0109
TSMC individual stock returns	0.0121	0.0061	0.0335	-0.0365	0.2693	-0.0007	0.0128
Residual predictive variance (h_t)	1.1188	0.7261	1.1328	0.1700	5.3927	**0.5598**	**0.8925**
	Dynamic Model Selection Coefficients						
Bitcoin Lagged returns	—	—	—	—	—	—	—
Bitcoin Momentum	-0.0268	0.0000	0.2480	0.0000	0.0000	0.0000	0.0000
VW CRSP Mkt excess returns	0.0608	0.0000	0.1095	0.2485	-0.2334	-0.2334	0.2334
SMB portfolio returns	0.0283	0.0000	0.1244	0.6826	-0.2871	-0.2871	0.2871
HML portfolio returns	—	—	—	—	—	—	—
RMW portfolio returns	0.0199	0.0000	0.1509	1.3414	0.0000	0.0000	0.0000
CMA portfolio returns	-0.0041	0.0000	0.0588	0.0000	0.0000	0.0000	0.0000
Momentum portfolio returns	-0.0009	0.0000	0.0238	0.1168	0.0000	0.0000	0.0000
Gold spot returns	—	—	—	—	—	—	—
Platinum spot returns	0.0037	0.0000	0.0753	1.5165	0.0000	0.0000	0.0000
Silver spot returns	—	—	—	—	—	—	—
Growth of Google Bitcoin searches	0.1588	0.1627	0.1535	0.4127	-0.2246	-0.2246	0.5499
VW returns on Chinese electricity stocks	-0.0059	0.0000	0.0831	0.0000	0.0000	0.0000	0.0000
VW returns on US electricity stocks	-0.0408	0.0000	0.1907	0.0000	-0.4479	-0.4479	0.4479
NVIDIA individual stock returns	0.1217	0.1264	0.0825	0.2261	-0.0944	-0.0944	0.3471
AMD individual stock returns	0.0201	0.0000	0.1416	1.0668	0.0000	0.0000	0.0000
TSMC individual stock returns	0.0026	0.0000	0.0531	1.0708	0.0000	0.0000	0.0000
Residual predictive variance (h_t)	1.0607	0.6863	1.1560	7.6214	0.4888	**0.4888**	**0.6488**

Table 3

Summary Statistics for Recursively Estimated DMA and DMS Predictive Regression Coefficients: Litecoin Returns

The statistics are based on weekly data, expressed in US dollars. The percentile statistics are across time and probability-weighted within time periods. We have boldfaced pairs of 5th-95th empirical percentile statistics that fail to include zero, which implies that in overall terms (under the assumption of independence over time), the series of recursively estimated slopes are statistically significant at a 10% size.

	Mean	Median	St. Dev.	Min	Max	5th percentile	95th percetile
	Dynamic Model Average Coefficients						
Bitcoin Lagged returns	0.2749	0.3213	0.1582	-0.0740	0.5482	**0.3150**	**0.3277**
Litecoin Momentum	-0.0002	0.0005	0.0363	-0.4601	0.1972	-0.1237	0.1246
VW CRSP Mkt excess returns	0.0142	0.0100	0.0185	-0.0352	0.0775	-0.0052	0.0251
SMB portfolio returns	-0.0032	-0.0051	0.0111	-0.0208	0.0516	**-0.0065**	**-0.0038**
HML portfolio returns	0.0117	-0.0006	0.0299	-0.0249	0.1326	-0.0138	0.0126
RMW portfolio returns	-0.0118	-0.0039	0.0248	-0.1619	0.0103	-0.0457	0.0379
CMA portfolio returns	0.0050	0.0016	0.0141	-0.0152	0.1211	-0.0038	0.0070
Momentum portfolio returns	-0.0117	-0.0082	0.0183	-0.0648	0.1401	-0.0254	0.0090
Gold spot returns	0.0210	0.0092	0.0273	-0.0059	0.1944	**0.0071**	**0.0113**
Platinum spot returns	-0.0135	-0.0083	0.0221	-0.1742	0.0089	-0.0900	0.0734
Silver spot returns	-0.0136	-0.0122	0.0104	-0.0472	0.0018	**-0.0200**	**-0.0044**
Growth of Google Litecoin searches	0.4333	0.3067	0.2763	0.0102	1.0738	**0.2724**	**0.3411**
VW returns on Chinese electricity stocks	-0.0034	-0.0001	0.0131	-0.0329	0.0285	-0.0050	0.0047
VW returns on US electricity stocks	-0.0163	-0.0115	0.0267	-0.1566	0.0092	-0.0865	0.0635
NVIDIA individual stock returns	-0.0037	-0.0013	0.0149	-0.0432	0.0330	-0.0130	0.0103
AMD individual stock returns	0.0256	0.0047	0.0422	-0.0090	0.2230	**0.0036**	**0.0058**
TSMC individual stock returns	0.0112	0.0054	0.0255	-0.0153	0.1070	**0.0021**	**0.0088**
Residual predictive variance (h_t)	0.6471	0.4688	0.7208	0.0928	4.1857	**0.1515**	**0.7861**
	Dynamic Model Selection Coefficients						
Bitcoin Lagged returns	0.2625	0.3161	0.1738	0.0000	0.5448	**0.6323**	**0.7775**
Litecoin Momentum	0.0006	0.0000	0.0099	0.0000	0.1639	0.0000	0.0000
VW CRSP Mkt excess returns	___	___	___	___	___	___	___
SMB portfolio returns	___	___	___	___	___	___	___
HML portfolio returns	___	___	___	___	___	___	___
RMW portfolio returns	___	___	___	___	___	___	___
CMA portfolio returns	___	___	___	___	___	___	___
Momentum portfolio returns	___	___	___	___	___	___	___
Gold spot returns	___	___	___	___	___	___	___
Platinum spot returns	___	___	___	___	___	___	___
Silver spot returns	___	___	___	___	___	___	___
Growth of Google Litecoin searches	0.4514	0.3923	0.2410	0.0000	0.8660	**0.7845**	**1.2063**
VW returns on Chinese electricity stocks	___	___	___	___	___	___	___
VW returns on US electricity stocks	___	___	___	___	___	___	___
NVIDIA individual stock returns	0.0005	0.0000	0.0056	0.0000	0.0683	0.0000	0.0000
AMD individual stock returns	0.0028	0.0000	0.0266	0.0000	0.2848	0.0000	0.0000
TSMC individual stock returns	___	___	___	___	___	___	___
Residual predictive variance (h_t)	0.5373	0.4408	0.4519	0.0864	2.8005	**0.2602**	**0.6214**

Table 4

Summary Statistics for Recursively Estimated DMA and DMS Predictive Regression Coefficients: Ripple Returns

The statistics are based on weekly data, expressed in US dollars. The percentile statistics are across time and probability-weighted within time periods. We have boldfaced pairs of 5th-95th empirical percentile statistics that fail to include zero, which implies that in overall terms (under the assumption of independence over time), the series of recursively estimated slopes are statistically significant at a 10% size.

	Mean	Median	St. Dev.	Min	Max	5th percentile	95th percetile
	Dynamic Model Average Coefficients						
Bitcoin Lagged returns	0.0046	0.0049	0.0099	-0.0404	0.0417	**0.0021**	**0.0077**
Ripple Momentum	0.0147	0.0071	0.1218	-0.1504	0.7138	-0.0187	0.0330
VW CRSP Mkt excess returns	0.0189	-0.0007	0.0343	-0.0130	0.1458	-0.0036	0.0022
SMB portfolio returns	0.0336	0.0189	0.0390	0.0008	0.1775	**0.0183**	**0.0194**
HML portfolio returns	-0.0329	-0.0139	0.0421	-0.1815	0.0025	-0.0495	0.0217
RMW portfolio returns	0.0035	-0.0009	0.0171	-0.0382	0.0354	-0.0111	0.0093
CMA portfolio returns	0.0273	0.0208	0.0293	-0.0056	0.1278	**0.0161**	**0.0256**
Momentum portfolio returns	0.0028	0.0021	0.0175	-0.0260	0.0716	-0.0003	0.0044
Gold spot returns	0.0080	0.0022	0.0112	-0.0099	0.0392	-0.0044	0.0087
Platinum spot returns	-0.0253	-0.0210	0.0211	-0.0637	0.0078	**-0.0243**	**-0.0177**
Silver spot returns	-0.0155	-0.0063	0.0187	-0.0562	0.0269	-0.0171	0.0045
Growth of Google Ripple searches	-0.0046	0.0101	0.0916	-0.5272	0.0991	-0.3332	0.3533
VW returns on Chinese electricity stocks	-0.0116	-0.0145	0.0117	-0.0309	0.0349	**-0.0178**	**-0.0112**
VW returns on US electricity stocks	0.0124	0.0045	0.0167	0.0008	0.0862	-0.0004	0.0049
NVIDIA individual stock returns	0.0050	0.0009	0.0129	-0.0292	0.0514	-0.0054	0.0072
AMD individual stock returns	-0.0216	-0.0106	0.0224	-0.0663	0.0020	-0.0158	0.0005
TSMC individual stock returns	0.0337	0.0116	0.0433	0.0022	0.1951	**0.0106**	**0.0126**
Residual predictive variance (h_t)	1.0846	1.0088	0.8466	0.1373	3.0424	**0.6063**	**1.4112**
	Dynamic Model Selection Coefficients						
Bitcoin Lagged returns	0.0027	0.0000	0.0208	0.0000	0.1798	0.0000	0.0000
Ripple Momentum	0.0147	0.0000	0.0985	0.0000	0.7723	0.0000	0.0000
VW CRSP Mkt excess returns	0.0015	0.0000	0.0201	0.0000	0.2724	0.0000	0.0000
SMB portfolio returns	0.0055	0.0000	0.0424	0.0000	0.3590	0.0000	0.0000
HML portfolio returns	-0.0117	0.0000	0.0753	-0.6193	0.0000	0.0000	0.0000
RMW portfolio returns	0.0003	0.0000	0.0042	0.0000	0.0561	0.0000	0.0000
CMA portfolio returns	0.0090	0.0000	0.0626	0.0000	0.5487	0.0000	0.0000
Momentum portfolio returns	0.0009	0.0000	0.0122	0.0000	0.1654	0.0000	0.0000
Gold spot returns	-0.0018	0.0000	0.0239	-0.3233	0.0000	0.0000	0.0000
Platinum spot returns	-0.0013	0.0000	0.0138	-0.0966	0.0816	0.0000	0.0000
Silver spot returns	0.0003	0.0000	0.0043	0.0000	0.0587	0.0000	0.0000
Growth of Google Ripple searches	-0.0057	0.0000	0.0770	-1.0411	0.0000	0.0000	0.0000
VW returns on Chinese electricity stocks	0.0018	0.0000	0.0157	-0.0315	0.1419	0.0000	0.0000
VW returns on US electricity stocks	0.0000	0.0000	0.0039	-0.0398	0.0343	0.0000	0.0000
NVIDIA individual stock returns	-0.0031	0.0000	0.0245	-0.2417	0.0000	0.0000	0.0000
AMD individual stock returns	0.0024	0.0000	0.0190	0.0000	0.1858	0.0000	0.0000
TSMC individual stock returns	0.0099	0.0000	0.0603	0.0000	0.4981	0.0000	0.0000
Residual predictive variance (h_t)	0.9857	0.9605	0.8300	0.1303	4.8363	**0.5787**	**1.3423**

Table 5

Summary Statistics for Recursively Estimated DMA and DMS Predictive Regression Coefficients: Litecoin Returns

The statistics are based on weekly data, expressed in US dollars. The percentile statistics are across time and probability-weighted within time periods. We have boldfaced pairs of 5th-95th empirical percentile statistics that fail to include zero, which implies that in overall terms (under the assumption of independence over time), the series of recursively estimated slopes are statistically significant at a 10% size.

	Mean	Median	St. Dev.	Min	Max	5th percentile	95th percetile
	Dynamic Model Average Coefficients						
Bitcoin Lagged returns	0.0873	0.0598	0.0718	0.0132	0.4169	**0.0557**	**0.0639**
Ethereum Momentum	-0.2164	0.0299	0.6004	-2.2577	0.1879	-0.2128	0.2725
VW CRSP Mkt excess returns	0.0063	0.0071	0.0134	-0.0449	0.0375	-0.0124	0.0266
SMB portfolio returns	-0.1636	-0.1296	0.1123	-0.3379	-0.0026	**-0.1409**	**-0.1182**
HML portfolio returns	-0.0185	-0.0163	0.0190	-0.0890	0.0152	-0.0480	0.0154
RMW portfolio returns	0.0222	0.0241	0.0156	-0.0050	0.0711	-0.0207	0.0275
CMA portfolio returns	0.0177	0.0055	0.0313	-0.0520	0.0725	-0.0074	0.0183
Momentum portfolio returns	-0.0180	-0.0075	0.0246	-0.0621	0.0221	-0.0109	0.0040
Gold spot returns	-0.0386	-0.0383	0.0258	-0.1049	0.0002	**-0.0581**	**-0.0184**
Platinum spot returns	0.0647	0.0574	0.0399	-0.0021	0.2326	**0.0437**	**0.0711**
Silver spot returns	0.0767	0.0677	0.0491	0.0189	0.3245	**0.0597**	**0.0758**
Growth of Google Ethereum searches	0.1345	0.1583	0.0711	0.0003	0.2795	**0.1575**	**0.1590**
VW returns on Chinese electricity stocks	-0.0475	-0.0461	0.0181	-0.0989	-0.0123	**-0.0622**	**-0.0299**
VW returns on US electricity stocks	-0.0171	-0.0131	0.0116	-0.0507	-0.0018	**-0.0205**	**-0.0057**
NVIDIA individual stock returns	0.0301	0.0371	0.0209	-0.0134	0.1065	**0.0295**	**0.0448**
AMD individual stock returns	0.0192	0.0111	0.0188	-0.0027	0.0790	-0.0074	0.0148
TSMC individual stock returns	-0.0051	-0.0058	0.0055	-0.0317	0.0089	-0.0147	0.0031
Residual predictive variance (h_t)	1.1403	1.1898	0.3933	0.3994	1.7550	**1.0981**	**1.2815**
	Dynamic Model Selection Coefficients						
Bitcoin Lagged returns	0.0873	0.0598	0.0718	0.0132	0.4169	**0.0517**	**0.0679**
Ethereum Momentum	-0.2164	0.0299	0.6004	-2.2577	0.1879	-0.4555	0.5152
VW CRSP Mkt excess returns	0.0063	0.0071	0.0134	-0.0449	0.0375	-0.0319	0.0461
SMB portfolio returns	-0.1636	-0.1296	0.1123	-0.3379	-0.0026	**-0.1522**	**-0.1069**
HML portfolio returns	-0.0185	-0.0163	0.0190	-0.0890	0.0152	-0.0796	0.0471
RMW portfolio returns	0.0222	0.0241	0.0156	-0.0050	0.0711	-0.0174	0.0308
CMA portfolio returns	0.0177	0.0055	0.0313	-0.0520	0.0725	-0.0202	0.0312
Momentum portfolio returns	-0.0180	-0.0075	0.0246	-0.0621	0.0221	-0.0144	0.0005
Gold spot returns	-0.0386	-0.0383	0.0258	-0.1049	0.0002	-0.0780	0.0015
Platinum spot returns	0.0647	0.0574	0.0399	-0.0021	0.2326	**0.0300**	**0.0848**
Silver spot returns	0.0767	0.0677	0.0491	0.0189	0.3245	**0.0517**	**0.0838**
Growth of Google Ethereum searches	0.1345	0.1583	0.0711	0.0003	0.2795	**0.1568**	**0.1597**
VW returns on Chinese electricity stocks	-0.0475	-0.0461	0.0181	-0.0989	-0.0123	-0.0784	0.0137
VW returns on US electricity stocks	-0.0171	-0.0131	0.0116	-0.0507	-0.0018	-0.0279	0.0017
NVIDIA individual stock returns	0.0301	0.0371	0.0209	-0.0134	0.1065	**0.0218**	**0.0525**
AMD individual stock returns	0.0192	0.0111	0.0188	-0.0027	0.0790	-0.0037	0.0184
TSMC individual stock returns	-0.0051	-0.0058	0.0055	-0.0317	0.0089	-0.0235	0.0119
Residual predictive variance (h_t)	1.1403	1.1898	0.3933	0.3994	1.7550	**0.4532**	**1.9264**

Table 6

Summary Statistics for Recursively Estimated DMA and DMS Predictive Regression Coefficients: Investment Grade US Corporate Bond Returns

The statistics are based on weekly data, expressed in US dollars. The percentile statistics are across time and probability-weighted within time periods. We have boldfaced pairs of 5th-95th empirical percentile statistics that fail to include zero, which implies that in overall terms (under the assumption of independence over time), the series of recursively estimated slopes are statistically significant at a 10% size.

	Mean	Median	St. Dev.	Min	Max	5th percentile	95th percetile
	Dynamic Model Average Coefficients						
Bitcoin Lagged returns	0.0116	0.0181	0.0176	-0.0477	0.0425	**-0.0406**	**-0.0293**
Ethereum Momentum	0.0089	0.0092	0.0173	-0.0394	0.0497	**-0.0204**	**-0.0160**
VW CRSP Mkt excess returns	0.0325	0.0289	0.0264	-0.0755	0.1287	**-0.0618**	**-0.0005**
SMB portfolio returns	0.0332	0.0240	0.0277	-0.0112	0.1097	**-0.0106**	**-0.0049**
HML portfolio returns	-0.0221	-0.0155	0.0609	-0.2595	0.1009	**-0.2029**	**-0.1502**
RMW portfolio returns	-0.0519	-0.0312	0.0591	-0.4175	-0.0065	**-0.3242**	**-0.1794**
CMA portfolio returns	0.0231	0.0212	0.0250	-0.1182	0.1500	**-0.0168**	**-0.0106**
Momentum portfolio returns	0.0223	0.0052	0.0467	-0.0337	0.3254	**-0.0279**	**-0.0196**
Gold spot returns	-0.0165	-0.0184	0.0433	-0.2248	0.0510	**-0.1517**	**-0.1005**
Platinum spot returns	-0.0699	-0.0679	0.0733	-0.2183	0.0332	**-0.2093**	**-0.1875**
Silver spot returns	0.0368	0.0397	0.0242	-0.0188	0.0963	**-0.0148**	**-0.0056**
Growth of Google Ethereum searches	0.0109	0.0088	0.0100	-0.0101	0.0323	**-0.0090**	**-0.0052**
VW returns on Chinese electricity stocks	0.0609	0.0486	0.0557	0.0051	0.4193	**0.0089**	**0.0168**
VW returns on US electricity stocks	0.0996	0.0521	0.0936	0.0083	0.3991	**0.0102**	**0.0167**
NVIDIA individual stock returns	-0.0236	-0.0157	0.0261	-0.1324	0.0209	**-0.1297**	**-0.0834**
AMD individual stock returns	0.0133	0.0132	0.0160	-0.0156	0.1048	**-0.0131**	**-0.0087**
TSMC individual stock returns	0.0027	0.0103	0.0356	-0.2063	0.0621	**-0.1384**	**-0.0648**
Residual predictive variance (h_t)	1.0816	0.9908	0.4320	0.4320	2.2924	**0.4832**	**1.9228**
	Dynamic Model Selection Coefficients						
Bitcoin Lagged returns	-0.0009	0.0000	0.0100	-0.1283	0.0000	0.0000	0.0000
Ethereum Momentum	-0.0006	0.0000	0.0063	-0.0676	0.0226	0.0000	0.0000
VW CRSP Mkt excess returns	0.0140	0.0000	0.0576	0.0000	0.3952	**0.0000**	**0.1848**
SMB portfolio returns	0.0214	0.0000	0.0623	-0.1272	0.2072	-0.0814	0.1371
HML portfolio returns	0.0161	0.0000	0.0810	-0.5953	0.1864	**0.0000**	**0.1329**
RMW portfolio returns	-0.0084	0.0000	0.0661	-0.5512	0.0000	0.0000	0.0000
CMA portfolio returns	0.0025	0.0000	0.0187	0.0000	0.1749	0.0000	0.0000
Momentum portfolio returns	0.0044	0.0000	0.0327	0.0000	0.2781	0.0000	0.0000
Gold spot returns	0.0025	0.0000	0.0199	0.0000	0.1989	0.0000	0.0000
Platinum spot returns	-0.0570	0.0000	0.0900	-0.2617	0.0000	**-0.2231**	**0.0000**
Silver spot returns	0.0047	0.0000	0.0278	0.0000	0.1955	0.0000	0.0000
Growth of Google Ethereum searches	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
VW returns on Chinese electricity stocks	0.0488	0.0000	0.1081	0.0000	0.5077	**0.0000**	**0.3241**
VW returns on US electricity stocks	0.0606	0.0000	0.1175	0.0000	0.3909	**0.0000**	**0.3334**
NVIDIA individual stock returns	-0.0112	0.0000	0.0360	-0.1545	0.0000	**-0.1180**	**0.0000**
AMD individual stock returns	0.0006	0.0000	0.0079	-0.0278	0.0643	0.0000	0.0000
TSMC individual stock returns	0.0262	0.0000	0.0460	-0.0207	0.1908	**-0.0027**	**0.1393**
Residual predictive variance (h_t)	1.0045	0.9218	0.4073	0.4020	2.2112	**0.4520**	**1.8392**

Table 7

Summary Statistics for Recursively Estimated DMA and DMS Predictive Regression Coefficients: Log Changes of the US Dollar Trade-Weighted Exchange Rate

The statistics are based on weekly data. The percentile statistics are across time and probability-weighted within time periods. We have boldfaced pairs of 5th-95th empirical percentile statistics that fail to include zero, which implies that in overall terms (under the assumption of independence over time), the series of recursively estimated slopes are statistically significant at a 10% size.

	Mean	Median	St. Dev.	Min	Max	5th percentile	95th percetile
	Dynamic Model Average Coefficients						
Bitcoin Lagged returns	-0.0090	-0.0114	0.0185	-0.0477	0.0773	**-0.0453**	**-0.0338**
Ethereum Momentum	-0.0450	-0.0378	0.0368	-0.1555	0.0402	**-0.1464**	**-0.1314**
VW CRSP Mkt excess returns	0.0257	0.0219	0.0299	-0.0197	0.1537	**-0.0174**	**-0.0130**
SMB portfolio returns	0.1060	0.0460	0.1080	-0.0001	0.3977	**0.0025**	**0.0092**
HML portfolio returns	-0.0460	-0.0224	0.0679	-0.2312	0.0493	**-0.2272**	**-0.2062**
RMW portfolio returns	0.0228	0.0138	0.0361	-0.0449	0.1012	**-0.0440**	**-0.0340**
CMA portfolio returns	0.0099	-0.0006	0.0451	-0.1216	0.1195	**-0.0708**	**-0.0500**
Momentum portfolio returns	0.0471	0.0397	0.0381	-0.1351	0.1445	-0.0337	0.0025
Gold spot returns	0.0278	0.0129	0.0797	-0.0147	0.8291	**-0.0124**	**-0.0045**
Platinum spot returns	-0.0224	-0.0020	0.0728	-0.2439	0.1180	**-0.2347**	**-0.2100**
Silver spot returns	0.0760	0.0348	0.1025	-0.2010	0.3667	**-0.0266**	**-0.0014**
Growth of Google Bitcoin searches	0.0164	0.0120	0.0182	-0.0302	0.0621	**-0.0263**	**-0.0112**
VW returns on Chinese electricity stocks	0.0385	0.0172	0.0867	-0.0089	0.6903	**-0.0055**	**-0.0007**
VW returns on US electricity stocks	-0.0632	-0.0616	0.0563	-0.1959	0.0836	**-0.1893**	**-0.1607**
NVIDIA individual stock returns	0.2392	0.2357	0.0480	0.0230	0.3894	**0.1396**	**0.1615**
AMD individual stock returns	-0.0157	-0.0152	0.0419	-0.1531	0.4464	**-0.1163**	**-0.0613**
TSMC individual stock returns	-0.1493	-0.0712	0.1634	-0.6132	-0.0047	**-0.5821**	**-0.5330**
Residual predictive variance (h_t)	1.1732	1.1326	0.3966	0.5733	2.7064	**0.6559**	**2.0560**
	Dynamic Model Selection Coefficients						
Bitcoin Lagged returns	0.0030	0.0000	0.0225	0.0000	0.1968	0.0000	0.0000
Ethereum Momentum	-0.0209	0.0000	0.0583	-0.2250	0.0000	**-0.1827**	**0.0000**
VW CRSP Mkt excess returns	0.0008	0.0000	0.0108	0.0000	0.1572	0.0000	0.0000
SMB portfolio returns	0.0596	0.0000	0.1425	0.0000	0.7773	**0.0000**	**0.4152**
HML portfolio returns	-0.0393	0.0000	0.1000	-0.5601	0.0000	**-0.2763**	**0.0000**
RMW portfolio returns	—	—	—	—	—	—	—
CMA portfolio returns	0.0199	0.0000	0.0524	0.0000	0.1899	**0.0000**	**0.1606**
Momentum portfolio returns	0.0157	0.0000	0.0573	0.0000	0.2589	**0.0000**	**0.2155**
Gold spot returns	0.0088	0.0000	0.1142	0.0000	1.6436	0.0000	0.0000
Platinum spot returns	-0.0394	0.0000	0.0999	-0.3589	0.1911	**-0.2950**	**0.0000**
Silver spot returns	0.0792	0.0000	0.1521	-0.4814	0.4537	**0.0000**	**0.4180**
Growth of Google Bitcoin searches	—	—	—	—	—	—	—
VW returns on Chinese electricity stocks	0.0093	0.0000	0.0752	0.0000	0.6683	0.0000	0.0000
VW returns on US electricity stocks	-0.0537	0.0000	0.0853	-0.2341	0.0000	**-0.2118**	**0.0000**
NVIDIA individual stock returns	0.2693	0.2586	0.0668	0.0000	0.4761	**0.1924**	**0.3940**
AMD individual stock returns	-0.0066	0.0000	0.0522	-0.1922	0.5244	**-0.1265**	**0.0000**
TSMC individual stock returns	-0.2047	-0.1671	0.1990	-0.7186	0.0000	**-0.5898**	**0.0000**
Residual predictive variance (h_t)	1.0572	1.0428	0.3059	0.5465	1.8945	**0.6148**	**1.7040**

Table 8

Recursive Out-of-Sample Realized Forecasting Performances of Alternative Models

The table reports two indicators of realized, OOS predictive accuracy: the root mean-squared forecast error and the OOS R-square. Four alternative forecasting models are compared, recursive OLS including all predictors jointly, (unfeasible) recursive OLS based on the best, full-sample predictor (that is also reported in the table), dynamic model averaging, and dynamic model selection. For each cryptocurrency/asset class, we have boldfaced the model yielding the lowest RMSFE and OOS R-square across the four predictive frameworks. When the best achievable OOS R-square is negative (an indication that a model cannot outperform the sample mean), it has been emphasized using boldfaced red. The recursive, expanding window OOS experiments are applied to the available sample after initializing the recursive estimated on the basis of 26 initial observations.

	Sample	Recursive OLS Estimation		Best Recursive OLS Predictor			Dynamic Model Averaging		Dynamic Model Selection	
		OOS R^2	RMSFE	Predictor	OOS R^2	RMSFE	OOS R^2	RMSFE	OOS R^2	RMSFE
	Cryptocurrency returns									
Bitcoin returns	Jan. 2011 - Jan. 2019	-1.6710	1.4777	Google searches	-0.0542	0.9284	-0.2781	1.0222	**-0.0061**	**0.9015**
Litecoin returns	July 2013 - Jan. 2019	-0.4839	1.2342	Google searches	0.2857	0.8564	0.3821	0.7665	**0.4609**	**0.7418**
Ripple returns	April 2015 - Jan. 2019	-1.0618	1.4548	Silver spot ret.	**-0.0027**	**1.0145**	-0.0918	1.0586	-0.0691	1.0475
Ethereum returns	June 2016 - Jan. 2019	-0.6398	1.3144	Google searches	0.0070	1.0125	-0.1391	1.0936	**0.0109**	**0.9989**
	Benchmark asset returns									
Inv. Grade US Corporate bond returns	Jan. 2011 - Jan. 2019	-0.2331	1.1091	US Electrical Power Stocks	-0.0358	1.0165	0.0456	1.0336	**0.0622**	**0.9789**
VW CRSP Mkt excess returns	Jan. 2011 - Jan. 2019	-0.4856	1.2174	1-month T-bill	-0.0295	1.0134	0.0044	1.0298	**0.0285**	**0.9936**
VW World Developed Market returns (ex-US)	Jan. 2011 - Jan. 2019	-0.4737	1.2125	1-month T-bill	-0.0032	1.0004	0.0149	1.0252	**0.0319**	**0.985**
Gold spot returns	Jan. 2011 - Jan. 2019	-0.3379	1.1553	Silver spot ret.	-0.0304	1.0139	**0.0197**	**1.0001**	0.0101	1.0044
Value-weighted US Dollar Exchange Rate Returns	Jan. 2011 - Jan. 2019	-0.3379	1.1553	US Electrical Power Stocks	0.0006	0.9996	-0.0192	1.0092	**0.0027**	**0.9555**

Table 9

Summary Statistics for Recursive, Expected Power Utility ($\gamma = 3$) Portfolio Weights

The statistics are based on recursive, weekly optimized weights from a power utility with $\gamma = 3$. The percentile statistics are across time. A "___" indicates that the asset was never demanded.

		Mean	Median	St. Dev.	Min	Max	5th percentile	95th percentile
		Dynamic Model Average Portfolio Weights						
Baseline (2011-2019)	CRSP VW Mkt	0.1118	0.0000	0.1784	0.0000	0.6666	0.0000	0.4755
	Developed (ex-US) VW Mkt	0.0422	0.0000	0.0938	0.0000	0.4839	0.0000	0.2761
	US IG Corporate Bonds	0.2214	0.1897	0.1944	0.0000	0.7814	0.0000	0.5621
	Gold	0.1012	0.0603	0.1213	0.0000	0.5552	0.0000	0.3797
	Long-short dollar trade	0.1312	0.0627	0.1689	0.0000	1.0000	0.0000	0.4740
	Cash	0.3921	0.4202	0.2296	0.0000	0.9785	0.0000	0.7558
Baseline + Bitcoin (2011-2019)	Bitcoin	0.2125	0.1828	0.1482	0.0000	0.9931	0.0343	0.5092
	CRSP VW Mkt	0.0903	0.0000	0.1554	0.0000	0.6511	0.0000	0.4527
	Developed VW Mkt	0.0277	0.0000	0.0801	0.0000	0.4669	0.0000	0.2209
	US Corporate Bonds	0.3133	0.3046	0.2492	0.0000	0.8542	0.0000	0.7090
	Gold	0.1419	0.1078	0.1313	0.0000	0.8046	0.0000	0.4231
	Long-short dollar trade	0.1232	0.0055	0.1749	0.0000	1.0000	0.0000	0.4892
	Cash	0.0911	0.0000	0.1917	0.0000	0.9171	0.0000	0.5500
Baseline + All Crypto (2017-2019)	Bitcoin	0.2459	0.0000	0.3653	0.0000	1.0000	0.0000	0.9999
	Ethereum	0.0544	0.0000	0.1418	0.0000	0.7348	0.0000	0.3770
	Litecoin	0.1569	0.0000	0.3412	0.0000	1.0000	0.0000	1.0000
	Ripple	0.0537	0.0000	0.1880	0.0000	1.0000	0.0000	0.3334
	CRSP VW Mkt	0.0504	0.0000	0.1769	0.0000	0.7995	0.0000	0.6873
	Developed VW Mkt	0.0371	0.0000	0.1835	0.0000	1.0000	0.0000	0.0758
	US Corporate Bonds	0.1376	0.0000	0.2973	0.0000	1.0000	0.0000	0.9728
	Gold	0.0723	0.0000	0.1498	0.0000	0.6607	0.0000	0.4142
	Long-short dollar trade	0.1918	0.0000	0.3314	0.0000	1.0000	0.0000	1.0000
	Cash	___	___	___	___	___	___	___
		Dynamic Model Selection Portfolio Weights						
Baseline (2011-2019)	CRSP VW Mkt	0.1050	0.0000	0.1811	0.0000	0.7190	0.0000	0.5254
	Developed (ex-US) VW Mkt	0.0405	0.0000	0.1037	0.0000	0.6983	0.0000	0.2895
	US IG Corporate Bonds	0.3371	0.3351	0.2400	0.0000	0.9375	0.0000	0.7849
	Gold	0.1233	0.0766	0.1455	0.0000	0.9526	0.0000	0.4069
	Long-short dollar trade	0.1669	0.1259	0.1769	0.0000	0.9502	0.0000	0.5023
	Cash	0.2272	0.1474	0.2495	0.0000	1.0000	0.0000	0.6990
Baseline + Bitcoin (2011-2019)	Bitcoin	0.1213	0.0987	0.1127	0.0000	0.8345	0.0000	0.3072
	CRSP VW Mkt	0.1051	0.0000	0.1795	0.0000	0.7189	0.0000	0.5241
	Developed VW Mkt	0.0388	0.0000	0.1028	0.0000	0.6983	0.0000	0.2846
	US Corporate Bonds	0.3596	0.3712	0.2508	0.0000	0.9249	0.0000	0.7654
	Gold	0.1200	0.0750	0.1404	0.0000	0.9551	0.0000	0.4066
	Long-short dollar trade	0.1408	0.0726	0.1739	0.0000	0.9502	0.0000	0.4838
	Cash	0.1143	0.0000	0.2037	0.0000	1.0000	0.0000	0.5519
Baseline + All Crypto (2017-2019)	Bitcoin	0.1801	0.0000	0.3131	0.0000	0.9999	0.0000	0.8659
	Ethereum	0.0969	0.0000	0.1677	0.0000	0.7405	0.0000	0.4892
	Litecoin	0.1603	0.0000	0.3403	0.0000	1.0000	0.0000	1.0000
	Ripple	0.0044	0.0000	0.0161	0.0000	0.1102	0.0000	0.0388
	CRSP VW Mkt	0.0603	0.0000	0.1961	0.0000	1.0000	0.0000	0.4885
	Developed VW Mkt	0.0406	0.0000	0.1759	0.0000	1.0000	0.0000	0.2141
	US Corporate Bonds	0.1608	0.0000	0.3001	0.0000	1.0000	0.0000	0.8954
	Gold	0.0893	0.0000	0.1668	0.0000	0.6237	0.0000	0.5207
	Long-short dollar trade	0.2072	0.0000	0.3389	0.0000	1.0000	0.0000	1.0000
	Cash	___	___	___	___	___	___	___

Table 9 (continued)

Summary Statistics for Recursive, Expected Power Utility ($\gamma = 3$) Portfolio Weights

		Mean	Median	St. Dev.	Min	Max	5th percentile	95th percentile
		Historical Sample Mean						
Baseline (2011-2019)	CRSP VW Mkt	—	—	—	—	—	—	—
	Developed (ex-US) VW Mkt	—	—	—	—	—	—	—
	US IG Corporate Bonds	1.0000	1.0000	0.0000	0.9999	1.0000	0.9999	1.0000
	Gold	—	—	—	—	—	—	—
	Long-short dollar trade	—	—	—	—	—	—	—
	Cash	—	—	—	—	—	—	—
Baseline + Bitcoin (2011-2019)	Bitcoin	0.6167	0.7318	0.3888	0.0000	1.0000	0.0000	1.0000
	CRSP VW Mkt	—	—	—	—	—	—	—
	Developed VW Mkt	—	—	—	—	—	—	—
	US Corporate Bonds	0.3833	0.2681	0.3888	0.0000	1.0000	0.0000	0.9999
	Gold	—	—	—	—	—	—	—
	Long-short dollar trade	—	—	—	—	—	—	—
	Cash	—	—	—	—	—	—	—
Baseline + All Crypto (2017-2019)	Bitcoin	0.0000	0.0000	0.0000	0.0000	0.0002	0.0000	0.0001
	Ethereum	0.3720	0.0002	0.4829	0.0000	1.0000	0.0000	0.9999
	Litecoin	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000
	Ripple	0.6279	0.9997	0.4829	0.0000	1.0000	0.0000	1.0000
	CRSP VW Mkt	—	—	—	—	—	—	—
	Developed VW Mkt	—	—	—	—	—	—	—
	US Corporate Bonds	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0001
	Gold	—	—	—	—	—	—	—
	Long-short dollar trade	—	—	—	—	—	—	—
	Cash	—	—	—	—	—	—	—

Table 10

Realized Performance of Recursive, Expected Power Utility Portfolio Weights

For three alternative values of the coefficient of relative risk aversion and three alternative asset menus/sample sizes, the table reports realized performance measures for optimal portfolio weights derived from the maximization of expected power utility. All measures are reported on a weekly basis. For each asset menu, the last column computed the weekly fee (economic value) that an investor ought to be ready to pay to switch out from a recursive strategy based on historical moments into DMA/DMS strategies that exploit predictability from a given set of predictors.

		Mean	Volatility	Sharpe ratio	Skewness	Kurtosis	Realized avg. utility	CER	Value of Predict.
					$\gamma = 3$				
Baseline (2011-2019)	DMA	0.0073	**0.1985**	0.0360	-0.1738	12.9479	-1.5951	-0.0401	
	DMS	**0.0362**	0.2136	**0.1687**	**1.8553**	16.1019	-11.1165	-0.3879	
	Historical Moments	0.0323	0.2055	0.1563	-0.1091	**4.3276**	**-0.5652**	**0.0406**	-0.0807
Baseline + Bitcoin (2011-2019)	DMA	0.0184	0.1981	0.0922	0.2105	9.6490	-1.0479	**0.0908**	
	DMS	**0.0421**	0.2260	**0.1857**	**1.7777**	14.2805	-9.4896	-0.3705	
	Historical Moments	0.0164	**0.1565**	0.1043	0.3596	**5.0106**	**-0.5245**	0.0764	0.0144
Baseline + All Crypto (2017-2019)	DMA	0.0711	0.2806	0.2531	3.4361	16.4236	-0.4935	**0.2066**	
	DMS	**0.0871**	0.3287	**0.2647**	**4.1405**	23.3581	-0.5119	0.1883	
	Historical Moments	0.0153	**0.2132**	0.0714	2.2090	**8.7942**	**-0.5341**	0.1676	0.0390
					$\gamma = 8$				
Baseline (2011-2019)	DMA	0.0033	**0.0979**	0.0329	-0.4054	12.8984	-0.5113	0.1835	
	DMS	0.0204	0.1307	0.1549	**2.2960**	18.2336	**-0.5040**	**0.1852**	
	Historical Moments	**0.0323**	0.2055	**0.1563**	-0.1091	**4.3277**	-5.2663	-0.0027	0.1879
Baseline + Bitcoin (2011-2019)	DMA	0.0121	**0.1492**	0.0803	-0.2103	8.8387	-10.0298	-0.0552	
	DMS	**0.0288**	0.1535	**0.1867**	**1.2436**	12.8310	-2.7433	0.0556	
	Historical Moments	0.0210	0.1561	0.1336	0.0096	**4.1684**	**-0.4463**	**0.2498**	-0.1942
Baseline + All Crypto (2017-2019)	DMA	0.0635	0.2777	0.2282	**3.2969**	16.4554	-0.3177	0.1921	
	DMS	**0.0890**	0.2757	**0.3222**	2.6509	12.7519	-0.3192	0.1915	
	Historical Moments	0.0160	**0.2154**	0.0736	2.1118	**8.4650**	**-0.2794**	**0.2086**	-0.0165
					$\gamma = 15$				
Baseline (2011-2019)	DMA	0.0012	**0.0552**	0.0189	-0.5093	13.1589	-0.1872	0.2335	
	DMS	0.0111	0.0738	0.1482	**2.1635**	18.5226	**-0.1854**	**0.2341**	
	Historical Moments	**0.0321**	0.2049	**0.1559**	-0.1139	**4.3369**	-16.2692	-0.0778	0.3119
Baseline + Bitcoin (2011-2019)	DMA	0.0067	0.0953	0.0688	-0.6730	8.0138	-2.9381	0.1668	
	DMS	0.0181	**0.0941**	**0.1909**	**1.8674**	15.0699	**-0.4139**	**0.2821**	
	Historical Moments	**0.0247**	0.1582	0.1552	-0.0227	**3.9987**	-16.3292	0.0784	0.2036
Baseline + All Crypto (2017-2019)	DMA	0.0563	0.2344	0.2396	**3.2218**	16.9876	**-0.6127**	**0.2577**	
	DMS	**0.0822**	0.2316	**0.3546**	1.6739	7.6092	-0.7038	0.2492	
	Historical Moments	0.0168	**0.2096**	0.0798	1.7699	**6.9814**	-0.7088	0.1488	0.1089

Figure 1

Recursive Dynamic Model Average Coefficient Estimates for Bitcoin Returns

The thicker (black) line reports the recursive estimates on a fixed scale [-2, 2] marked on the left axis, the lighter (blue) curve concerns estimates on a variable (right) scale to enhance readability.

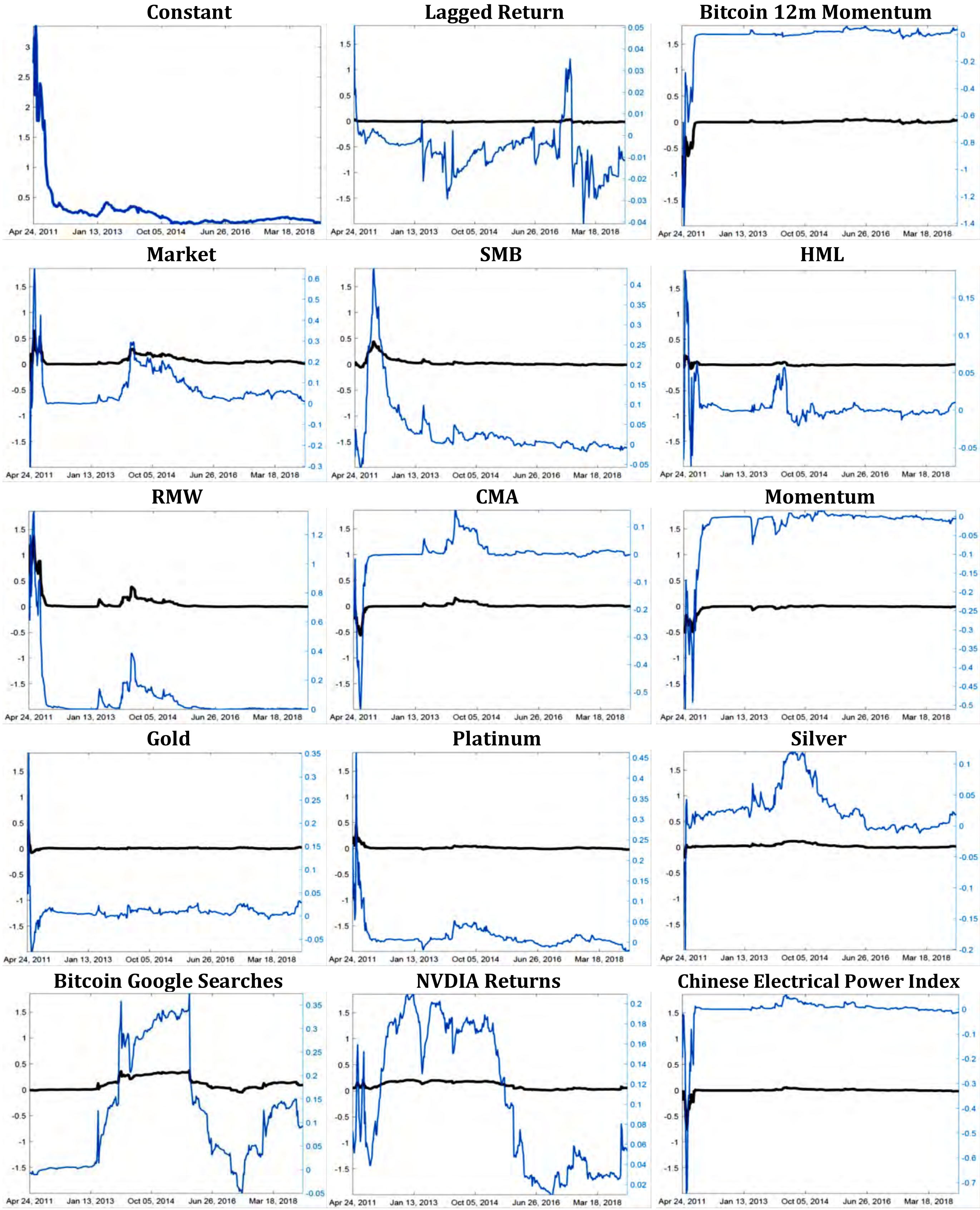

Figure 2

Recursive Dynamic Model Average Probability Estimates for Bitcoin Returns

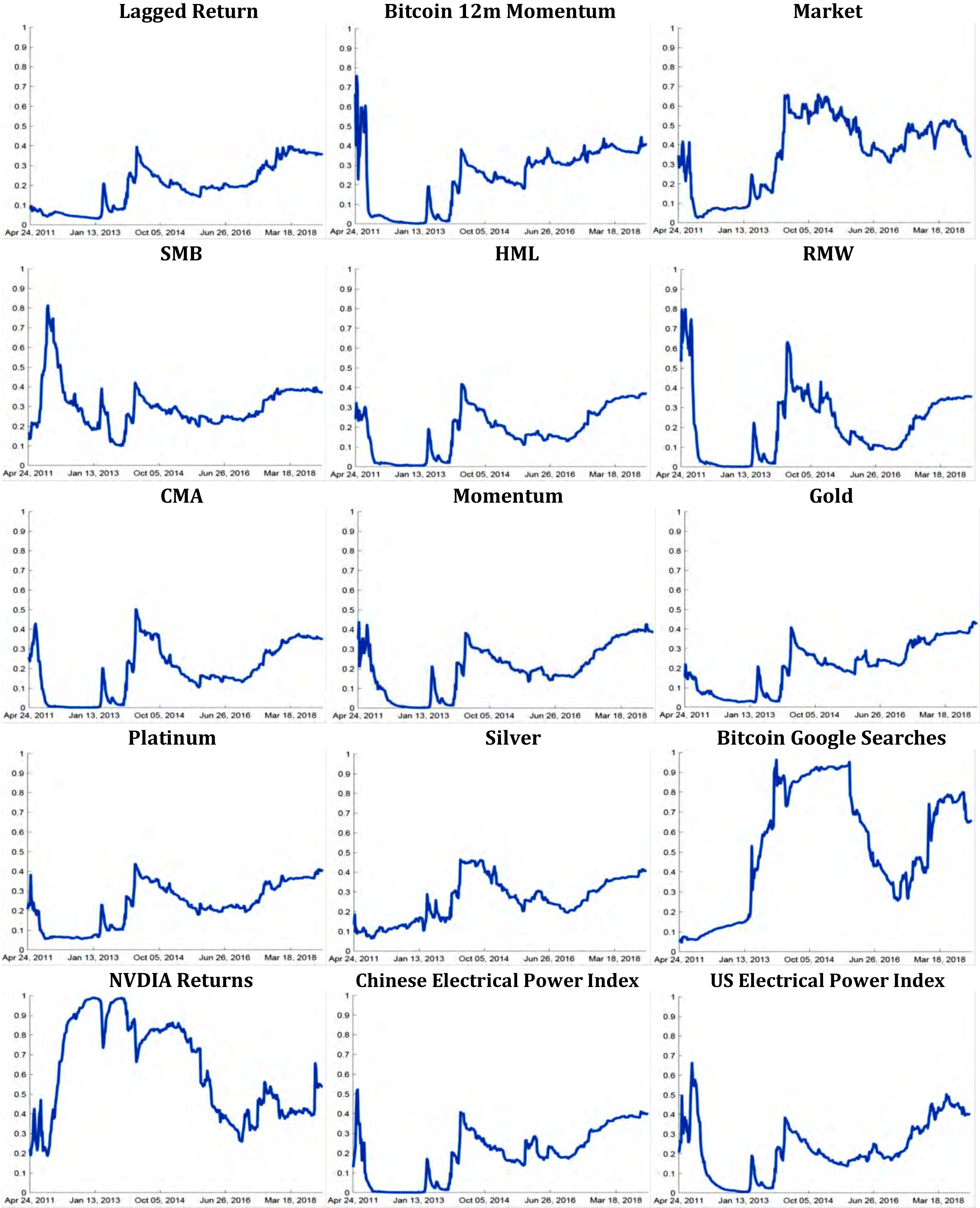

Figure 3

Recursive Dynamic Model Selection Coefficient Estimates for Bitcoin Returns

The thicker (black) line reports the recursive estimates on a fixed scale [-2, 2] marked on the left axis, the lighter (blue) curve concerns estimates on a variable (right) scale to enhance readability.

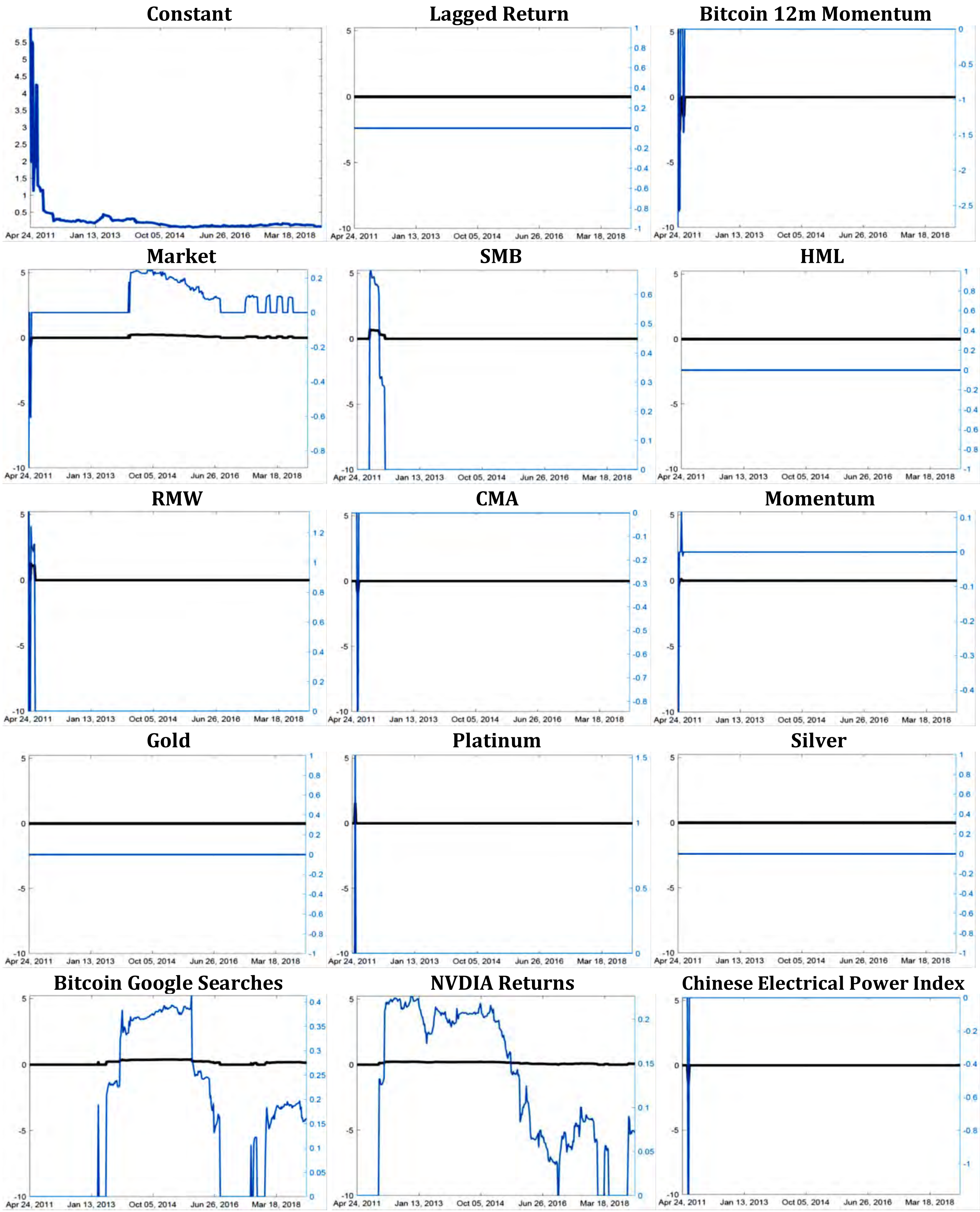

Figure 4

Recursive Dynamic Model Average Coefficient Estimates for Litecoin Returns

The thicker (black) line reports the recursive estimates on a fixed scale [-2, 2] marked on the left axis, the lighter (blue) curve concerns estimates on a variable (right) scale to enhance readability.

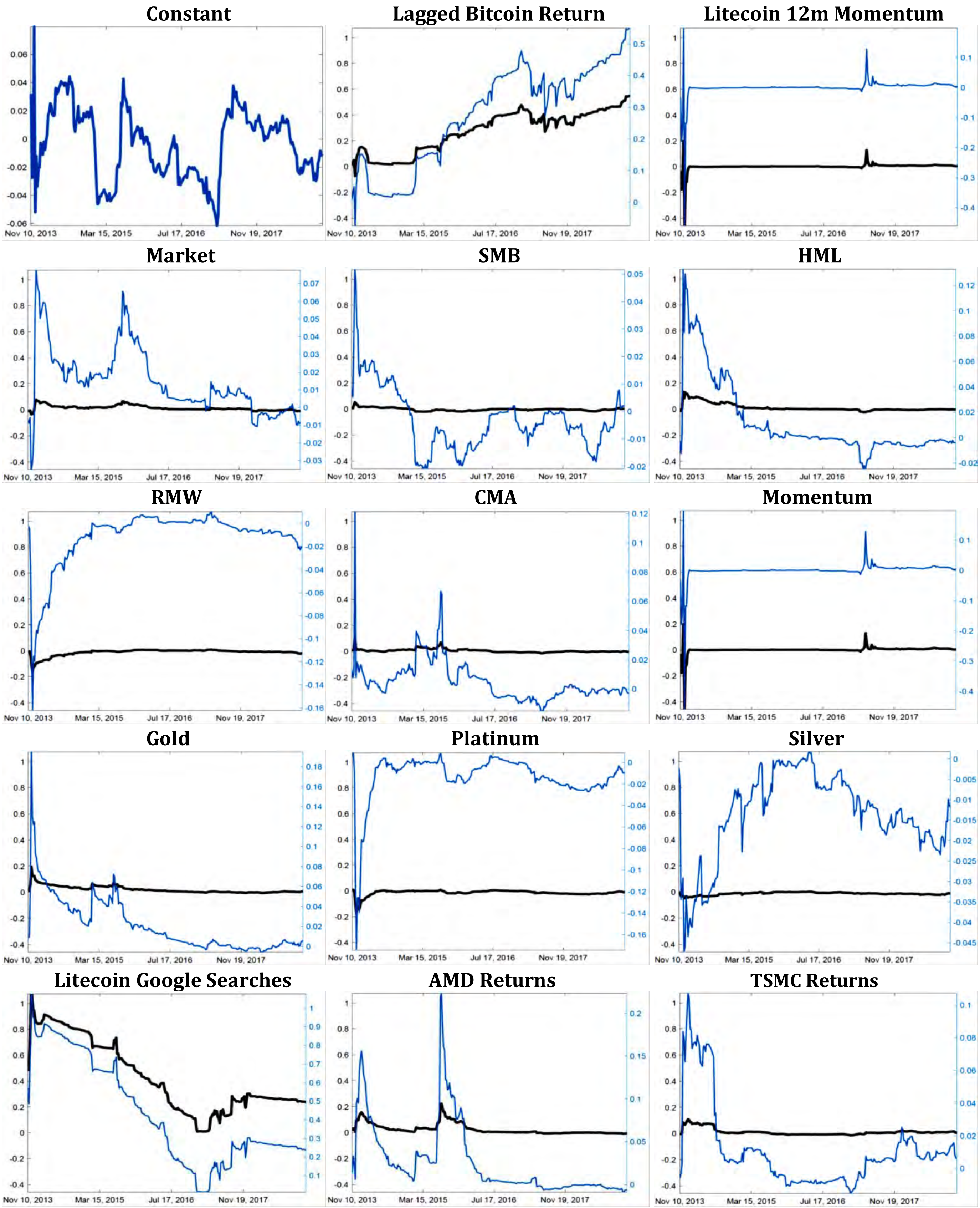

Figure 5

Recursive Dynamic Model Average Probability Estimates for Litecoin Returns

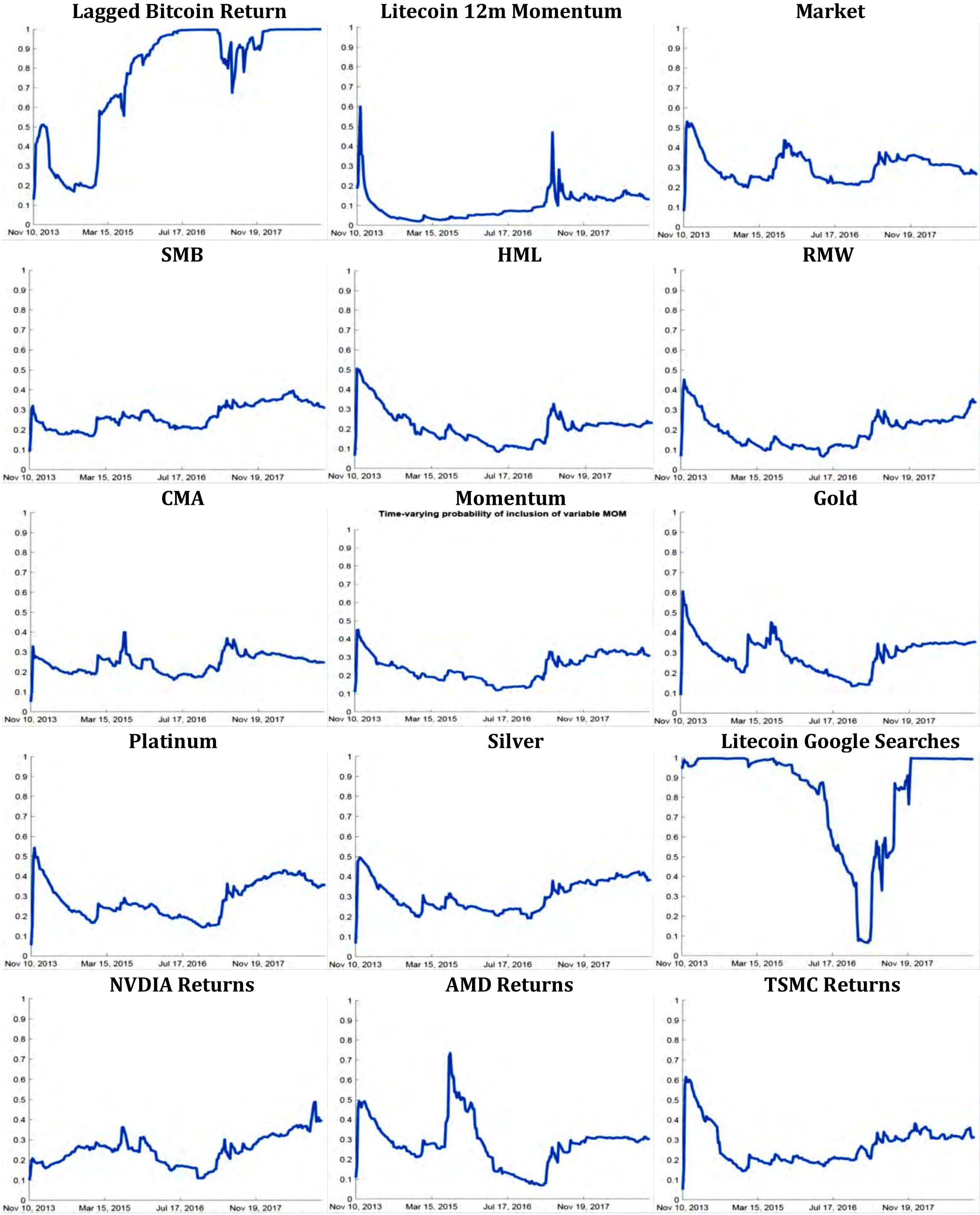

Figure 6

Recursive Dynamic Model Selection Coefficient Estimates for Litecoin Returns

The thicker (black) line reports recursive estimates on a fixed scale [-0.5, 1.1] marked on the left axis, the lighter (blue) curve concerns estimates on a variable (right) scale to enhance readability.

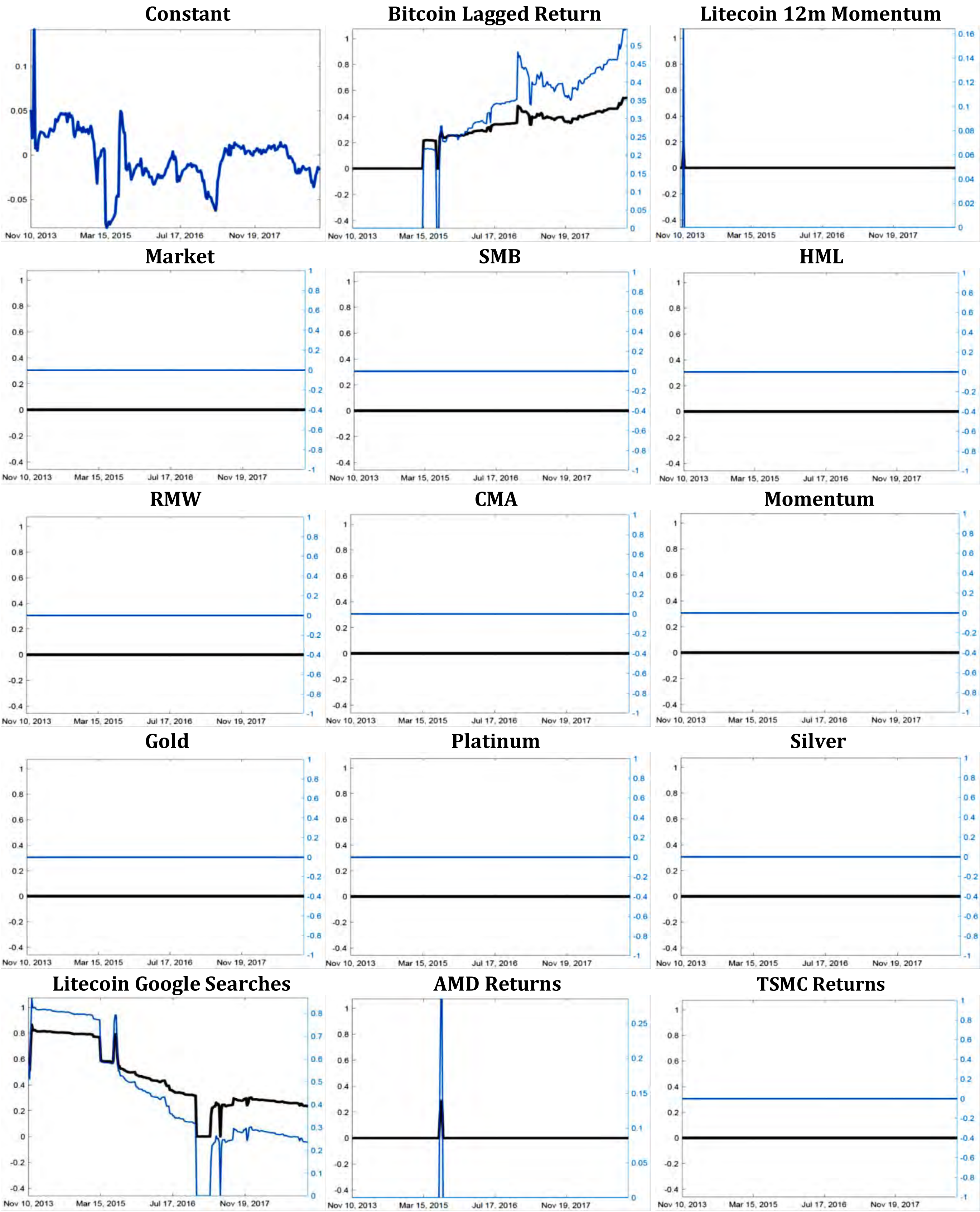

Figure 7

Recursive Dynamic Model Average Coefficient Estimates for Ripple Returns

The thicker (black) line reports recursive estimates on a fixed scale [-6, 1.5] marked on the left axis, the lighter (blue) curve concerns estimates on a variable (right) scale to enhance readability.

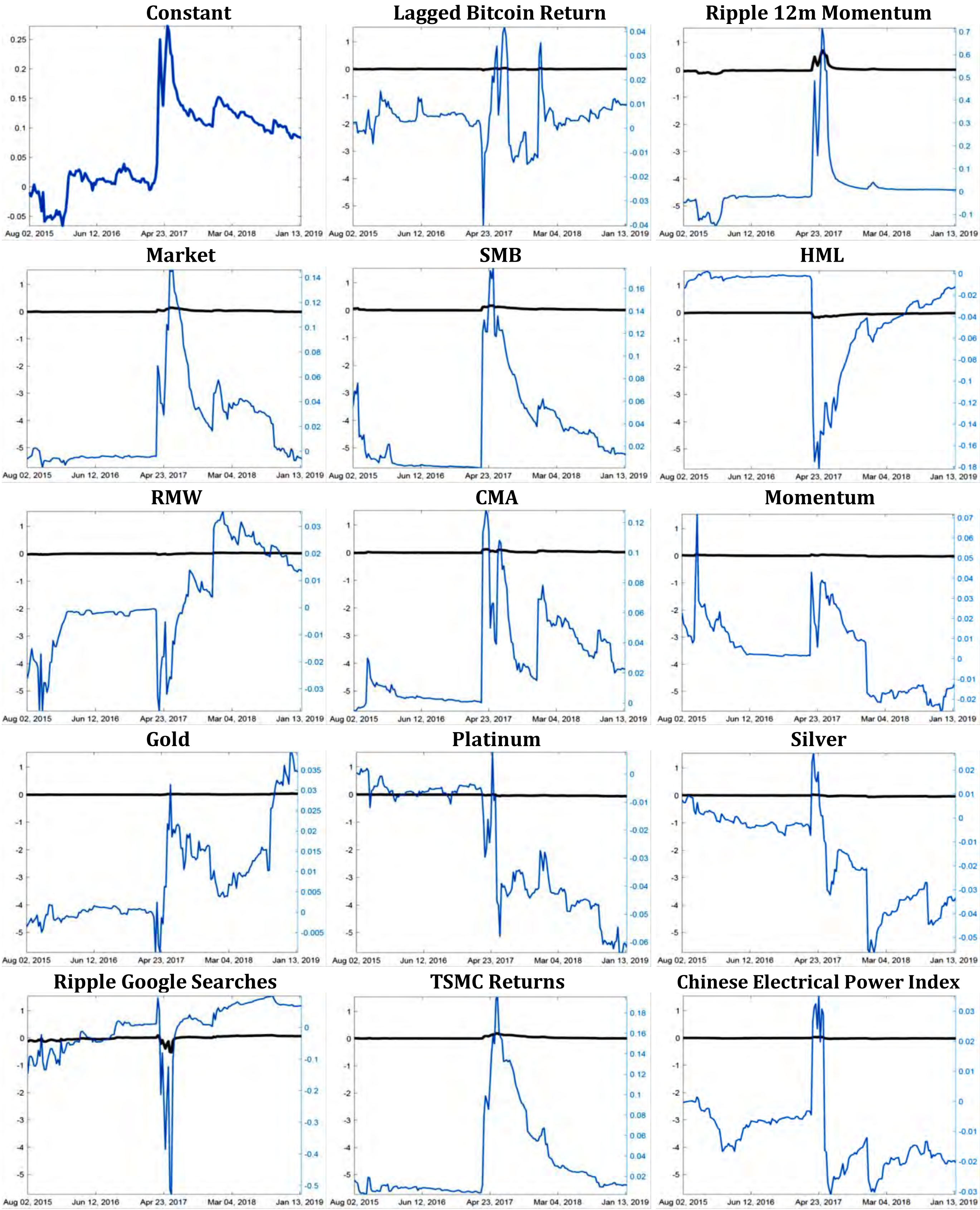

Figure 8

Recursive Dynamic Model Selection Coefficient Estimates for Ripple Returns

The thicker (black) line reports recursive estimates on a fixed scale [-6, 1.5] marked on the left axis, the lighter (blue) curve concerns estimates on a variable (right) scale to enhance readability.

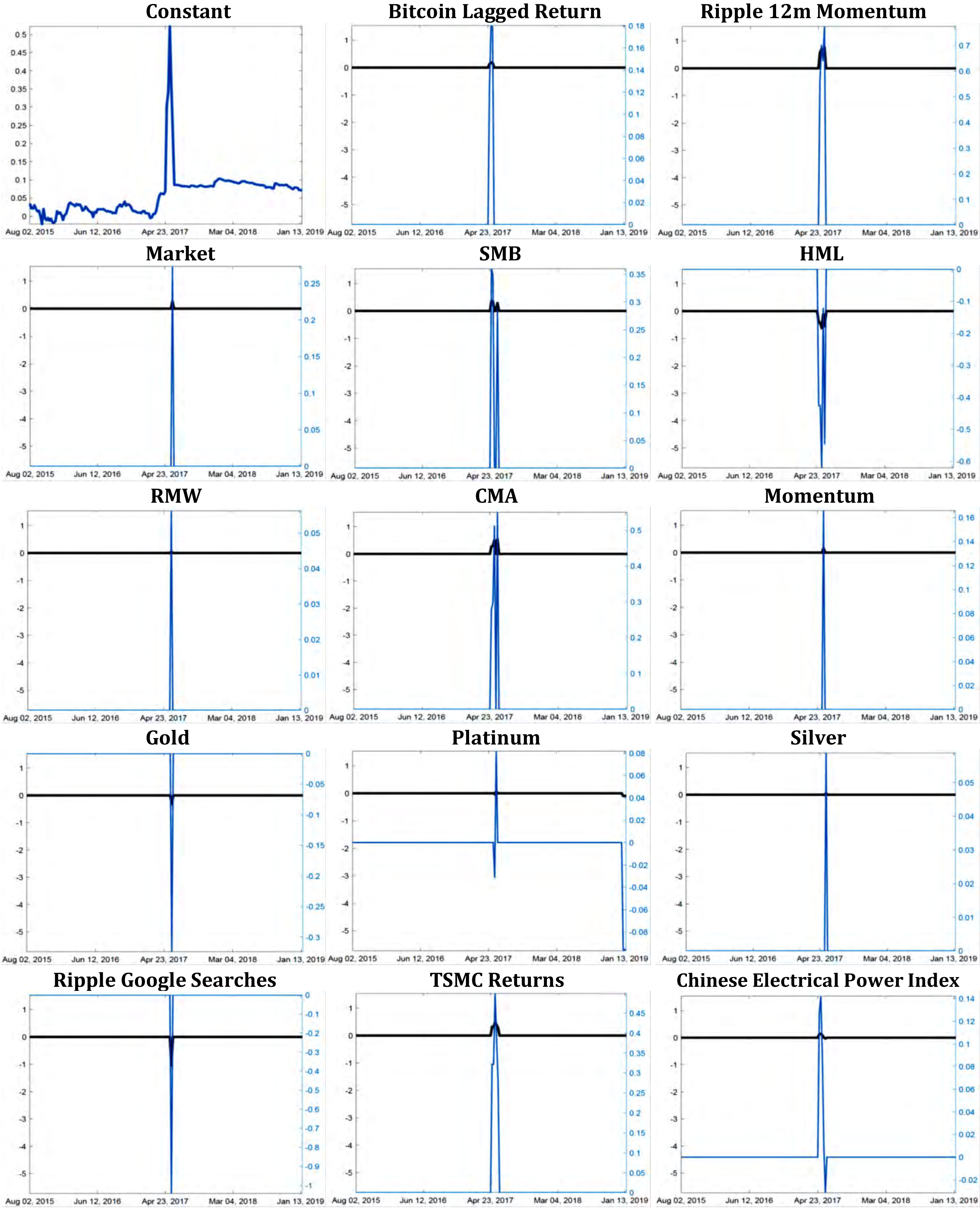

Figure 9

Recursive Dynamic Model Average Coefficient Estimates for Ethereum Returns

The thicker (black) line reports estimates on a fixed scale [-7.4, 0.8] marked on the left axis, the lighter (blue) curve concerns estimates on a variable (right) scale to enhance readability.

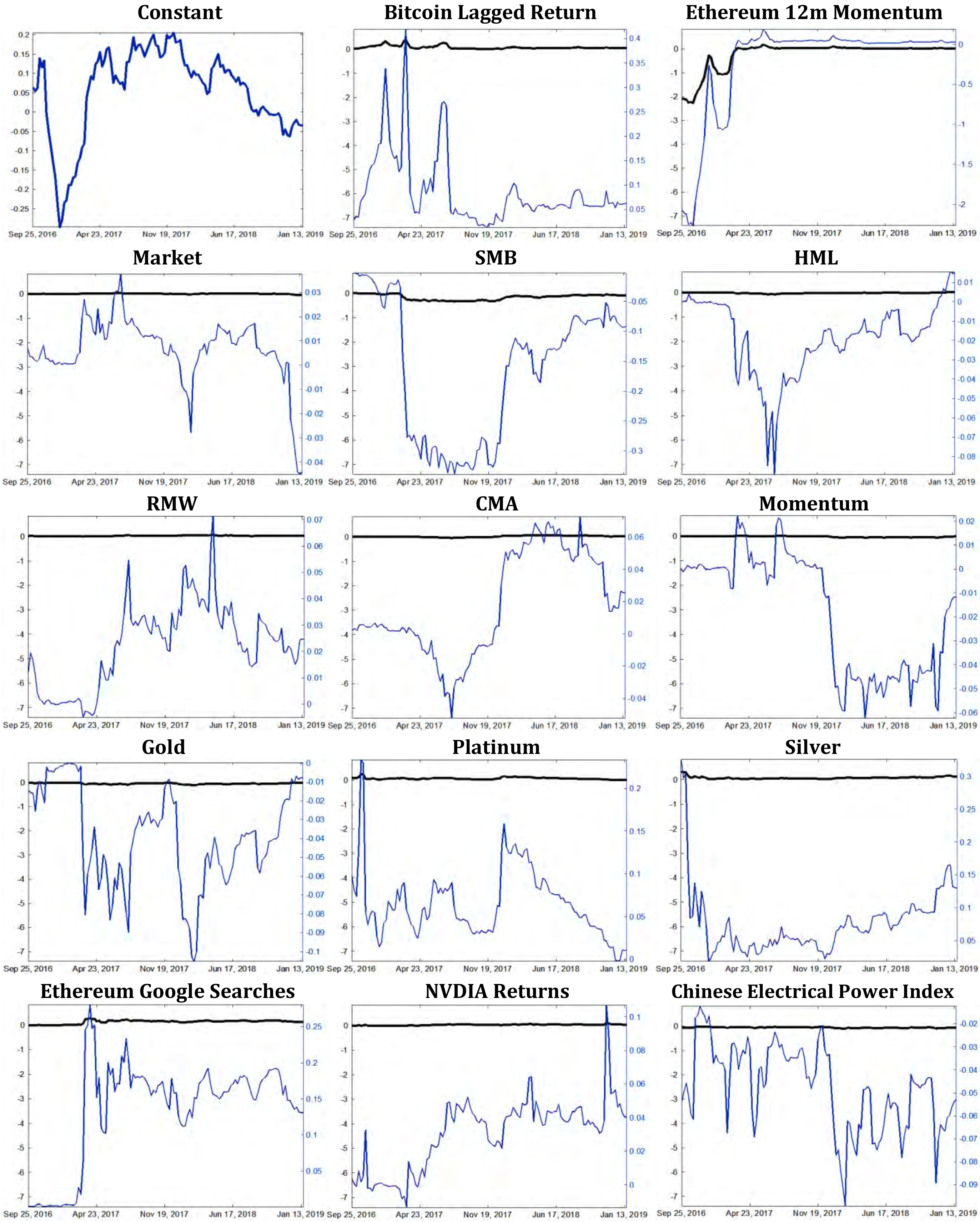

Figure 10

Recursive Dynamic Model Average Probability Estimates for Ethereum Returns

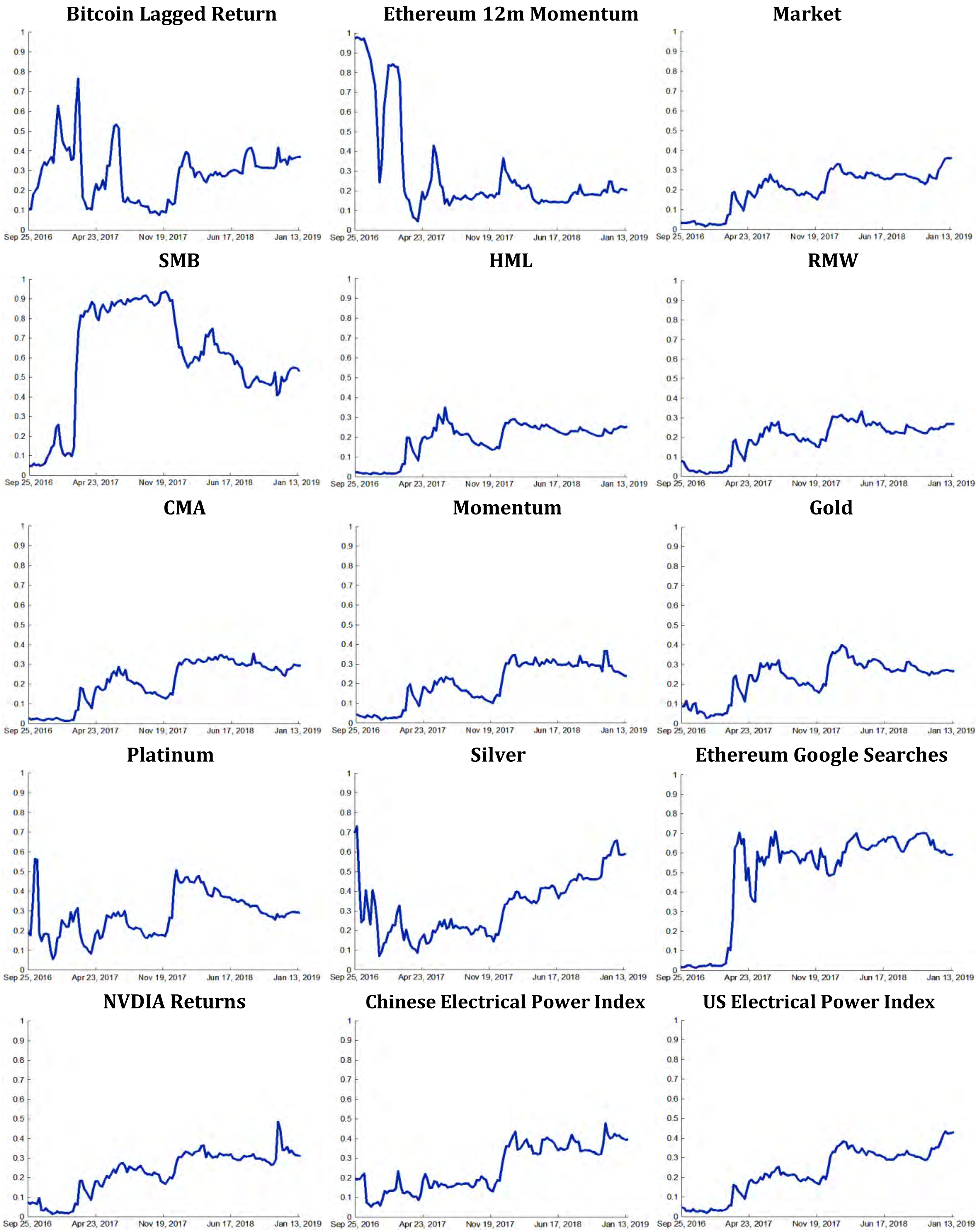

Figure 11

Recursive Dynamic Model Average Coefficient Estimates for the Variance of Residuals

The color-coded shapes at the bottom of the horizontal axis show the sub-samples covered by the Litecoin, Ripple, and Ethereum plots for Bitcoin; by the Ripple and Ethereum plots for Litecoin; by the Ethereum plot in the case of Ripple.

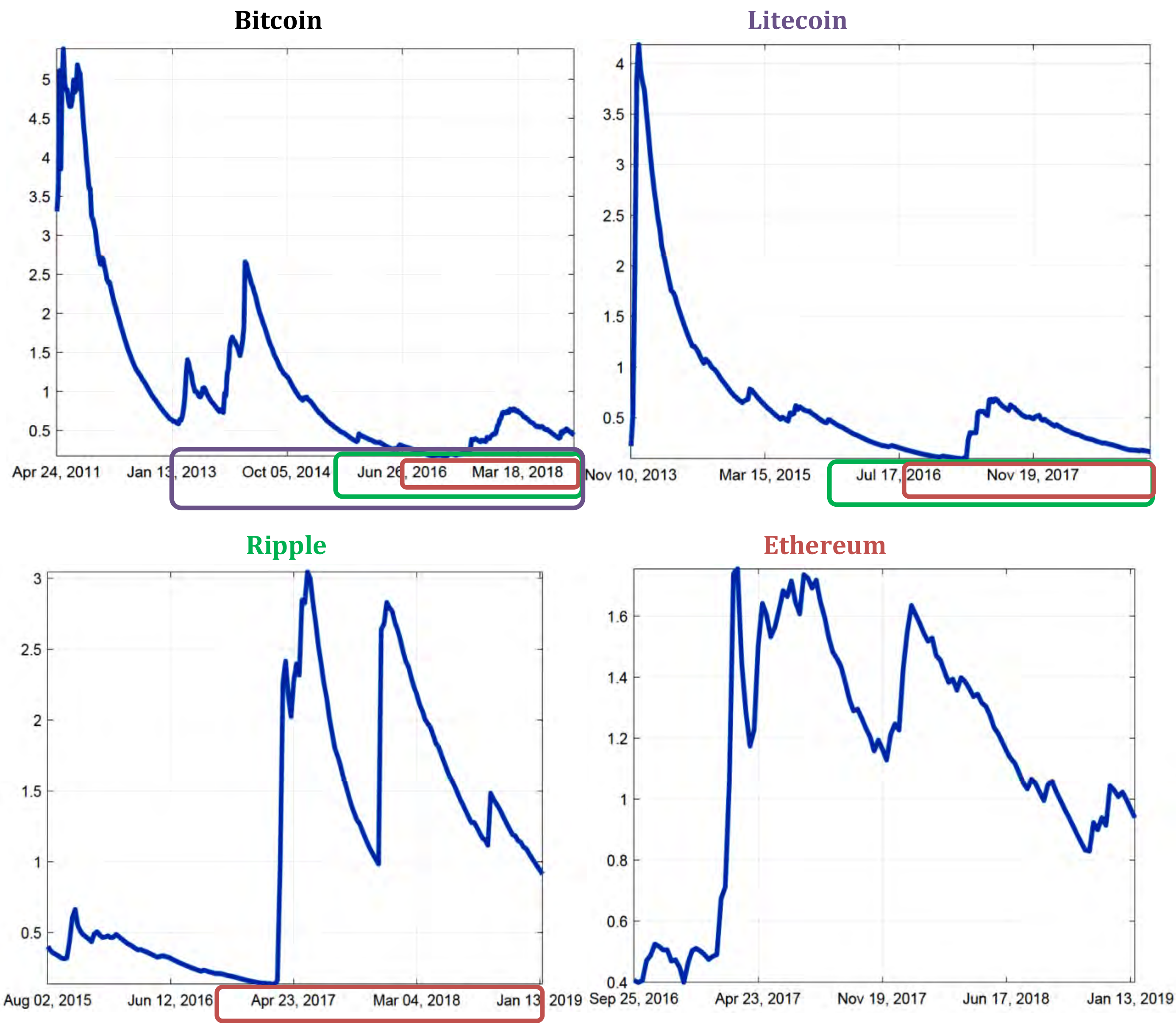

Figure 12

Recursive OLS Coefficient Estimates for Bitcoin Returns

The thicker (black) line reports the estimates on a fixed scale [-2, 2] marked on the left axis, the lighter (blue) curve concerns estimates on a variable (right) scale to enhance readability.

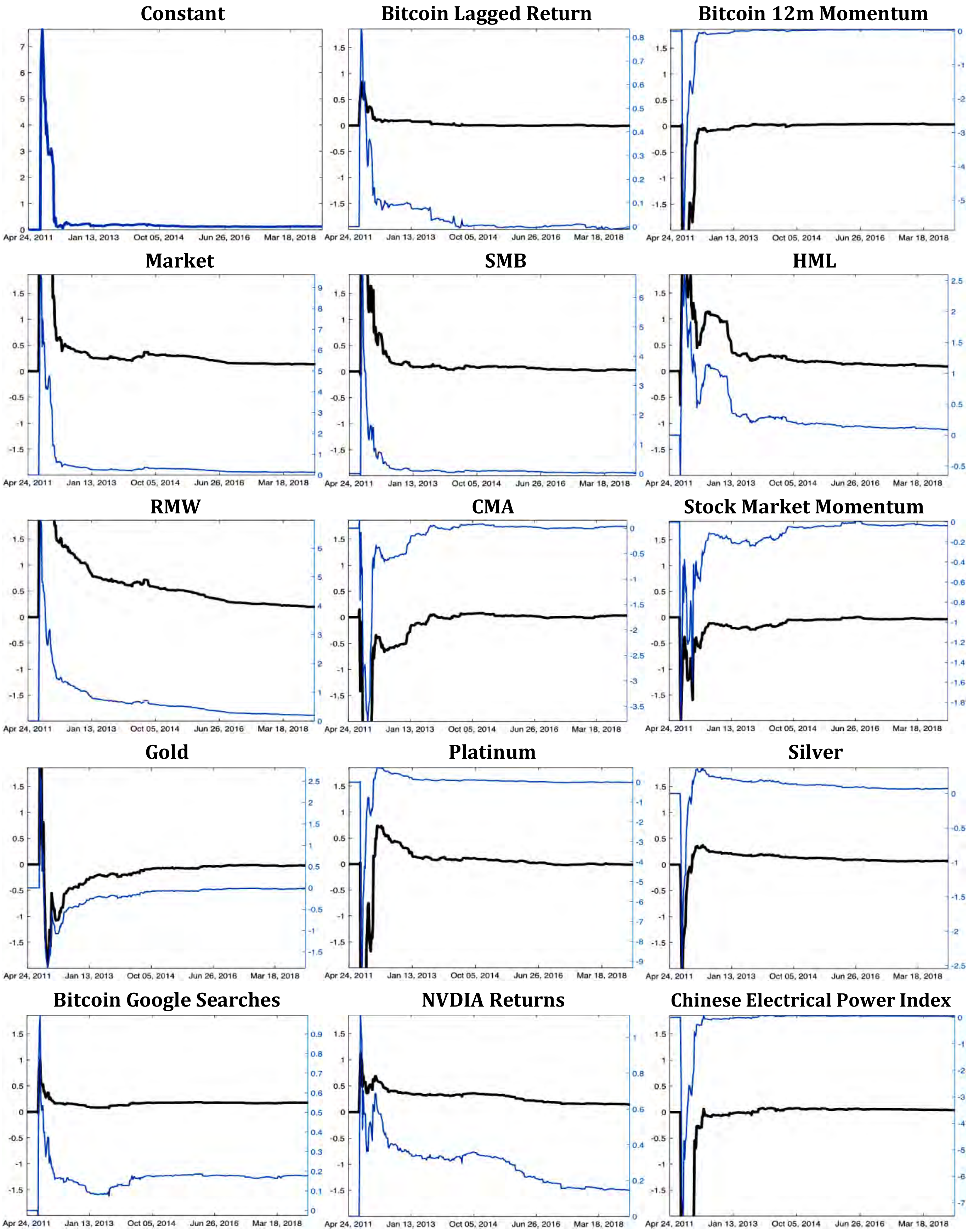

Figure 13

Comparison of Recursive DMA and OLS Coefficient Estimates for Litecoin Returns

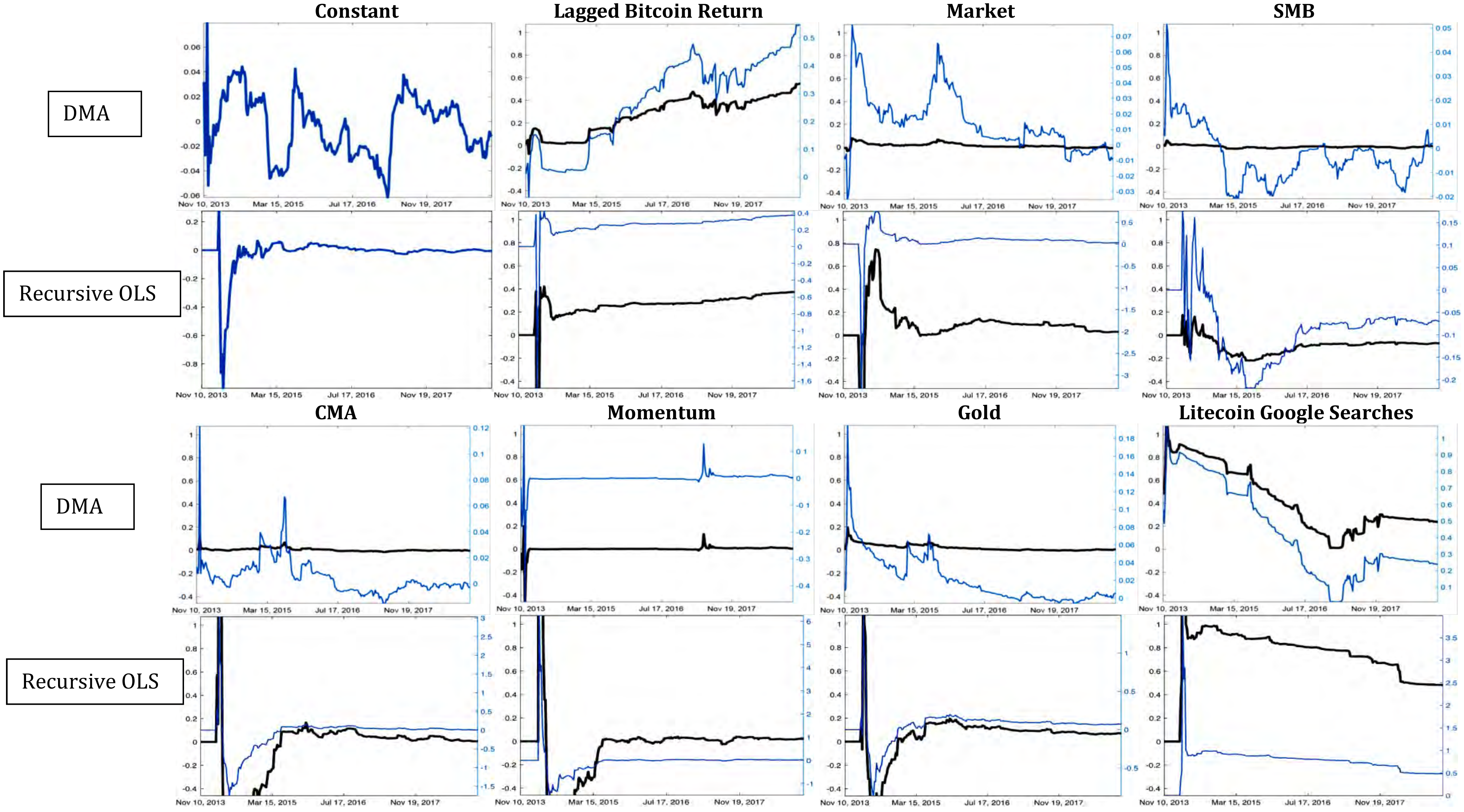

Figure 14

Recursive DMA Coefficient Estimates for US Investment Grade Corporate Bonds

The thicker (black) line reports estimates on a fixed scale [-0.6, 0.5] marked on the left axis, the lighter (blue) curve concerns estimates on a variable (right) scale to enhance readability.

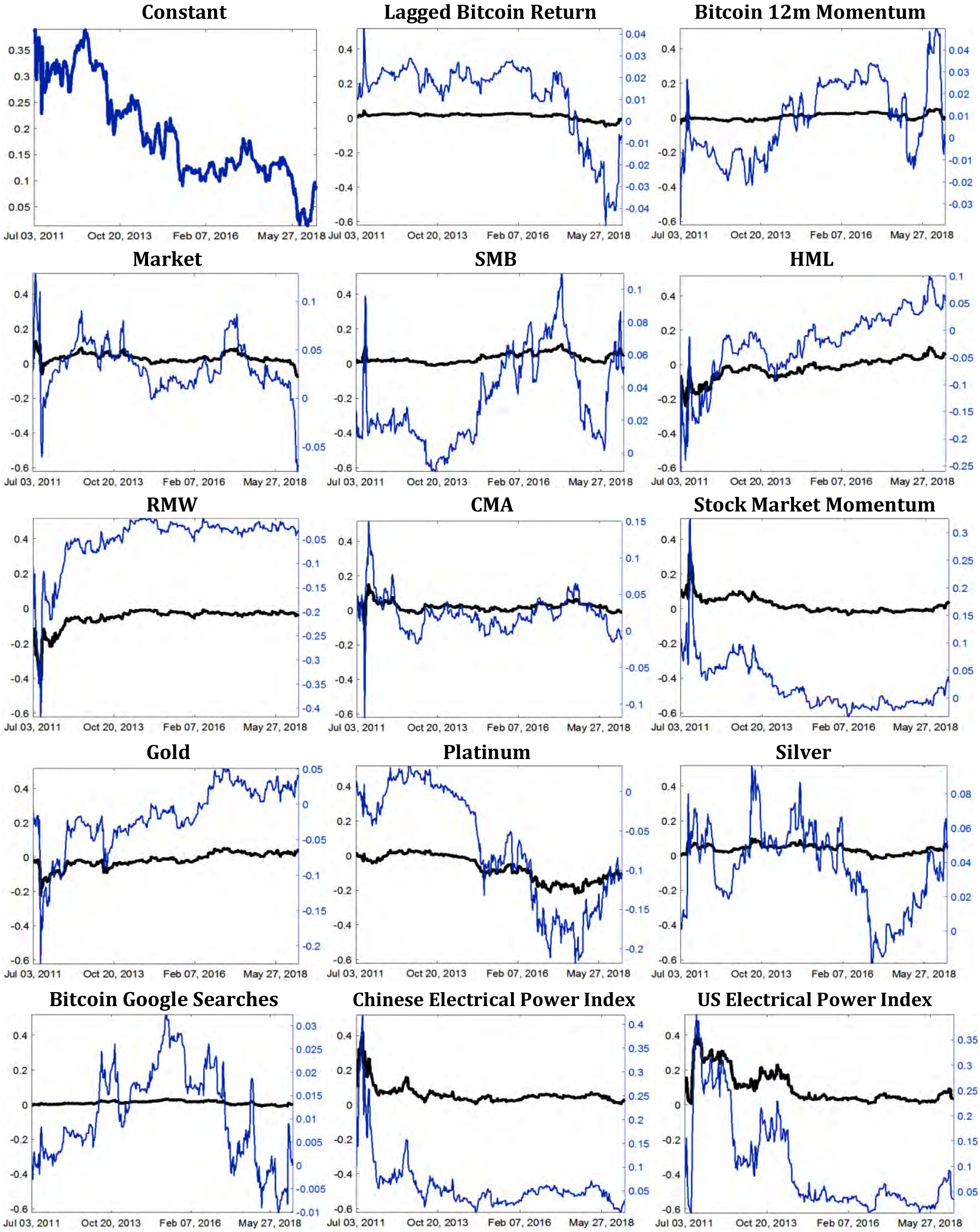

Figure 15

Recursive Dynamic Model Average Probability Estimates for US Investment Grade Corporate Bonds Returns

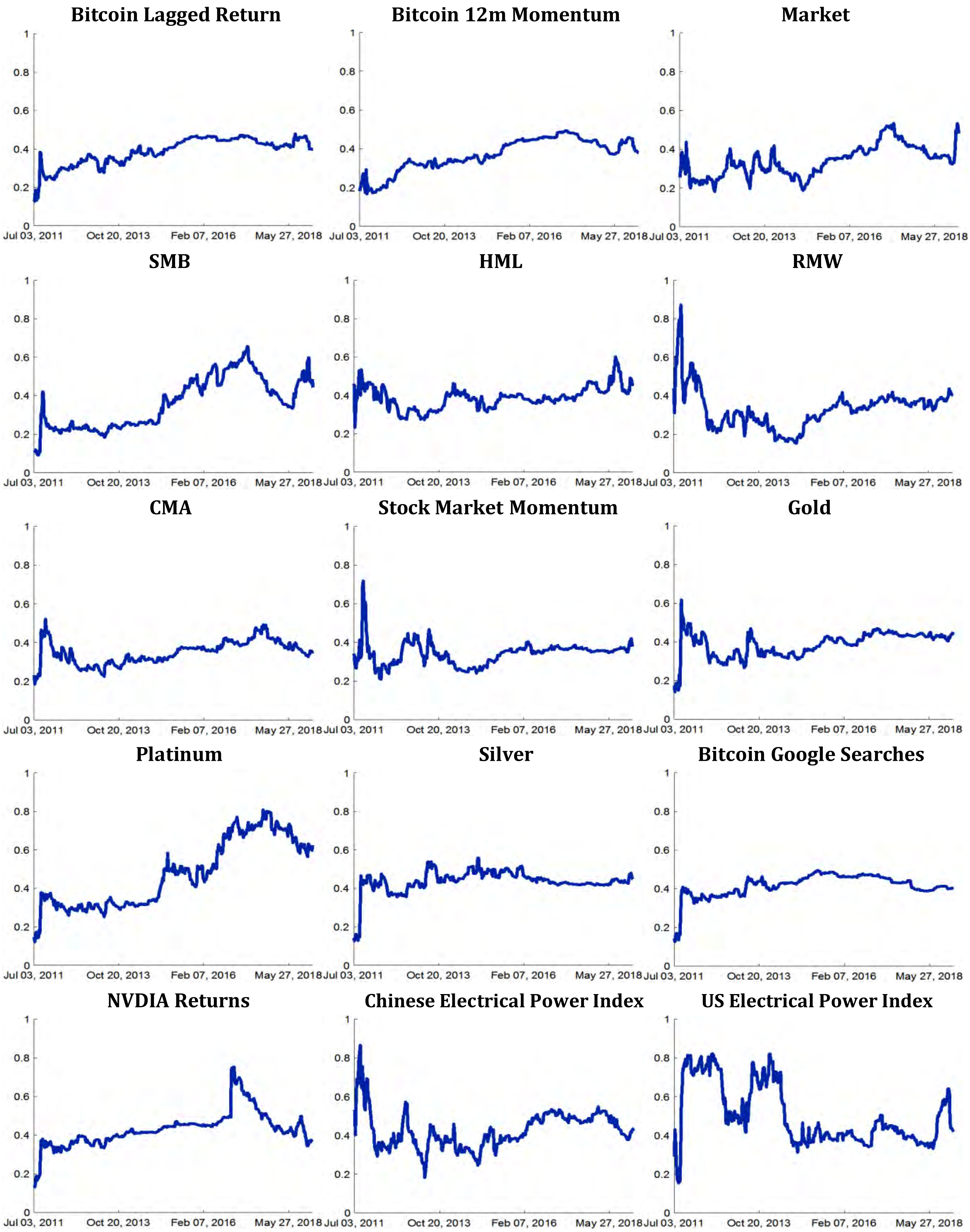

Figure 16

Recursive Dynamic Model Average Estimates for VW Dollar Exchange Rate Returns

The thicker (black) line reports the estimates on a fixed scale [-0.8, 1.2] marked on the left axis, the lighter (blue) curve concerns estimates on a variable (right) scale to enhance readability.