WORKING PAPER SERIES

WORKING PAPER
N. 90
OCTOBER 2018

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Can Investors Benefit from Hedge Fund Strategies? Utility-Based, Out-of-Sample Evidence*

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November 2021

Abstract

We report systematic, out-of-sample evidence on the benefits to an already well-diversified investor that may derive from further diversification into various hedge fund strategies. We investigate dynamic strategic asset allocation decisions that take into account investors' preferences, realistic transaction costs, return predictability, and the parameter uncertainty that such predictability implies. Our results suggest that not all hedge fund strategies benefit a long-term investor who is already well diversified across stocks, government and corporate bonds, and REITs. However, when parameter uncertainty is accounted for, the best performing models offer net positive economic gains to investors with low and moderate risk aversion. Most of the realized economic value fails to result from a mean-variance type enhancements in realized performance but comes instead from an improvement in realized higher-moment properties of optimal portfolios.

Keywords: Strategic asset allocation, hedge fund strategies, predictive regressions, out-of-sample performance, certainty equivalent return.
JEL classification: G11, G17, G12, C53.

* The authors are solely responsible for any errors or omissions. The views expressed herein are those of the authors and do not necessarily reflect the views of the Commission or the authors' colleagues on the staff of the Commission. We thank session participants at the EFA 2018, at the EFMA 2019 conferences, at the 1st Bristol Financial MarketsConference in October21, the University of Liverpool Management School, and in particular Dimitris Papadimitriou and Bakhteat Talukdar (two discussants), as well as participants of the Research Incubator organized by the SEC's Office of Asset Management. The authors are solely responsible for any errors or omissions. Orlov is an employee of the Commodity Futures Trading Commission. The views expressed in this article are made in Orlov's personal capacity and are based upon the author's personal analysis and independent understanding of the subject matter, and do not necessarily represent the views of the CFTC, its Commissioners, or the United States Government. Nor does the article or the personal views expressed herein constitute or imply any CFTC or United States Government endorsement of, or preferential treatment toward, any non-federal entity.

1 Introduction

A number of leading scholars have recently voiced the view that hedge funds would not—and could not—represent a separate, financially relevant asset class on their own.[1] Former hedge fund manager Simon Lack (2012) has pointedly written that "[i]f all the money that's ever been invested in hedge funds had been put in Treasury bills instead, the results would have been twice as good" (p. 1). The academic literature reflects this chasm. Ackermann, McEnally and Ravenscraft (1999), Brown, Goetzmann and Ibbotson (1999), and Liang (1999) showed that in the aggregate, hedge funds (henceforth, HFs) realize positive risk-adjusted performance, which is a condition to generate economic value in a mean-variance framework; however, Griffin and Xu (2009) find little evidence that HFs, on average, deliver abnormal performance. At the individual fund level, Chen and Liang (2007) show that HFs time the equity market and Kosowski, Naik and Teo (2007) show that abnormally high performance of top HFs cannot be explained by pure luck. Yet, Fung, Hsieh, Naik and Ramadorai (2008) find that only a quarter of all funds of HFs produce significantly positive alphas and Dichev and Yu (2011) report that the HF returns are not much higher than the risk-free rate once investor capital flows into and out of funds are taken into account.

In spite of the raging debate, investors kept pouring wealth into the HF industry with renewed vigor after the 2007-2009 Global Financial Crisis, and the assets under management by the overall industry have increased by $524.8 billion in 2020 only and at the end of 2020 the total assets of hedge funds have exceeded $4.1 trillion (BarclayHedge Research, 2021).[2] Are investors just after a mirage? Are they just lured by the record performances allegedly achieved by a few lonely but famed HFs during the 1980s and 1990s, when the industry was nascent and many of the very HF "stars" were still small and riding green pastures, free of strategy over-crowding? To try and tackle these questions, our paper presents comprehensive, out-of-sample evidence on the potential benefits accrued to investors who diversify their portfolios of bonds, stocks, and

[1] For instance, John Cochrane has been quoted by Lim (2013) to have stated: "Hedge funds are not a new asset class. They trade in exactly the same securities you already own."
[2] For comparison, hedge funds assets were less than $40 billion in 1990 (see Agarwal, Mullally and Naik, 2015).

publicly traded real estate to include HF strategies.

HFs are alternative investment vehicles that are subject to limited regulation and thus can take advantage of sophisticated strategies that rely on leverage, short-selling, and derivatives (see, e.g., Agarwal, Mullally and Naik, 2015, and Getmansky, Lee and Lo, 2015, for an introduction and references to seminal papers). Major investors in HFs include foundations, public and private pension funds, university endowments, and funds of HFs, but the ability of relatively small investors to expand their asset menus to include HF strategies has recently been facilitated by the advent of investable HF indices. Therefore whether or not HFs do create economic value (at least) in stylized portfolio choice problems and under fairly realistic assumptions has become a pressing research question, of general interest.

Although HFs tout their sophisticated strategies and promise to deliver superior returns that are largely immune to adverse market developments, it remains important to provide systematic, optimization-based evidence on whether investors can actually reap risk-adjusted benefits from diversifying into this asset class. In fact, a literature exists that has investigated the null hypothesis that HFs could not add significant (often risk-adjusted) economic value. In many respects, the seminal paper is Ackermann et al. (1999) which assessed the portfolio value of HFs using Elton, Gruber and Rentzler's (1987) mean-variance methodology for estimating the contribution of an alternative investment to an existing portfolio. They reported that the correlations between HF returns and eight international stock and bond indices were sufficiently low, and the Sharpe ratio of HF was sufficiently high to augment the overall Sharpe ratio. Similarly, Agarwal and Naik (2000) found that a portfolio comprising of passive asset classes and investing in mainly nondirectional HF strategies provided better a ex-ante risk-expected-return tradeoff than just investing passively in a broad range of asset classes comprising of equities, bonds, currencies, and commodities. These conclusions were discussed by a number of other papers set up in a Markowitz's static mean-variance framework, wherein HFs are usually assigned considerable weight at the expense of bonds, see e.g., Amenc, El Bied and Martellini (2003), Terhaar, Staub and Singer (2003), and recently Mladina (2015). However, there are severe doubts as to whether a standard mean-variance framework and the Sharpe ratio as a leading performance index to rank funds may be suitable to HF strategies. Although adding HFs to an asset menu leads to mean–variance improvements, Amin and Kat (2003a) have shown that including them in a given (not optimized) portfolio may frequently lead to lower skewness and higher kurtosis, which are then impossible to gauge in a two-moment set up. Cremers, Kritzman and Page (2005) have rejected the validity of mean-variance analysis applied to HFs due to the strong and statistically significant non-normalities of HFs and experimented instead with the maximization of the log utility of wealth (which turns out to support the selection of the maximum growth portfolio).[3] Recognizing the significant tail risk that HFs expose to, Agarwal and Naik (2004) have proposed to assess the economic value of HFs in a mean-conditional Value at-Risk (M-CVaR) framework. Recently, Karehnke and de Roon (2020) have provided formal nonparametric tests to analyze the cost of skewness for investors and finnd that about 11% of the hedge funds provide both mean-variance and skewness benefits for stock and bond investors.

In this paper, we also take steps from a need to go past the risk-return characterization of HFs, and we contribute to the literature relative to each of these studies along two or more of the following dimensions: (i) we perform a dynamic, long-horizon portfolio optimization that admits cash outflows (in the stylized form of consumption streams) under constant relative risk aversion preferences that do not only integrate mean and variance in expected utility optimization, but focus instead on the entire predictive density of future outcomes, (ii) following Barberis (2000) we take parameter uncertainty into account in the sense that we compute optimal consumption and portfolio shares under the Bayesian predictive density of excess asset returns; (iii) we account for transaction costs on an ex-ante basis (in fact, these are pegged to make HFs more expensive to trade), i.e., allowing the investors not to trade when the costs incurred would exceed the expected utility gain from trading; (iv) we measure the welfare benefits of HFs as an asset class relying on realized utility differential measures of risk-adjusted performance, and (v) we conduct an out-of-sample (OOS) analysis.

[3] Since the seminal work by Agarwal and Naik (2004), it is well understood that HFs may exhibit non-normal payoffs for reasons such as their use of options, or of option-like dynamic strategies. The payoffs on a large number of equity-oriented hedge strategies resemble those of writing put options.

Importantly, because we embrace a dynamic portfolio approach which estimates hedging demands and features long-horizon investors, in this paper we also take into account the existence (if any) of linear predictability in the returns of the assets in the menu of choice. In this, we follow Bali, Brown and Caglayan (2012, 2014) and Wegener, von Nitzsch and Cengiz (2010) who have stressed that while HFs are not market-neutral as they are exposed to systematic, macroeconomic-type risks, such as the default premium and nominal interest rate shocks that predict performance. In fact, as emphasized by Amenc, El Bied and Martellini (2003), Avramov, Barras and Kosowski (2013), and Avramov, Kosowski, Naik and Teo (2011), HF returns are exposed to a large number of rewarded risk factors and, as such, we should expect them to be predictable because, as argued by Ferson and Harvey (1991), most of the predictability in financial returns can be attributed to predictable shifts in risks and the market-wide reward for risks.[4] For instance, HFs rely heavily on leverage, which might be highly sensitive to business cycle conditions. To this purpose, we use simple but popular vector autoregressive (VAR) models, as in Campbell et al. (2003). The investor maximizes expected power utility defined over a monthly consumption stream over a given investment horizon, *H*. It is important to produce utility-based evidence as even a superior risk-return trade-off of a HF strategy in a static perspective may not improve an investor's risk-adjusted expected performance in the light of the remaining assets in the menu of choice (Amin and Kat, 2003a).[5] With this goal in mind, we conduct a wide range of recursive OOS experiments and assess the realized performance of portfolios using two metrics, the certainty equivalent return (CER) and the Sharpe ratio. The CER, defined as the riskless return that an investor is willing to accept in order to forego a risky portfolio/strategy/asset menu, is the most appropriate measure for ranking alternative models because it is a function not only of the underlying return generating process but also of the

[4] Amenc et al. (2003) find evidence of predictability in HF index returns using the (lagged) yield on 3-month T-bills, the dividend yield, the default spread, the term spread, the US and world equity factors, and changes in a volume-weighted basket of currencies vs. the US dollar. Avramov et al. (2013) examine whether conditional strategies based on simple trading rules can successfully exploit predictability from the default spread, the dividend yield, the VIX index, and the net aggregate flows into the HF industry.

[5] Similarly, Bollen (2013) has suggested that while market-neutral (i.e., zero-R^2) hedge funds are characterized by high Sharpe ratios, they likely expose the investors to substantial downside risk.

investor's preferences. We also report the Sharpe ratio for completeness but note that it may lead to inaccurate rankings due to (spurious) serial correlation in HF returns, which can be attributed to return smoothing and the presence of illiquid securities in HF portfolios (see Getmansky, Lo and Makarov, 2004; Khandani and Lo, 2011).

Our analysis is performed in two steps. In the first step, we compute the optimal portfolio-consumption rules for an investor who diversifies across stocks, government bonds, corporate bonds, and REITs; we refer to this setup as the *baseline asset menu*. For each of three values of the relative risk-aversion coefficient (2, 5, and 10), we entertain a total of 64 VAR models, which correspond to all possible combinations that can be built assuming either one or two autoregressive lags, two different sample selection methods (i.e., rolling vs. expanding windows), and using up to four predictors (i.e., the default and term structure spreads, the 3-month short rate, and the dividend yield) which are widely used in the literature on return predictability. Macroeconomic variables and uncertainty proxies such as these were recently shown to have explanatory power for HF returns (see Avramov et al., 2011; Bali et al., 2014), which is why we use these same predictors (in addition to HF strategy-specific predictors, following Fung and Hsieh, 2004) in our VAR models in the second step of the analysis. Moreover, we account for transaction costs and also compute Bayesian optimal decisions to also compare them with simpler portfolio rules that ignore parameter uncertainty.

The model yielding the highest CER within the baseline asset menu is expanded in a second step to one (out of ten) HF strategy at the time using Hedge Fund Research style indices; we refer to this environment as the *extended asset menu*. In each case, we also re-optimize the structure of the models to include HF strategy-specific predictors. Using the resulting realized OOS CER estimates, we evaluate whether extending the asset menu to include HF strategies is desirable to long-term, risk-averse investors who are already well diversified across a broad spectrum of classical asset classes. This approach also allows us to identify which hedge strategy, if any, provides the highest realized utility gains relative to the optimal baseline portfolio.

The key results of our analysis can be summarized as follows. In both the baseline and extended asset menus, the optimal portfolio weights are somewhat skewed (especially when parameter uncertainty is ignored) towards real estate but appear to be rather stable over time and to seldom require leverage and short-selling even though we admit short strategies. We present evidence that the inclusion of HF strategies leads to a moderate but plausible demand for HFs especially in the Bayesian case, which may lead an aggressive investor to leverage her portfolio by shorting 1-month T-bills and US Treasuries. Moreover, the implied hedging demands for HF strategies tend to be positive because their returns show a mild first-order serial correlation while they have negative coefficients on past lags of the predictors. As a result, long positions in HFs can be used to hedge intertemporal stochastic variations in investment opportunities and even anticipated changes in parameter estimates. If investor were able to detect top performing models for the prediction of risk premia on the different asset classes, most HF strategies and, as a result, also the composite HFR index would outperform a classical asset menu on a risk-adjusted basis, even taking the resulting sample uncertainty and transaction costs into account.

Most of the OOS economic value reported in this paper fails to result from a mean-variance order improvement: in fact, when combined with classical assets, most (all) HF strategies yield realized mean returns (Sharpe ratios) that are approximately equal to the baseline allocation. For instance, for an investor with a constant relative risk aversion coefficient of 5, while traditional assets lead to a mean return of 12.1% per year and to an annualized Sharpe ratio of approximately 0.41, when the HFR composite index is used to expand the asset menu, the realized annual mean return climbs to 12.7% and the Sharpe ratio ticks up to 0.44. Crucially, HF strategies substantially improve the higher-moment properties of the optimal portfolio: kurtosis declines from 6.7 to 3.80 and skewness stays essentially constant at -0.6.

This logic geared at empirically assessing the existence of a three-way trade-off, if any, between Sharpe ratios, certainty equivalent returns, and skewness and kurtosis can be best visualized with the help of the three pictures in Figure 1. The plots represent the annualized Sharpe ratio (on the horizontal axis), the annualized percentage CER (vertical axis), and the Jarque-Bera (henceforth, JB) statistic of non-normalities (a composite of realized skewness and kurtosis, the size of the circles) as perceived by a generically risk-averse investor (not necessarily mean-variance, for instance under power utility preferences) derived from three alternative frameworks.[6] In the leftmost panel, we have the case of a Gaussian IID model with constant investment opportunities, in which skewness and kurtosis are be fixed at their Gaussian levels (zero and three, respectively), so that the ray/diameter of the circles is fixed and by necessity the CER must be monotone increasing in the Sharpe ratio.[7] This means that in the absence of predictability, there cannot be any trade-offs between achieving high Sharpe ratios at the cost of non-normal portfolio returns and pursuing CER maximization. One can read our paper, as an attempt to refute this scenario when HF strategy excess returns belong to the asset menu and a rather pervasive search over potential optimizing models of constant vs. time-varying investment opportunities is performed. The middle picture in Figure 1 concerns an almost opposite case in which as the Sharpe ratio increases, the skewness and kurtosis properties of the optimal portfolio worsen (i.e., skewness declines and kurtosis incrases), the JB statistic worsens so that the CER declines (this can happen intentionally through portfolio manipulation or not, see, e.g., Goetzmann et al., 2007). Therefore, the circles are located along a monotone decreasing functions and their ray/diameter grows as we move towards the right, in correspondence to increasing Sharpe ratios. This case is a distinctive possibility in our empirical tests, i.e., that—as often reported in the literature—HF may represent Sharpe ratio-enhancing alternative assets however ridden of asymmetric and non-linear patterns that can make for low realized OOS CERs. Finally, the third, rightmost plot represents the plausible case in which the empirical properties of the data may prevent us from establishing any specific links between the realized Sharpe ratio, skewness, and kurtosis of the optimal portfolio weights. This means that different set ups (in our application, asset menus) may lead to heterogeneous trade-offs among mean, variances,

[6] The JB statistic Is defined as $JB \equiv (T/6)(\widehat{Skew}^2+0.25(\widehat{Kurt}-3)^2)$ where T is the number of observations, $\widehat{Skew}$ is the sample skewness, and $\widehat{kurt}$ is the sample kurtosis. As shown by Schuhmacher, Kohrs, and Auer (2021), the existence of skewness in returns does not allow a rejection of mean-variance analysis because (in the presence of a risk-free asset), the return distribution of every portfolio is determined by its mean and variance if and only if asset returns follow a generalized location-scale skew-elliptical distribution which is clearly not normal. In this case, our notion of non-normality will need to be re-interpreted as deviation from such special elliptical benchmark and JB statistic corrected to account for some degree of "allowed" skewess.

[7] To our argument, the shape of such a relantionship is irrelevant and in no way the link should be linear and described by a 45-degree line. In fact, note that the scales on the axes are quite different. The plot implies that under Gaussian IID investment opportunities, the investor turns into a Sharpe ratio optimizer.

skewness, kurtosis, and certainty equivalent returns.

Under the more realistic assumption that an investor could not know in advance what the best performing model (in terms of realized CER) would have to be ex-post, so that she had to pick at random a median model, the investor would have not fared always so well unless she had known—unrealistically—which specific hedge strategy to pick. Indeed, betting on the composite HFR index or on a fund-of-funds strategy leads to median realized CERs that are negative and therefore dominated by the simplest of the portfolio strategies: 100% in cash at all times. While the realized CERs of the median predictability model are promising for a few strategies because they exceed the performance of the median model applied to a traditional asset menu, other strategies lead to a non-positive CER. Depriving investors from the possibility to fine tune the predictability model hurts in particular the strategies that trade equities.

Our paper draws primarily on two strands of the literature. The first strand attempts to explain HF returns using style analysis, multifactor, and nonlinear models (see, e.g., Fung and Hsieh, 2002a, 2004; Hamza, Kooli and Roberge, 2006; Bali et al., 2012, 2014). A second strand of literature focuses on the performance evaluation and optimal portfolio decisions involving hedge strategies (see, e.g., Agarwal and Naik, 2004, Mladina, 2015, Panopoulou and Vrontos, 2015). Finally, there is extensive research on the underlying biases in the data on HF returns and the perils these would pose to a meaningful assessment of the risk-adjusted benefits (see, e.g. Agarwal, Fos and Jiang, 2013, Aiken, Clifford and Ellis, 2013). Our specific contribution is that we pursue a dynamic, optimizing consumption-portfolio approach that recognizes the existence of predictability in HF returns as well as in all other asset classes typically available to an investor. In doing so, we echo the recommendation by Amin and Kat (2003a) to distinguish between an analysis of "(…) whether in terms of risk and return hedge funds offer investors value for money." and an integrated portfolio view as "It is important to note from the outset, however, that strictly speaking this is a different question than whether hedge funds should be included in an investment portfolio. The fact that an investment offers a superior risk-return profile does not automatically mean investors should buy into it as it may not fit their preferences and/or fit in with other available alternatives." (p. 253).[8]

Bali, Brown, and Demirtas (2013) have analyzed whether HFs provide a signficant economic improvement over standard portfolios of stocks and bonds, also as a result of HFs' extensive use of derivatives, short selling, and leverage to yield dynamic trading strategies that create significant nonnormalities in realized portfolio returns. Because traditional performance measures fail to provide an accurate characterization of the relative strength of portfolios including HFs, they also employ utility-based nonparametric and parametric performance measures, such as (almost) stochastic dominance and manipulation-proof indices. Differently from Brown et al.'s work, we adopt a more tightly parameterized expected power, CRRA set up that however allows us to perform systematic OOS recursive experiments that take into explicit account predictability, transaction costs (ex-ante, hence creating a no-trade region), and parameter uncertainty, through the recursive calculation of Bayesian portfolio rules.[9,10]

The next section describes the research design that allows us to exploit the predictability in asset returns, determine optimal consumption-portfolio rules, and measure OOS performance. Section 3 describes the data on the baseline assets, HF indices, and predictor variables. Section 4 systematically selects the best-performing model from within the baseline asset menu. Section 5 computes optimal allocations with HF strategies and studies which strategies, if any, can improve the realized utility of the investor. Section 6 concludes.

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[8] Amin and Kat (2003a) find that the majority of individual HFs as well as HF indices cannot in isolation produce efficient payoffs, but that they are able to do so when combined in a portfolio with the S&P 500 index, which suggests that a relatively well-developed portfolio approach to the problem is advisable.

[9] Hoevenaars, Molenaar, Schotman and Steenkamp (2008) have also reported that HFs have a substantial impact on the portfolio rules. However, we perform an OOS recursive back-test and employ a utility-based metric (CER) to compare the benefits of diversifying into the HF strategies.

[10] There is also one small literature that has investigated the effects of the (sizeable, between 3 and 4% on average, see Ibbotson, Chen, and Zhu, 2011; Jurek and Stafford, 2015) fees charged by HFs on typical inferences on their value to portfolio diversification. In fact, in our design, we have used HF returns net-of-fees and transaction costs along with returns on other, more classical asset classes that are treated in more heterogeneous ways: for instance, stock and bond returns are gross-of-fees, while REIT returns are net of management fees and costs internalized by the trusts. Yet, in large portions of our analysis we further impose transaction costs to trade HFs, which represents a double counting of fees and trading costs that is aimed at taking their illiquidity into account.

2 Research design

We compute the optimal portfolio-consumption rules using the dynamic programming discrete-time solution methods applied (among many other papers) in Barberis (2000). At least in the frequentist portion of our empirical work, we perform recursive, realized OOS evaluations while adjusting for small-sample bias following Engsted and Pedersen (2012). The investor is assumed to have a five-year investment horizon. The return generating process has either a VAR or Gaussian IID structure, and model estimation and optimization take place within rolling and expanding window schemes. Moreover, we entertain both the case in which our "representative" investor behaves like a frequentist who computes the predictive density of future asset returns by simply plugging estimated parameter values in the assumed data generating process as well as the Bayesian set up in which the investor incorporates estimation uncertainty by taking integrating the posterior density of the unknown parameters out to be obtain a predictive density for asset returns that takes parameter uncertainty into account. This section gives a summary of these methodologies.

2.1 *Predictability of asset returns*

2.1.1 Vector autoregressive models and bias correction in the frequentist case

The dynamics of investment opportunities are described by a range of reduced-form, pth-order VAR(p) processes. All variables, including the predictors, are modeled as endogenous. A vector of state variables $\boldsymbol{z}_{t+1}$ is defined as

$$\boldsymbol{z}_{t+1} \equiv \begin{bmatrix} r_{1,t+1} \\ \boldsymbol{x}_{t+1} \\ \boldsymbol{y}_{t+1} \end{bmatrix}, \tag{1}$$

where $r_{1,t+1}$ is the log return on a benchmark short-term security, $\boldsymbol{x}_{t+1}$ is an (n - 1) vector of log excess returns on the risky asset, and $\boldsymbol{y}_{t+1}$ is an m vector of predictor variables. The stochastic evolution of $\boldsymbol{z}_{t+1}$ in a VAR(1) model is given by[11]

$$\boldsymbol{z}_{t+1} = \boldsymbol{\Phi}_0 + \boldsymbol{\Phi}_1 \boldsymbol{z}_t + \boldsymbol{v}_{t+1}, \tag{2}$$

[11] All higher-order VAR can be re-written as a VAR(1) by way of a companion form representation (see e.g., Hamilton, 1994, p. 259). When applied to the companion form, formulas (2) – (4) yield the corresponding, relevant quantities (e.g., moments) for a generic VAR(p).

where $\boldsymbol{\Phi}_0$ is the $(n+m)$ vector of intercepts, $\boldsymbol{\Phi}_1$ is the $(n+m)$x$(n+m)$ coefficients matrix, and $\boldsymbol{v}_{t+1}$ is a vector of Gaussian white noise processes distributed as

$$\boldsymbol{v}_{t+1} \overset{i.i.d.}{\sim} \mathcal{N}(\mathbf{0}, \textstyle\sum_v), \qquad \textstyle\sum_v \equiv Var_t(\boldsymbol{v}_{t+1}) = \begin{bmatrix} \sigma_1^2 & \boldsymbol{\sigma}'_{1x} & \boldsymbol{\sigma}'_{1y} \\ \boldsymbol{\sigma}_{1x} & \boldsymbol{\Sigma}_{xx} & \boldsymbol{\Sigma}'_{xy} \\ \boldsymbol{\sigma}_{1y} & \boldsymbol{\Sigma}_{xy} & \boldsymbol{\Sigma}_{yy} \end{bmatrix} \tag{3}$$

The shocks are zero mean, homoskedastic normal variables, which are contemporaneously correlated but IID over time. Normality is therefore induced in the unconditional distribution of $\boldsymbol{z}_t$, where the mean $\boldsymbol{\mu_z}$ and the covariance matrix $\boldsymbol{\Sigma_{zz}}$ are described by

$$\boldsymbol{\mu_z} = (\boldsymbol{I}_{(n+m)} - \boldsymbol{\Phi}_1)^{-1}\boldsymbol{\Phi}_0, \qquad vec(\textstyle\sum_{\boldsymbol{z}}) = (\boldsymbol{I}_{(n+m)^2} - \boldsymbol{\Phi}_1 \otimes \boldsymbol{\Phi}_1)^{-1} vec(\boldsymbol{\Sigma}_v). \tag{4}$$

In the frequentist case, and differently from much earlier researcher, we take into account the instability of the VAR parameters and adjust the estimates for small-sample bias as in Engsted and Pedersen (2012). Using Pope's (1990) formula, Engsted and Pedersen quantify the bias in the standard, OLS estimate $\widehat{\boldsymbol{\Phi}}_1$ of the slope parameters of the VAR in (2) as

$$\boldsymbol{Bias}_T = -\frac{\boldsymbol{b}}{T} + O\left(T^{-\frac{3}{2}}\right), \tag{5}$$

where T is the number of observations used in estimation and

$$\boldsymbol{b} = \boldsymbol{\Sigma}_v \left[\left(\boldsymbol{I}_{(n+m)} - \boldsymbol{\Phi}'_1\right)^{-1} + \boldsymbol{\Phi}'_1\left(\boldsymbol{I}_{(n+m)} - (\boldsymbol{\Phi}'_1)^2\right)^{-1} + \sum_{i=1}^{n+m} \lambda_i \left(\boldsymbol{I}_{(n+m)} - \lambda_i \boldsymbol{\Phi}'_1\right)^{-1} \right] \boldsymbol{\Sigma}_z^{-1}, \tag{6}$$

$\boldsymbol{\Sigma}_v$ and $\boldsymbol{\Sigma}_{\boldsymbol{z}}$ are defined in (3) and (4), respectively, and λ_i is the ith eigenvalue of $\boldsymbol{\Phi}_1$.[12]

Starting from the OLS $\widehat{\boldsymbol{\Phi}}_1$, the bias-correction procedure is implemented in four steps. First, as long as there are no unit roots in $\widehat{\boldsymbol{\Phi}}_1$, we compute the bias $\boldsymbol{B}_T$ by substituting $\widehat{\boldsymbol{\Phi}}_1$ for $\boldsymbol{\Phi}_1$ in (6). Second, we subtract the result from the OLS estimate to arrive at the bias-corrected $\widetilde{\boldsymbol{\Phi}}_1$. Third, we check whether the latter contains unit roots and, if so, find the maximum value $\kappa \in [0, 0.01, 0.02, \ldots, 0.99]$ that multiplies $\boldsymbol{B}_T$ such that the bias-corrected $\widetilde{\boldsymbol{\Phi}}_1$ lies again in the stationarity region (see Kilian, 1998); otherwise, we set $\kappa = 1$. Finally, we calculate the bias-adjusted estimate of the intercept $\boldsymbol{\Phi}_0$ by imposing that the unconditional mean vector of $\boldsymbol{z}_t$ coincides with its full-

[12] The rate of convergence of the approximation error in (5) is equal to $T^{-3/2}$ and is comparable to that of either a bootstrap or Monte Carlo bias-adjustment simulation.

sample mean:[13]

$$\widetilde{\boldsymbol{\Phi}}_0 = \left(\boldsymbol{I}_{(n+m)} - \widetilde{\boldsymbol{\Phi}}_1\right) \cdot \widehat{\boldsymbol{\mu}}_{\boldsymbol{z}}. \tag{7}$$

To establish a benchmark against which to assess the VAR-based results, we also computed optimal portfolios on the basis of a Gaussian IID model for excess returns in which returns evolve according to

$$\boldsymbol{x}_{t+1} = \boldsymbol{\Phi}_0 + \boldsymbol{v}_{t+1}, \qquad \boldsymbol{v}_{t+1} = \overset{i.i.d.}{\sim} \mathcal{N}(\boldsymbol{0}, \textstyle\sum_{\boldsymbol{v}}). \tag{5}$$

This model implies no predictability and, equivalently, constant investment opportunities. Under this dynamics for excess returns, the investor will choose the same portfolio allocation regardless of the investment horizon. Bias correction has no effect in this framework as both the bias-corrected and the unadjusted estimates of $\boldsymbol{\Phi}_0$ coincide.

2.1.2 Bayesian posterior estimation

In a Bayesian set up, the investor shall treat the data as fixed and given and the parameters (say, $\boldsymbol{\Phi}_0$, $\boldsymbol{\Phi}_1$, and $\sum_{\boldsymbol{v}}$) as random. Her goal then becomes to estimate the posterior, joint density of the parameters of interest, $p(\boldsymbol{\Phi}_0, \boldsymbol{\Phi}_1, \sum_{\boldsymbol{v}} | \boldsymbol{z}_2, \boldsymbol{z}_2, \ldots, \boldsymbol{z}_T)$ (the first observation is used to condition the analysis because of the Markov nature of the model). To this purpose, it is convenient to re-write the model as

$$\boldsymbol{Z} = \boldsymbol{Y}\boldsymbol{C} + \boldsymbol{V}, \tag{6}$$

where $\boldsymbol{Z}$ is a (T-1)x(n+m) matrix with $\boldsymbol{z}_2'$, $\boldsymbol{z}_3'$, …, $\boldsymbol{z}_T'$ as rows, $\boldsymbol{Y}$ is a (T-1)x(n+m+1) matrix with [1 $\boldsymbol{z}_2'$], [1 $\boldsymbol{z}_3'$], …, [1 $\boldsymbol{z}_T'$] as rows, $\boldsymbol{C}$ is a (n+m+1)x(n+m) matrix of coefficients with $\boldsymbol{\Phi}_0'$ in its top row and $\boldsymbol{\Phi}_1'$ below it, and $\boldsymbol{V}$ is a (T-1)x(n+m) matrix with $\boldsymbol{v}_2'$, $\boldsymbol{v}_3'$, …, $\boldsymbol{v}_T'$ as rows. Using a classical, uninformative priors approach, we assume inverse Gamma-Wishart priors

$$p(\boldsymbol{C}, \boldsymbol{\Sigma}_{\boldsymbol{v}}) \propto |\boldsymbol{\Sigma}_{\boldsymbol{v}}|^{-(n+m+1)/2}, \tag{7}$$

so that simple calculations show that the posterior $p(\boldsymbol{C}, \boldsymbol{\Sigma}_{\boldsymbol{v}}^{-1} | \boldsymbol{z}_2, \boldsymbol{z}_2, \ldots, \boldsymbol{z}_T)$ is proportional (up to a constant of integration) to $p(vec(\boldsymbol{C}) | \boldsymbol{\Sigma}_{\boldsymbol{v}} \boldsymbol{z}_2, \boldsymbol{z}_2, \ldots, \boldsymbol{z}_T)\ p(\boldsymbol{\Sigma}_{\boldsymbol{v}}^{-1} | \boldsymbol{z}_2, \boldsymbol{z}_2, \ldots, \boldsymbol{z}_T)$ where

$$\boldsymbol{\Sigma}_{\boldsymbol{v}}^{-1} | \boldsymbol{z}_2, \boldsymbol{z}_2, \ldots, \boldsymbol{z}_T \sim Wishart(T - n - m - 1, (\widehat{\boldsymbol{V}}'\widehat{\boldsymbol{V}})^{-1}) \tag{8}$$

[13] Through a simulation study, Engsted and Pedersen (2012) find that the bias-correction procedure outlined above leads to an improvement upon the initial OLS estimates in terms of mean square error, variance and bias, and that the improvement is more significant as the samples are smaller.

$$vec(\boldsymbol{C})|\boldsymbol{\Sigma}_v, \boldsymbol{z}_2, \boldsymbol{z}_2, \ldots, \boldsymbol{z}_T \sim N(vec(\widehat{\boldsymbol{C}}), \boldsymbol{\Sigma}_v \otimes (\boldsymbol{Y}'\boldsymbol{Y})^{-1}), \tag{9}$$

and $\widehat{\boldsymbol{V}} \equiv \boldsymbol{Z} - \boldsymbol{Y}\widehat{\boldsymbol{C}}$ is the matrix of residuals.

2.2 *Portfolio selection*

2.2.1 Preferences and optimal portfolio-consumption choice

As in Barberis (2000), the investor can allocate her savings among *n* securities, with the resulting gross portfolio returns given by

$$R_{p,t+1} = exp\left(\sum_{i=2}^{n} \alpha_{i,t}\left(R_{i,t+1} - R_{1,t+1}\right) + r_{1,t+1}\right), \tag{10}$$

where $R_{\mathrm{i,t+1}}$ is the gross return on the risky asset *i* which has been assigned a weight $\alpha_{i,t}$, and $R_{1,t+1}$ is the gross return on a benchmark, short-term security.[14] In line with the portfolio choice literature (see Brandt, 2009), we assume that the investor maximizes time-separable, CRRA power utility preferences, here written in recursive form,

$$U\left(C_t, E_t(U_{t+1})\right) = \left[(1-\delta)C_t^{1-\gamma} + \delta\left(E_t\left(U_{t+1}^{1-\gamma}\right)\right)\right]^{\frac{1}{1-\gamma}}, \tag{11}$$

where δ is the subjective discount factor and $\gamma > 0$ is the coefficient of constant relative risk aversion. γ also determines the investor's consumption substitution patterns across time as (11) implies that the constant elasticity of intertemporal substitution is simply $\psi = \gamma^{-1}$.[15] It is well-known (see Ang, 2014) that power utility makes an investor's expected utility dependent on features of the entire distribution of the realized consumption/wealth process, including moments of order higher than mean and variance. This resonates well with the great emphasis that has been placed on the (allegedly, poor) skewness, kurtosis, and left tail risk properties of HF strategy returns (see, e.g., Amin and Kat, 2003b; Agarwal, Ruenzi and Weigert, 2017).

On each period *t*, the investor allocates her savings $(W_t - C_t)$ across the assets in her menu, thus

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[14] We express all returns in nominal terms because of a relatively low and constant realized inflation in our sample ought to have been discounted in assets prices and it is unlikely to affect the key findings.

[15] It is well known that while γ mainly drives optimal portfolio allocation, ψ determines the optimal consumption-savings ratio. Because here we just focus on realized portfolio performances, preliminary experiments on the benchmark asset menu revealed that ψ indeed exercised a rather modest effect on optimal recursive weights so that setting $\psi = \gamma^{-1}$ implies a relatively minor simplification.

facing the intertemporal capital accumulation (budget) constraint:

$$W_{t+1} = (W_t - C_t)R_{p,t+1} = W_t(1-c_t)R_{p,t+1}, \tag{12}$$

where $c_t \equiv C_t/W_t$ is the optimal consumption-wealth ratio. Moreover, we assume that the investor rebalances her portfolio at regular intervals defined by K - 1 points ($\tau_1, \tau_2, \ldots, \tau_{K-1}$) solves a dynamics programming problem. Of course, when K = 1, there is no rebalancing and the portfolio problem simplifies to a buy-and-hold one. As in Barberis (2000), we employ standard state-space discretization techniques and then solve the problem iteratively backwards (see also the Appendix to Guidolin and Timmermann, 2008, also with reference to the details of the discretization grid) starting from $\tau_{K-1} = t + [(K - 1/K)]H$, where H is the investment horizon, going back to $\tau_0 = t$. The control variables of the problem are the portfolio weigths ($\alpha_0, \alpha_1, \ldots, \alpha_{K-1}$) and the consumption share share selections, $c_0, c_1, \ldots, c_{K-1}$. Needless to say, when $K = 1$, the dynamic program turns into a buy-and-hold portfolio problem. Therefore the initial, time t portfolio problem is:

$$\max_{\substack{\alpha_0,\alpha_1,\ldots,\alpha_{K-1} \\ c_0,c_1,\ldots,c_{K-1}}} E_t\left[\sum_{j=0}^{K-1}(1-\delta)^{\tau_j - t}\frac{C_{\tau_j}^{1-\gamma}}{1-\gamma}\right], \tag{13}$$

subject to (15) and the dynamic wealth constraint that for $k \geq 1$

$$W_{\tau_k} = W_{\tau_{k-1}} exp\left(\sum_{i=2}^{n}\alpha_{i,\tau_{k-1}}\left(R_{i,\tau_k} - R_{1,\tau_k}\right) + R_{1,\tau_k}\right), \tag{14}$$

in which wealth acts as the endogenous state variable.[16] However, after defining the derived utility of wealth function as

$$V(W_{\tau_k}, \boldsymbol{z}_{\tau_k}, \tau_k) \equiv \max_{\substack{\alpha_k,\alpha_{k+1},\ldots,\alpha_{K-1} \\ c_k,c_{k+1},\ldots,c_{K-1}}} E_{\tau_k}\left[\sum_{j=k}^{K-1}(1-\delta)^{\tau_j - t}\frac{C_{\tau_j}^{1-\gamma}}{1-\gamma}\right], \tag{15}$$

we exploit the Bellman's principle by which $V\left(W_{\tau_k}, \boldsymbol{z}_{\tau_k}, \tau_k\right) = \max_{\alpha_k, c_k} E_{\tau_k}\left[V\left(W_{\tau_{k+1}}, \boldsymbol{z}_{\tau_{k+1}}, \tau_{k+1}\right)\right]$ and the fact (to be proven by induction) that under CRRA preferences the value function factors in a

[16] In a Gaussian IID framework, this first-order conditions of the programme (15)-(16) imply constant consumption shares and that the optimal portfolio rule is fully myopic with constant portfolio weights over time.

particularly convenient way, as

$$V(W_{\tau_k}, \boldsymbol{z}_{\tau_k}, \tau_k) = \begin{cases} \dfrac{W_{\tau_k}^{1-\gamma}}{1-\gamma} Q(\boldsymbol{z}_{\tau_k}, \tau_k) & if\ \gamma > 0, \gamma \neq 1 \\ \\ lnW_{\tau_k} Q(\boldsymbol{z}_{\tau_k}, \tau_k) & if\ \gamma = 1 \end{cases}, \tag{16}$$

so that the Bellman's equation is simply

$$Q(\boldsymbol{z}_{\tau_k}, \tau_k) \equiv \max_{\alpha_k, c_k} E_{\tau_k} \left[c_{k+1} exp\left(\sum_{i=2}^{n} \alpha_{i,k}(R_{i,\tau_{k+1}} - R_{1,\tau_{k+1}}) + R_{1,\tau_{k+1}} \right) \right]^{1-\gamma} + E_{\tau_k}[Q(\boldsymbol{z}_{\tau_{k+1}}, \tau_{k+1})], \tag{17}$$

taken with respect to $\boldsymbol{z}_{\tau_{k+1}} \sim N(\widetilde{\boldsymbol{\Phi}}_0 + \widetilde{\boldsymbol{\Phi}}_1 \boldsymbol{z}_{\tau_k}, \widetilde{\boldsymbol{\Sigma}}_{\boldsymbol{v}})$ in the frequentist case and with respect to the predictive density for the vector $\boldsymbol{z}_{\tau_{k+1}}$ obtained from the posterior $p(\boldsymbol{C}, \boldsymbol{\Sigma}_{\boldsymbol{v}}^{-1} | \boldsymbol{z}_2, \boldsymbol{z}_2, \dots, \boldsymbol{z}_T)$ by integrating it with respect to the conditional density $\boldsymbol{z}_{\tau_{k+1}} \sim N(\widetilde{\boldsymbol{\Phi}}_0 + \widetilde{\boldsymbol{\Phi}}_1 \boldsymbol{z}_{\tau_k}, \widetilde{\boldsymbol{\Sigma}}_{\boldsymbol{v}})$,

$$p(\boldsymbol{z}_{\tau_{k+1}} | \boldsymbol{z}_2, \dots, \boldsymbol{z}_{\tau_{k-1}}, \boldsymbol{z}_{\tau_k}) = \int_{\boldsymbol{C}, \boldsymbol{\Sigma}_{\boldsymbol{v}}^{-1}} p(\boldsymbol{z}_{\tau_{k+1}} | \boldsymbol{C}, \boldsymbol{\Sigma}_{\boldsymbol{v}}^{-1}; \boldsymbol{z}_2, \dots, \boldsymbol{z}_{\tau_k})\, p(\boldsymbol{C}, \boldsymbol{\Sigma}_{\boldsymbol{v}}^{-1} | \boldsymbol{z}_2, \dots, \boldsymbol{z}_{\tau_{k-1}}, \boldsymbol{z}_{\tau_k}) \mathrm{d}(\boldsymbol{C}, \boldsymbol{\Sigma}_{\boldsymbol{v}}^{-1}) \tag{18}$$

characterized in equations (8)-(9) in the Bayesian case, when parameter uncertainty is taken into account. Because in the latter case, the posterior may considerably depart from a Gaussian distribution and on the basis of the evidence of similar applications in the literature, the backward iterative solution is applied to the case of H = 5 years (i.e., 60 months), with 59 monthly rebalancing points. The expectation in (15) and (17) is approximated by simulation (drawing from the predictive density in (18)) using 40,000 independent trials, boosted using antithetic variate methods when K = 60 (monthly rebalancing), to keep computationalm feasibility, while we manage to use 100,000 independent trials when K = 1 (buy-and-hold strategy). δ is assumed to equal 0.223% in annualized terms (i.e., $(1-\delta) = 0.000186$ at a monthly basis.[17]

2.3 Imputing transaction costs

Following Balduzzi and Lynch (1999) and Guidolin andn Hyde (2012), in our baseline exercise, we assume that the investor faces transaction costs that are proportional to wealth, so that her

[17] Even though $(1-\delta)$ does not to equal the cash (potentially, risk-free) cash rate in this partial equilibrium framework, we stick to the convention of equaling subjective and market-implied, riskess discount rate, in this case identified with the 1-month T-bill rate.

law of motion for wealth is

$$W_{\tau_k+1} = W_{\tau_k}(1-c_{\tau_k})(1-tc_{\tau_k})R_{p,\tau_k+1}, \tag{19}$$

where tc_{τ_k} is transaction cost paid per dollar of wealth. The law of motion for wealth in (19) implicitly assumes that consumption at time τ_k and any transaction costs to be paid at the same time are obtained by liquidating costlessly the risky and the riskless assets in the proportions $\{\boldsymbol{\alpha}_{\tau_k}\}_{k=0}^{K-1}$. This assumption is sensible for liquid assets, especially when they pay coupons or dividends that can be readily used to pay for transaction costs. This certainly the case for the baseline asset menu of stocks, bonds, and REITs and—in spite of their reduced liquidity—also when the asset menu will be expanded to include hedge fund strategies provided the resulting optimal portfolio will not be entirely invested in long positions in less liquid HF strategies. As for tc_{τ_k}, we assume that there is both a fixed and a variable component to transaction costs. Therefore we model tc_{τ_k} as a function of the difference between the end- and the beginning-of-period wealth allocation to the assets, $\{\boldsymbol{\alpha}_{\tau_k} - \boldsymbol{\alpha}_{\tau_{k-1}}\}_{k=1}^{K-1}$:

$$tc_{\tau_k} = \kappa_f I_{\left\{\exists i \ni \alpha_{i,\tau_k} \neq \alpha_{i,\tau_k-1}\right\}} + \kappa_v \sum_{i=1}^{N} |\alpha_{i,\tau_k} - \alpha_{i,\tau_k-1}| \tag{20}$$

where$I_{\left\{\exists i \ni \alpha_{i,\tau_k} \neq \alpha_{i,\tau_k-1}\right\}} = 1$ when the condition $\alpha_{i,\tau_k} \neq \alpha_{i,\tau_k-1}$ is satisfied for at least one *i* = 1, 2, …, *N* (i.e., there is trading in asset *i* between t $\tau_k - 1$ and τ_k), and 0 otherwise. The first term is a fixed fraction of total investor's wealth that represents the fixed cost of rebalancing the portfolio, regardless of the size of the rebalancing. The second term is proportional to the change in the value of the asset holdings. Interestingly, under the new wealth process in (19)-(20), the inherited portfolio allocation from the previous period, $\boldsymbol{\alpha}_{\tau_{k-1}}$ (which is simply $\boldsymbol{\alpha}_{t-1}$ in the case of buy and hold problems), becomes a state variable when either κ_f or κ_v is greater than zero, since its value determines the transaction costs to be paid at time t.[18] Importantly, such framework for

[18] As shown in Guidolin and Hyde (2012), also in this case, the Bellman equation may be solved by backward recursion, using Monte Carlo methods. The only difference with respect to the case of $\kappa_f = \kappa_v = 0$ is that a Monte Carlo approximation of the expectation will have to recognize that the of the portfolio shares also affects $E_{\tau_k}[Q(\boldsymbol{z}_{\tau_{k+1}}, \tau_{k+1})]$. This turns the maximization in a fixed-point problem that can be

transaction costs characterizes them as costs imposed ex-ante on the problem, i.e., the investor acknowledges the presence of the costs and solves her problem being aware that whenever a portfolio re-shuffling implies transaction costs that exceed the expected utility gain derived from the change in asset allocation (taking into account that a portfolio revision today also affects future wealth and potential transaction costs to be borne), the investor will refrain from trading. This sophisticated scheme of transaction cost imputation may create a no-trade region that reduces portfolio turnover. Because marginal trades often relying on modest moves in either the perceived future investment opportunities or on minor changes in estimated parameters, tend to lead to fragile asset allocations that are often punished in OOS experiments, the ex-ante nature of the transaction costs in our paper makes it even possible that realized performances (including mean returns) may ex-post increase when transaction costs are modelled, even though trading will imply that the very costs need to be paid.

In our application, we set κ_f = 0.025%, κ_v = 0.15% in the case of the traditional, more liquid assets that in fact can be traded through virtually costless exchange traded funds over all of our OOS period. These selections are approximately ¼ of the values assumed by Balduzzi and Lynch (1999) as justified by the subsequent research they has reported a visible decline in trading costs on all major US markets and security types, including equity and bond indices. We double these values for κ_f and κ_v when these are applied to HF strategies, because these need to be traded using less liquid and virtually costlier over-the-counter index-linked notes. In the following, transaction costs will be always taken into account. We shall consider the case in which there are no transaction costs as a robustness check, which is of interest to either Readers that put a strong belief on a different modelling of transaction costs or as a way to estimate the maximum economic value of HF strategies irrespective of the higher transaction costs they imply.

2.4 Out-of-sample performance measurement

We assume a 5-year investment horizon (H) that balances the usefulness of a relatively long investment horizon with the need to have an adequate OOS period available to assess alternative

easily solved on a sufficiently dense grid.

portfolio strategies. We perform recursive OOS experiments using the optimal weights obtained by numerically solving the problem in (15) under the conditions in (16) which have been shown to hold under power utility. We take the period 2004:01–2019:12 as our (pseudo) OOS period in which we estimate realized performances recursively. Importantly, such a period includes both pre- and post-Great Financial Crisis data but excludes the Covid-19 market turmoil that might have advised us to adopt an explicit regime-switching strategy. We assume that investors may choose to either follow a 5-year buy-and-hold strategy (optimal only when investment opportunities are constant but often efficient because it leads to lower turnover) or rebalance their portfolio on a monthly basis, which is optimal under time-varying investment opportunities. If rebalancing is pursued, investors are given the possibility to fully exploit the predictability captured by the vector $\boldsymbol{z}_t$: the first investor allocates her wealth starting in 2004:01 according to the corresponding optimal weights and adjusts her exposure at the beginning of each of the next 60 months. The second investor acts in the same way as the first, although starting and ending one month later, etc. This is repeated until $T - H$. When the investor simply implements a buy-and-hold strategy, the weights computed at time t are held for H months before the optimal portfolio structure is re-estimated in the light of new data.

We assess the realized OOS performance at the end of the investment horizon for each investor using two metrics: the CER and the Sharpe ratio. The CER is the riskless return that makes adopting a portfolio rule as attractive as cashing in a safe return equal to the CER. A negative CER would signal the investor's willingness to pay to avoid a risky strategy:

$$\sum_{t=1}^{T-H}(1-\delta)^t\, E_t\left[\frac{\hat{C}_t^{1-\gamma}(\widehat{\boldsymbol{\alpha}}_t)}{1-\gamma}\right] = \sum_{t=1}^{T-H}(1-\delta)^t\, E_t\left[\frac{\tilde{C}_t^{1-\gamma}}{1-\gamma}\right], \tag{21}$$

where $\tilde{C}_t \equiv (1-\beta CER_H^{1-\gamma})/[1-\left(\beta CER_H^{1-\gamma}\right)^{(H-t+1)/\gamma}]$ is the consumption stream derived from a riskless strategy paying a monthly return of CER_H, for the entire holding period H.

For completeness, we also evaluate model performance using a more conventional H-period Sharpe ratio, which is the ratio of the excess mean return to the standard deviation of the portfolio being evaluated:

$$SR_H \equiv \frac{R_{p,H} - R_{1,H}}{\sigma_{p,H}}, \tag{22}$$

where $R_{p,H}$ and $R_{1,H}$ are the cumulative returns (over H months) on the portfolio and on the benchmark, short-term security, respectively, while $\sigma_{p,H}$ is the volatility of cumulative portfolio excess returns. We deem the CER the most appropriate realized performance measure because it is a function not only of the underlying return generating process but also of the investor's preferences. Moreover, the Sharpe ratio may be biased by high serial correlation in HF returns due to illiquidity and returns smoothing (Getmansky et al., 2004; Khandani and Lo, 2011).

The OOS recursive design proceeds as follows. In a first step, we compute the optimal portfolio-consumption rules for an investor who has no access to HFs but is otherwise well diversified across stocks, long-term government bonds, corporate bonds, and real estate (i.e., the baseline asset menu). Assuming either one or two lags in the VAR models, using up to (i.e., also the realized OOS performance of lighter models that include less predictors is examined) four predictors selected among the default spread, the term structure spread, the 3-month nominal rate, and the dividend yield, and two different sample selection methods (rolling and expanding window), we estimate a total of 192 models. The first vector of optimal portfolio weights is estimated using data for a 1994:01–2003:12 sample. For the next-period estimation, one additional set of monthly observations (referring to 2004:01) is added to the initial sample. This process is repeated recursively until the last available observation (2019:12) is included in the analysis. While under the expanding window scheme the sample size increases with each new estimation, in the rolling-window scheme, it is kept constant at 10 years (120 observations) by rolling the sample forward and discarding the oldest observations.[19]

The OOS experiment is repeated considering three alternative risk aversion coefficients (γ = 2, 5, 10). The VAR model that provides the investor with the highest CER is taken as optimal and

[19] The use of 10 years of data in the rolling window scheme addresses the investors' need of protection against structural breaks in the underlying predictive relations (Stock and Watson, 1996). We have experimented with longer windows with qualitative similar results but a loss of OOS statistically accurate evidence as the recursive scheme implied that the first allocation that can be assessed is determined by how many initial observations are required. Shorter windows, and in particular the classical 5-year moving window are instead unfeasible because of the relatively large parametric size of VAR(2) models.

assumed to be the one against which the same investor benchmarks all possible modifications to her baseline asset menu. In a second step, we extend the best-performing model (as defined by the number of lags, estimation scheme, predictors included, etc.) by including one HF strategy at a time along with a set of predictor variables tailored to each strategy. This occurs separately for each of three values of γ. We thus estimate 17 VAR models for each of the ten HF strategies, each of the three values of the coefficient of risk aversion and each of the two types of recursive OOS experiments performed (i.e., rolling and expanding windows); this yields a total of 1020 alternative VARs.[20]

Finally, by comparing the CER obtained in this second stage with the CER of the initial best-performing VAR, we are able to give a data-driven answer to our research question — that is, whether or not extending strategic asset allocation to include HFs is desirable to a long-term investor who is already well diversified across a broad spectrum of both classical and alternative asset classes. Additionally, our research design allows us to answer the question of which HF strategy yields the highest utility gains.[21]

3 The data

This section summarizes the data on the baseline and extended asset menus and the corresponding predictors, and describes how our choice of the HFR indices tries to minimize the effect of the biases prevalent in HFs data. The selection of the January 1994 - December 2019 sample is driven by the availability and characteristics of the HFs data.[22]

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[20] The vector $\boldsymbol{z}_t$ in (2) includes the baseline assets, the HF strategy under investigation, the best-performing predictors for the baseline menu, and up to four strategy-specific predictors, for a total $n + m$ that ranges between 5 and 13.The 17 VAR models cited in the text encompass the 16 combinations of the four predictor variables (including the case when only the baseline asset predictors are included, without any specific variable for the hedge strategy) and a pure AR process without additional predictors ($\boldsymbol{y}_{t+1}$=1 for all t).

[21] For comparison purposes, optimal allocation and realized performances are also reported for a Gaussian IID underlying return generating process (i.e., constant investment opportunities). Our CER spread estimates are of course model-driven, even though the combination of an extensive search over predictors and the addition of HF-specific predictors ought to guarantee some degree of robustness.

[22] Our sample is sufficiently well-balanced as it encompasses major macroeconomic and idiosyncratic events that affected all asset classes under consideration (e.g., the 1997-1998 Asian crisis, the 1998 Russian default and LTCM fall, the technology bubble, the 2008-2009 financial crisis and the subsequent recovery, and the 2013 taper tantrum). Optimizing portfolio choices in such a context enables us to

3.1 *Baseline asset menu*

We use the 1-month T-bill rate, the CRSP value-weighted equity index (inclusive of dividends), the CRSP/Ibbotson 10-year US government bond index, the FTSE NAREIT Composite Index, and the Barclays Long U.S. Corporate Total Return Index to proxy for our five baseline asset classes.[23] The Barclays long-term corporate bond index tracks the performance of US corporate bonds with maturities of 10 years or greater, and the FTSE NAREIT index is an indirect index built on all tax-qualified REITs.[24]

In a manner consistent with the literature, we use four predictors to model the time variation in investment opportunities as defined by our baseline asset menu. In line with Avramov et al. (2013) and Campbell et al. (2003), we employ the dividend yield, whose forecasting ability with respect to equity returns has been demonstrated at least since Rozeff (1984) and Campbell and Shiller (1988). The predictive power of the dividend yield extends also to other asset classes, including corporate bonds (Fama and French, 1989) and REITs (Karolyi and Sanders, 1998; Fugazza et al., 2009). Second, following Fama (1981), we use the short-term riskless interest rate proxied by the 3-month Treasury constant maturity rate. Third, we rely on the term spread, which is calculated as the difference between the 10-year Treasury constant maturity rate and the corresponding 3-month rate. The predictive power of the term spread concerns not only excess bond returns (Fama, 1990), but also the state of the economy at large and thus other asset returns (Campbell, 1987). Finally, we include the default spread computed as the yield differential between Moody's seasoned Baa and Aaa corporate bond portfolio rates. Keim and Stambaugh (1986) find that default spreads are able to predict corporate and government bonds as well as stock returns, while Fugazza et al. (2009) and Ling, Naranjo and Ryngaert (2000) point to the predictive power of both the term spread and the default spread for to REIT excess returns.

Panel A of Table 1 presents key descriptive statistics for the baseline menu asset returns and

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evaluate how well the models perform in bull and bear markets and whether they can adjust over time.

[23] The data on monthly asset returns and predictor variables are obtained from CRSP, Datastream, Bloomberg, the web site of NAREIT, and the Federal Reserve Bank of St. Louis' FRED.

[24] The use of FTSE NAREIT returns is in line with the literature on real estate predictability (see, e.g., Fugazza, et al. 2009), which opts for an indirect measures over direct-appraised and transactions data.

their predictors. The monthly (annualized) average excess log-returns in our sample are 0.53% (6.41%), 0.28% (3.37%), 0.39% (4.65%), and 0.83% (9.92%) for stocks, government bonds, corporate bonds, and REITs, respectively. Not surprisingly, higher monthly mean returns correspond to higher estimates of volatility: 4.50% (15.58%), 2.03% (7.04%), 2.62% (9.08%), and 5.72% (19.82%). Interestingly enough, stocks show the lowest unconditional monthly Sharpe ratio (0.12), corporate bonds the highest (0.15), with long-term Treasuries and REITs falling in between (0.14). The fact that bonds command higher Sharpe ratios than stocks is driven by the inclusion of the post-crisis, 2009-2014 period in our sample, with declining rates and FED-driven support to the bond market. Kurtosis is well in excess of three for all returns, skewness is on average negative, and as a result, the Jarque-Bera test points to the rejection of normality for all asset classes and predictors.

3.2 *Extended asset menu*

Perhaps the most important issue endemic to HF data is the selection bias that stems from the lack of reporting standards and, consequently, from the discretionary decisions by HF managers as to whether to report the returns and to which databases (see, e.g., Fung et al., 2008, Akien et al., 2012). This leads to a limited comparability among various hedge strategy performance indices, which is further exacerbated by the providers' disparate choices with respect to weighting and fund inclusion thresholds and characteristics (e.g., Titman and Tiu, 2010). Although Agarwal et al. (2013) suggest that the incentives underlying the choice of whether to submit data to an index provider may skew an index return either upward (i.e., returns are *more* likely to be disclosed after a positive track record) or downward (i.e., returns are *less* likely to be disclosed after a positive track record to preserve confidentiality or to avoid broadening the investor base), Aiken et al. (2013) compare reporting and non-reporting funds and conclude that the net selection bias is positive — i.e., it leads to an overestimation of HF returns.[25]

[25] Other biases that can significantly distort the true representation of hedge funds returns include backfiling (or instant history), survivorship, liquidation, and incubation biases, see Agarwal et al. (2015) for a discussion and review of the literature. However, Edelman, Fung and Hsieh (2013) have recently issues some re-assurances on the reliability of standard data sets as they find that the performance measures for mega hedge fund management companies that collectively manage over 50% of the

Because our goal is establish estimated bounds to the economic value of HF strategies, and the literature has generally concluded that on a net basis, these biases may tilt upwards the recorded HF returns (and bias downward the estimable volatility, because of the smoothing effects of positively serially correlated returns) in commercial data bases, we use the HFRI style indices distributed by Hedge Fund Research (HFR) as proxies for HF strategies. HFRI data are (i) net-of-fees, (ii) available starting from 1990 on a monthly basis for most of the main strategies and sub-strategies, (iii) compiled using data on both surviving and non-surviving funds, (iv) encompassing both closed and open funds, and (v) obtained after imposing either a minimum threshold of $50 million of assets under management (AUM) or a track record of more than a year. Importantly, no backfilling bias plagues the HFRI indices. Moreover, HFR provides, within the limits of the AUM thresholds imposed, a rather comprehensive coverage of the HF universe. To some extent (see the discussion in Tuchschmid et al. 2010), HFRI indices are investable via synthetic replication products (see Boigner and Gadzinski, 2013). Our sample starts in in 1994 because after that year the HFRI's survivorship bias is virtually non-existent, as the track record of non-surviving funds has been retained since that year (e.g., Liang 2000). The selection bias in HFRI is less severe than with other sources of hedge index returns and HFR makes documented efforts (e.g., by directly contacting the managers and investors) to minimize liquidation bias.

In our empirical work, we focus on ten HF style indices that have been most frequently used in the literature (see, e.g., Fung and Hsieh, 2002a, Agarwal and Naik, 2004, Boyson, Stahel and Stulz, 2010, Panopoulou and Vrontos, 2015). Our dataset includes the two flagship indices (Fund Weighted Composite Index and Fund of Funds Composite Index), all four main strategies (Equity Hedge, Event Driven, Global Macro and Relative Value), and four sub-strategies—Equity Hedge Equity Market Neutral, Event Driven Merger Arbitrage, Event Driven Distressed/Restructuring and Relative Value Fixed Income Convertible Arbitrage.[26]

industry's assets that do not report to commercial databases are similar to those of funds reporting.

[26] Definitions and methodologies of construction of each composite index and each style category can be found at https://www.hedgefundresearch.com/indices. HFRI Indices are investable via synthetic replication products and there is some evidence that such products are traded over-the-counter, see Tuchschmid, Wallerstein and Zaker (2010).

As hinted at in Section 2, we include in our analysis predictors that are tailored to each strategy in addition to the four predictors used for the benchmark assets (i.e., the dividend yield, the short-term bill rate, the default spread, and the term spread), which recent literature has shown to have forecasting power for HF excess returns as well (see Avramov et al., 2011, Bali et al., 2012, 2014). To model the time-varying risk premia in the excess returns on the two flagship HFRI indices we follow Fung and Hsieh (2004) and use the Fama-French size factor (SMB), the CBOE S&P 500 BuyWrite Index (henceforth BMX, consistent with the corresponding Bloomberg ticker), Carhart's momentum factor, and a commodity trend-following factor.[27] For the HFRI Macro strategy, we employ the SMB, the BMX and the commodity and currency trend-following factors. To forecast the Equity Hedge, Equity Market Neutral, Event Driven, Merger Arbitrage and fixed income Relative Value/Arbitrage excess returns we use the SMB, BMX, Fama-French value factor (HML), and the momentum factor (see, e.g., Fung and Hsieh, 2002a, Agarwal and Naik 2004, Wegener et al. 2010), except for replacing the momentum factor with a bond trend-following factor when predicting the Event Driven strategy. Distressed/ Restructuring and Fixed Income Convertible Arbitrage are found to be best predicted by the default spread (as in Bali et al. 2014), the SMB, and by bond and short-term interest rate trend-following factors (e.g., Fung and Hsieh, 2002b, and Hamza et al., 2006).

The descriptive statistics for the extended asset menu and the HF predictors are reported in Panel B of Table 1. Monthly average excess log-returns on the 10 HF strategies range from 0.21% (funds of funds, FoF) to 0.57% (event driven). Equity hedge strategies have the highest monthly standard deviations (2.58%), while equity market neutral strategies have the lowest (0.85%). The merger arbitrage strategy registers the highest Shapre ratio (0.39) and FoF the lowest (0.13). Yet, for 9 of the 10 strategies/indices under investigation, the full-sample Sharpe ratio exceeds the highest Sharpe ratio for traditional assets (0.15). Also, in panel B, excess returns are highly non-normal (consistently with well-known evidence, e.g., Mitchell and Pulvino, 2001) and are

[27] BuyWrite is an option strategy combining a long position on the S&P 500 index with a short position on the near-term call on the same index. From the put-call parity, the strategy is equivalent to writing a put option on the S&P500 and investing the premium at the risk-free rate.

characterized by positive excess kurtosis and negative skewness (Anson, Ho and Silberstein, 2007). The returns on the four trend-following factors, the SMB factor, and the macro HF index display positive skewness. Interestingly, the short-term rate trend-following factor posted triple-digit monthly returns during the financial crisis, which shows it can capture flight-to-quality phenomena.

Table 2 reports pairwise linear correlations for the portfolio return series under consideration. Excess returns are generally weakly correlated not only in the baseline asset menu but also in the extended menu, suggesting that the diversification into of HFs may improve the overall portfolio performance. However, we observe that event driven and a few other HF strategies' correlation with stocks exceeds 0.8, which may attenuate the potential benefits of extending the baseline asset menu, at least in their case.

4 Preliminary results for the baseline asset menu

4.1 *Linear predictability of returns*

Table 3 presents the parameter estimates, as well as the correlation matrix of the residuals for a full VAR(1) model estimated on the full, 1994:01–2019:12 sample. For every asset and predictor, the table reports the bias-adjusted estimates, the original OLS estimates, and the associated *t*-statistics. Two remarks are in order. First, small-sample bias is particularly severe for the dividend yield coefficient in the VAR equation for excess stock returns, where the corrected coefficient (0.917) is less than half the (biased) OLS estimate (1.890). Other parameter estimates are similarly affected (e.g., the short rate in the stock equation and again the dividend yield estimated coefficient in the excess REIT equation), although the differences are not always statistically significant. With very few exceptions, the bias-adjusted estimates tend to be smaller in absolute value vs. the unadjusted ones. Second, and especially when bias adjustment is performed, the overall evidence of linear predictability of excess returns is rather weak, as evidenced by the low R^2 for most of the equations for excess returns in the VAR (the exception is short-term bill returns, as one may expect). In fact, the majority of the *t*-statistics lie outside the rejection region for the 5% significance level. Notable exceptions are excess stock returns, which seem to be well predicted by past values of the dividend yield and the term spread, and excess long-term government and corporate bond returns, which are positively driven by one lag of the term spread. These results are similar to the results in Campbell et al. (2003) and should be assessed in light of the burgeoning literature pointing to the disappearance of linear predictability in the returns of stocks and bonds (e.g., Pesaran and Timmermann 2002, Welch and Goyal, 2008). Finally, all four predictor variables can be approximately described by unit root AR(1) processes, even though, as a whole, the estimated VAR(1) model is stationary. The distortions implied by the high persistence of the predictors are partially mitigated by the bias-correction technique.

4.2 *Strategic asset allocation: total and hedging demands*

Table 4 presents sample means, standard deviations, and the 90% empirical ranges for the monthly recursive portfolio weights computed following the recursive scheme described in Section 3. These weights summarize how a risk-averse investor with intermediate risk aversion (γ = 5) should have optimally allocated her wealth across 1-month T-bills, stocks, long-term government and corporate bonds, and REITs in the 16 years between 2004 and 2019. Since it is unfeasible to report the statistics for all 192 VAR models estimated for this "reference" investor, we have selected the ten models that are found to yield the highest CERs (computed in the next section). Table 4 shows that when the horizon is short or the investor is myopic (so that short- and long-run portfolio shares are the same as no predictability may be exploited), the average weights are strongly tilted in favor of public real estate while, at least in some of the top ten strategies, 1-month T-bills are occasionally shorted to create leverage and invest in both stocks and REITs;[28] the demand for all types of bonds, especially Treasuries, tends to be modest and the latter are used in some strategies to leverage the portfolio. The sample historical mean of weights turns a bit different under a 5-year horizon (with monthly rebalancing) because under many (but

[28] In Table 4, the hedging demands of myopic investors (i.e., operating under constant, IID investment opportunities) are not identically zero because of approximation errors implied by the fact that under power utility, the portfolio problem is solved numerically, approximating expectations by Monte Carlo methods.

not all) strategies, the demand for stocks and corporate bonds turns decidedly positive and large, which therefore reflects sizeable hedging demands for these assets; because the demand for real estate is hardly affected by the horizon, this shift towards and stocks and corporate debt is financed by shorting Treasury bonds, independently of their maturity. In any event, especially when H = 60 months, (average) optimal portfolio shares vary widely across model specifications. For instance, according to an expanding VAR(2) including the default spread and the dividend yield as predictors, a long-horizon investor should build a portfolio which, on average, is long 168% in stocks, 167% in REITs, 82% in corporate bonds, and is short 164% in T-bills and 152% in long-term Treasuries. On the other hand, when in this specification the VAR lag order is reduced to one, the same investor would have held—both under a short and long investment horizon—a portfolio massively tilted towards REITs (87% for T = 60 months), with smaller stakes in equities (9%), cash (2%), and puny shares in corporate and Treasury bonds (1%); moreover, her hedging demands would have been negligible on average (between -4 and 1 percent only).

The average hedging demands (i.e., average differences between long- and short-horizon optimal weights that hedge portfolio performance against future changes in investment opportunities) are also quite heterogeneous across different frameworks that capture predictability. However, they generally tend to be moderate in the case of stocks and REITs. One aspect of our research design that contributes to this effect is the small-sample bias correction of the intercept and of the slope parameter estimates reported in Table 3, that we saw to be of a first-order impact in the cases of equities and real estate. These parameters, together with the correlation matrix of the residuals terms, govern the relative speed of reversion to the mean by excess returns (see Barberis, 2000). Because such reduced forecast follows a negative excess return shock, this means that the long-run mean-reversion speed of the asset class is reduced. For the 192 VARs models entertained in our paper, average hedging demands are usually zero or negative for 1-month T-bills and government bonds, and positive for corporate bonds and stocks. Interestingly, and consistent with Hoevenaars et al. (2008), in the case of REITs, the difference between total and myopic demands tends to be almost null, pointing to a flat term structure of risk.

Table 4 shows that recursive optimal weights change not only across models but also over time: both the reported standard deviations and 90% empirical ranges suggest that there are periods in which average leverage is magnified and the sign of the weights changes frequently. Such a pattern is in line with what has been systematically documented in studies on linear predictability (e.g., Brandt and Santa-Clara, 2006, and Fugazza et al., 2009, when real estate is included). In contrast, the Gaussian IID models produce the least volatile allocations. This is explained by the fact that in this case, the time variation in weights derives only from the updating of the sample estimates and not from the fact that the investment opportunities are explicitly recognized to be time-varying.

Much of our discussion so far has concerned the case of $\gamma = 5$, but the same qualitative insights also apply to the recursive OOS results obtained assuming either $\gamma = 2$ or $\gamma = 10$.[29] The most notable differences concern the fact that while the sign of the average total portfolio allocations to the five asset classes is preserved across the three values of γ, the size is directly proportional to the investor's risk tolerance ($1/\gamma$). More conservative investors ($\gamma = 10$) are generally less leveraged and tend to tilt their portfolios towards long-term government bonds while shunning stocks and demanding a less extreme share of REITs. Mildly risk-averse investors ($\gamma = 2$) instead attach large and positive weights (usually above 100%) to these risky assets, also borrowing in the corporate credit market. Dispersion measures are also proportional to the investor's risk tolerance, suggesting bigger spikes in the time series of portfolio weights for less risk-averse investors.

4.3 *Realized portfolio performance and optimal allocation*

Table 5 presents the realized performance measures obtained in the recursive OOS experiment for an investor with intermediate risk aversion ($\gamma = 5$).[30] The top panel analyzes the buy-and-hold case while the bottom panel pertains to the recursive, monthly rebalancing strategy. The

[29] These results are not tabulated due to space considerations but are available in an Internet Appendix, or from the Authors upon request.

[30] Tabulated results for $\gamma = 2$ and $\gamma = 10$ are in the Internet Appendix. Our findings are robust across different levels of the relative risk-aversion.

table reports the annualized mean, volatility, Sharpe ratio and CER of the portfolio rules implied by the VARs and the Gaussian models. The last two columns report the skewness and kurtosis of realized portfolio returns, thereby providing a more comprehensive view of the realized distribution of returns. The realized CER rankings in Table 5 are used to determine the best-performing model against which we benchmark the marginal contributions of HF strategies.

Table 5 shows that an investor who chooses to follow a simple buy-and-hold strategy attains, on average, lower CERs (0.26% per year) than an investor who rebalances on a monthly basis (2.97% per year). In fact, there is not a large difference between the typical CERs of the the two Gaussian IID models (0.45% for the expanding and 0.43% for the rolling window) when a buy-and-hold strategy is pursued, while the best-performing VAR produces a barely higher CER of 0.48% per year,. These CERs are all lower than the annual average yield on 1-month T-bills. This is expected: a fixed proportions buy-and-hold strategy is indeed optimal only when investment opportunities are constant and Table 3 did offer persuasive evidence that the investment oppourtinities are predictable, at least to some extent. This result is driven by the considerably negative skewness and high positive kurtosis of realized portfolio returns, which are fully taken into account by the investor's power utility function. Because of this poor performance of buy-and-hold portfolio vs. the monthly rebalanced ones, in the rest of this paper we shall focus our attention on the more realistic case with rebalancing, even though complete (and leading to qualitatively similar results, just generally disappointing in terms of the CERs obtained) calculations are available upon request for the buy-and-hold case.

Whereas buy-and-hold strategies optimized on the ten best VARs result in disappointing average performances characterized by rather large realized volatility, monthly rebalancing enables the investor to substantially improve CERs also because of a somewhat lower realized volatility as reflected in the narrower 90% bootstrapped confidence intervals.[31] The bottom panel of Table 5 shows however that—because an investor is allowed to discount her own future learning on the Gaussian IID distribution of excess asset returns (see Barberis, 2000)—it is the two constant

[31] The 90% confidence intervals are calculated by means of a block bootstrap technique with a block size equal to 12 monthly observations and 10,000 simulated paths.

investment opportunities models (expanding and rolling window Gaussian IID) that achieve the highest CERs of 9.4% and 6.4% per year, respectively. These are followed by a range of VAR(1) models, mostly estimated on a 10-year rolling window of data, and generally including a relatively large number of predictors (always the default spread and quite often the dividend yield); these lead to realized OOS CERs between 5.5 and 6.5 percent per year. These findings generally align with earlier linear predictability OOS studies applied to US data (see, e.g., Brennan, Schwartz and Lagnado, 1997; Guidolin and Hyde, 2012) that, on traditional asset menus composed of stocks, bonds, and real estate, found that the economic value of predictability may turn elusive especially when transaction costs are accounted for, as we do in this paper.

5 Main results: portfolio selection extended to hedge fund strategies

5.1 *Linear predictability of hedge fund returns*

Table 6 exemplifies our procedure by presenting parameter estimates and the residual correlation matrix for the best-performing VAR model for $\gamma = 5$ as specified in Table 3 now extended to include, for starters, excess returns on the HF weighted composite index (FWC).[32] The model is a VAR(1) estimated using an expanding window scheme on a sample up to December 2019 and includes six predictors: the term spread, the short-term rate, the dividend yield, the S&P 500 BuyWrite index returns, HML, and momentum. Table 6 reports the bias-adjusted estimates, the original OLS estimates, and the associated t-statistics. Bias adjustment plays an important role in this extended asset space, just as it does in the baseline menu. This is particularly evident from the dividend yield coefficients in the equations for the risky excess returns: for example, in the excess stock returns equation, the biased coefficient is almost four times the bias-corrected coefficient. We expect that such a reduction in value (relative to the biased estimates), when combined with the negative residual correlations between unexpected stock returns and unexpected changes in the dividend yield, would translate into a slower rate of mean-reversion which, in turn, is likely to command smaller hedging demands for most risky

[32] This procedure is repeated for each of the ten hedge fund strategies and the three values of the relative risk-aversion coefficient.

assets. As a result, correcting for the bias makes stocks (and to some extent, REITs) less attractive for hedging intertemporal stochastic changes in future returns. While a similar effect is generated by bias-adjusting the dividend yield coefficient in the FWC_{t+1} equation, the opposite effect is obtained in the $Corp_{t+1}$ and $REIT_{t+1}$ equations, where the speed of mean-reversion grows. The unadjusted OLS coefficients for the term spread are also considerably biased, especially in the case of government and corporate bond excess returns. Focusing on the FWC_{t+1} equation, which also flaunts the highest R^2 among the excess returns equations, now also the lagged stategy excess return implies a positive and statistically significant coefficient. This finding may be explained by positive serial correlation stemming from exposure of the hedge strategy to security payoffs that are not actively traded, as documented by a literature since at least Getmansky et al. (2004). Interestingly, the returns on the BuyWrite strategy seem to be explained, at least partially, by past HF index excess returns.

5.1 *Strategic asset allocation: total and hedging demands*

We compute monthly recursive OOS portfolio weights for the extended asset menu. Similarly to Section 4.2, we discuss in detail the results for an investor characterized by intermediate risk aversion (γ = 5) who allocates her wealth across 1-month T-bills, stocks, long-term government and corporate bonds, REITs, and a HF strategy. To individually assess the economic value of each of the HF strategies, we include them in the asset menu one at the time, from a starting pool of ten strategies.[33] For each of the resulting ten portfolios, Tables 7 through 9, as well as additional tables in an Internet Appendix (see, e.g., A1 and A2), report sample means, standard deviations, and the 90% realized range for monthly recursive OOS portfolio weights for the ten models that provide the investor with the highest CERs when the asset menu is expanded to each of the ten HF strategies, one at the time.

[33] Tabulated results for investors with γ = 2 and γ = 10 are available from the authors upon request and in an on-line Appendix. Much of the discussion for γ = 5 applies to the cases of γ = 2 and γ = 10 with the usual caveat: the average portfolio leverage and standard deviations turn out to be proportional to risk tolerance ($1/\gamma$). Interestingly, at least in the case of γ = 5 and 10, the best models give positive weight to HF strategies across the board of our OOS period, which allows us to skim over the fact that a few strategies—barring their outright replica (see, e.g., O'Doherty et al., 2016)—would be hard to short because HFR indices are not (always and reliably, see Getmansky et al., 2015) traded as shortable exchange traded notes.

Comparing Tables 6-8 with Table 4, we note that the inclusion of HF strategies in the portfolios prompts a dectetable (yet, never massive) shift away from public real estate and into long-term Treasuries and—especially when the excess returns on relative value HF strategies (RVR) are considered—hedge fund shares. In general, even over and beyond what is displayed in these tables, it is a few, distinctive HF strategies that attract the largest mean portfolio weights over our sample (especially RVR, equity market neutral, and funds of funds styles). All in all, even when the average demands for HFs fails to massive, the four traditional risky assets are affected only to a modest degree relative to the baseline portfolio in Table 4, even though the extended portfolios are on average not as long in REITs as we had previously reported. While under the baseline asset menu, the government bonds are largely neglected or even shorted, the opposite happens with three out of the ten HF strategies investigated, i.e., in the case of fixed income relative value (Table 8), event driven, and distressed restructuring strategies. Mean hedging demands for HFs are positive (but modest) because their excess returns are generally predictable and mean-reverting at a speed that tends to be slower than other asset classes, so that this alternative asset class can be (weakly) used to hedge intertemporal stochastic variations in investment opportunities. The resulting term structure of risk is negatively sloped so that long-horizon investments in HFs may be perceived as (slightly) less risky than short-horizon investments. Fund of funds, equity market neutral and merger arbitrage strategies are the only strategies showing negative, albeit still modest, hedging demands.

In some additional detail, Table 6 shows the results for an investor who is allowed to trade the HFRI Fund Weighted Composite Index (FWC) along with the baseline menu of assets. This is of course key evidence, because it may be argued that FWC represents a weighted average return for the whole HF industry. All the top strategies do exploit linear predictability, as none of these models consists of Gaussian IID models (this also true of Tables 8 and 9 and will be commented further below). In the long run, the optimal weights in this alternative strategy are positive, between 3 and 5% and such allocation exceeds the short-horizon weights due to a positive intertemporal hedging demand (of 2-3% on average). Positions in the other five assets are still heavily skewed towards REITs (with a weight of around 80%), with any remaining wealth distributed almost equally between stocks and long-term bonds (5-9% with small and generally negative hedging demands); a negligible weight goes to corporate bonds and cash tends to be ignored but not used (with only some exceptions at the long horizon) to leverage.[34] Of course, HFs themselves may provide (especially when their market beta is positive, as often found in the empirical literature, see e.g., Patton, 2008) exposures to equity and Treasury-bond risks, besides other types of tail risk-type and non-linear exposures, as documented by Agarwal, Arisoy and Naik (2018). For instance, based on the best-performing model (VAR(1) with an expanding window and the default spread and the short-term rate included as predictors, a long-horizon investor should build a portfolio which, on average, is long 3% in the composite HF strategy, 84% in REITs, 6% in stocks, 5% in government bonds, 2% in corporate bonds, and ignore T-bills altogether. Such an average allocation is rather stable over the entire OOS period with the exception of a few spikes during the financial crisis, as documented in unreported plots and shown by the tame realized standard deviationand empirical 90% interval ranges in Table 6.

When fund of HFs (FFP) is the strategy selected to be tested in addition to the baseline asset portfolio, Table 7 shows that a long-horizon investor should, on average, allocates a more substantial share to HFs but this exact share becomes more dependent on the selected VAR and estimation sample selection scheme, ranging for instance between -31% and 26% for a 5-year horizon investor. However, the implied hedging demands remain on average quite modest. Total portfolio demands for the other five assets are similar to the baseline case, although in this case leverage is occasionally (i.e., in a few strategies) obtained by shorting long-term government bonds and especially T-bills. The best-performing model is still an expanding VAR(1) model that includes the default spread and momentum as predictors. This strategy requires a long-horizon investor to buy fund of HF strategy (8%), REITs (77%), stocks (9%), long-term Treasuries (3%), and cash and corporate bonds in equal proportions (1.5%). As in Table 6, portfolio weigths are

[34] Such large shares invested in real estate are not completely bizarre. For instance, in a CAPM perspective, the official statistics for the US, reveal that in 2020 residential real estate made up about 83.98% of total household non-financial assets, 30.64% of total household net worth, and 26.64% of household total assets (Financial Accounts of the US, First Quarter, 2020, https://www.federalreserve.gov/releases/z1/20200611/z1.pdf).

subject to limted turnover during the OOS period. The insight that funds of funds may generate limited economic value and as such may play a limited role in the portfolios of rational, long-horizon investors echoes earlier findings by Amin and Kat (2003a) and Liang (2004).

Table 8 presents results for the fixed income relative value/arbitrage strategies (RVR). This strategy has been selected and reported in detail because it implies non-negligible optimal weights to be assigned on average to the strategy. In particular, the average hedging demands characterizing RVR excess returns tend to be positive, which is due to the negative first-order serial correlation of RVR excess returns (-0.50) which, combined with a negative slope coefficient in the VAR equation, translates current negative innovations in RVR into higher predicted returns. The best-performing VAR (which lists four predictors, i.e., the default spread, the short-term rate, SMB, and the returns on a portfolio of commodity look-back straddles) is the top performer in terms of realized CER among not only the 17 models estimated for the RVR strategy, but also as compared to all models of all HFs investments. According to this rich VAR, a long-horizon investor should allocate 15% of her wealth to RVR HF strategies, 68% to REITs, 9% in stocks, 7% in long-term government bonds, and 1% in corporate bonds.

As expected, the inclusion of Global Macro HFs (MAC) reduces exposures to stocks, REITs, and corporate bonds at both long and short horizons, as this strategy tends to also use traditional asset classes to take positions supported by macro views.[35] Hedging demands are on average negative, indicating an increasing term structure of risk. Under the best-performing VAR, which includes the three baseline predictors and Fung and Hsieh's (2004) currency trend-following factor, the weights change rather erratically over time, although they fluctuate within relatively tight ranges. For an investor wishing to add equity hedge strategies (EQH) to her portfolio, the equity exposure is given by the sum of the weights to this alternative asset class and stocks. Allocation to stocks is, on average, almost negligible and is used primarily to compensate large negative positions in EQH during certain periods.

[35] Tabulated results and plots of optimal portfolio weights for Global Macro, Equity Hedge, Event Driven, Merger Arbitrage, Event Driven Distressed/Restructuring, fixed income Relative Value and Convertible Arbitrage and Equity Hedge Equity Market Neutral strategies are available from the Auhors upon request. See, e.g, Tables A3 and A4 in the on-line Appendix.

In the case of event driven strategies (EVD), the optimal investment in HFs is positive, in fact, under some strategies, in excess of 10% of the total wealth, and short positions are frequently assumed in T-bills and corporate bonds to allow the investor to express the same demand for REITs found in Table 4, through leverage. In the case of merger arbitrage (MEA), the best-performing model is the no-predictability benchmark, which implies that both short- and long-horizon investors should follow the same portfolio rules. Optimizing investors go long in MEA, REITs and government bonds while shorting to some extent T-bills and corporate bonds. These allocations are the least volatile over the entire OOS period, as one would expect for a Gaussian IID model.

When distressed/restructuring strategies (DSE) are added to the baseline asset menu, the best-performing VAR is represented by an expanding VAR(1) model including 3 predictors and the optimal portfolio weights call for going long in DSE, REITs, stocks, and to some extent, government bonds; these positions are financed by shorting T-bills to some extent. Diversifying into convertible arbitrage strategies (COA) HF strategies results in relatively small portfolio leverage on average, although the weights and level of leverage vary substantially over time. These results approximately also extend to equity market neutral (EMN) HF strategies. The best-performing model in this case implies an optimal portfolio which, on average, is long 8% in EMN, 69% in REITs, 7% in stocks, 5% in Treasuries, and 13% in corporate bonds, and short 2% in T-bills.

5.2 *Realized portfolio performances and the economic value of hedge funds*

Our key set of results concerns the realized OOS performance of the extended portfolios for each of the three levels of relative risk aversion. For each of the 10 HF strategies under analysis, we have analyzed a total of 18 models. These include the Gaussian IID model, the purely autoregressive model, and the 16 VARs which can be built for all possible combinations of the four new predictors tailored to each HF strategy. Through a comparison of the estimated CERs of these 180 models with the CER of 9.41% obtained from the best performing model applied to the baseline asset menu, we are able to determine whether or not an allocation to HFs is attractive to a long-term investor who is already well diversified across a broad spectrum of classical asset classes. In the following, we do not formally test whether HF-strategies, as a whole, dominate traditional portfolios: this would be subject to a clear multiple, overlapping hypotheses testing problem best solved adopting techniques from model confidence set estimation and reality check testing in econometrics. Instead, we ask a more modest question which, however, represents a necessary condition to the measurement of the economic value of HFs to investors: do HF strategies exist that generate risk-adjusted performances in excess of the traditional strategies?

When the composite, value-weighted basket of HF strategies is added to the baseline menu of an intermediate risk-averse investor, the best-performing model is represented by an expanding VAR(1) that provides an annualized CER, net of transaction costs, of 9.31% (see Table 9) which is approximately the same as the baseline asset allocation in Table 5 (9.41%).[36] In fact, realized skewness and kurtosis of portfolio returns under the optimal strategy are -0.66 and 3.80, respectively, while the annualized mean return and volatility are 12.66% and 20.71%. Because of the monthly rebalancing, even though HFs returns per se are characterized by a volatility that is inferior to equity markets, the resulting market timing strategy may lead to rather risky realized OOS performance, at least in terms of recorded second moment. This model leads to a relatively "normal", unremarkable annualized Sharpe ratio of 0.44 vs 0.41 under the benchmark but compensates a power utility investor with slightly "better" realized higher-order moments. Therefore, at least as far as the weighted composite HF index is concerned, adding aggregate, asset-weighted HF strategies to the asset menu does not seem to create major economic value, also because—as commented earlier—the impact of the availability of HFs in terms of weights assigned to them is rather limited, also to a long-horizon investor.

In Table 9, a clear pattern emerges among the 18 models entertained with reference to the aggregate of the HF strategies: leaner models tend to provide the highest welfare measures. Nine models out of 18 produce CERs with a 90% confidence interval upper bound that exceeds the

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[36] In Table 9, as in all tables reporting the realized performances for asset menus that include HF strategies, the performance statisics that improve over the top performance for the benchmark asset menu (i.e., raise the mean, lower volatility, increase Sharpe ratio and/or CER, increase skewness, decrease kurtosis) are boldfaced.

benchmark CER of 9.41% from Table 5, suggesting that there anyway chances for an investor to profit from investing in FWC. Yet, the median CER across all expanding VAR remains negative, -10.10% per year (vs. 3.15% for the baseline asset menu): picking at random some predictability model to be applied to an asset menu that includes a HF index will not bring a positive risk-adjusted performance to investors. Yet, Table 10 puts in no way an end our empirical task: because this strategy is a weighted basket of a number of HF strategies, we can expect to find specific HF strategies to give both higher and smaller CERs vs. the best-performing model for FWC.

The results in Table 10 suggest similar results and conclusions for the case of fund of funds strategies. The finding that diversified baskets of HF strategies fail to generate substantial economic value is in line with the bulk of the early, mean-variance based literature (see, e.g., Amin and Kat, 2003a). The highest CER (9.44%), obtained by an expanding VAR(1) model in which the predictors are the default spread and the returns on a basket of lookback straddles on interest rates, is approximately identical to the CER yielded by the benchmark asset menu but it is characterized by a substantially lower Sharpe ratio (0.30) vs. the benchmark. The median strategy delivers instead a CER of 7.93% that is more than double the CER obtained by the median model in Table 5, and this represents an element of good news in favor of fund-of funds strategies. Notably, the table shows that adding funds-of-funds to the menu of choice stabilizes the performance (both in terms of realized variance and kurtosis) and induces some degree of positive skewness; however, presumably because of their double layer of fees, the realized mean is somewhat penalized, to the point that realized Sharpe ratios decline vis-à-vis Table 5, and the CER turns out to be essentially identical.[37]

On the contrary, and as an example of one specific type of strategy, making an investment in the RVR index provides substantial economic value to an investor regardless of the model selected,

[37] However, as γ increases, an investor will care less for the mean and more for the risk of the resulting distribution of realized portfolio returns (hence, wealth and consumption flows), so that that the distance between the top CERs including FFP strategies and those obtained from the baseline asset menu widens. We can speculate that for very high values of γ, FFP and FWC strategies may stabilize performance so much that they will generate large, positive economic value.

except for the no-predictability benchmark (see Table 11). While the lowest-ranked VAR model yields a CER of --0.42%, the highest-ranked model provides a CER of 10.7% — the largest value among all HF strategies tested in this paper (which is why RVR is presented in detail) and well exceeding the baseline asset menu. The best-performing model produces a mean annual return and volatility that are above/below the median values (13 and 23% per year), while the resulting Sharpe ratio (0.41) is below the median (0.19). Table 11 displays the usual mechanism through which a hedge strategy produces economic value: by reducing realized volatility, limiting kurtosis and generating positive skewness in returns. The only difference is that RVR does that in a large enough magnitude to generate higher CER net of transaction costs vs. the traditional asset menu. In fact, the median across expanding VAR models in this case leads to a 5.57% CER, with 90% empirical confidence bounds of 3.63 and 7.39 percent per year, so that the *median* lower bound outperforms the *median* investment scheme under the baseline asset menu in the sense that the lower bound when RVR belongs to the asset menu exceeds the upper bound under an asset menu that excludes hedge funds.

Figures 2 and 3 summarize the main results in Tables 5 and 9-11 concerning the comparison of economic value estimates obtained with and without HF strategies, and extend our presentation of results to all strategies including those collected in the on-line Appendix. Figure 2 compares the CER, mean returns, Sharpe ratios, skewness, and kurtosis of the top performing model for each hedge strategy, and plots them against the realized portfolio outcomes of the benchmark in Table 5. Figure 3 performs the same comparison with reference to median statistics for the expanding sample VAR models, selected because they represent the top performing model in Table 5. In the top portion of Figure 4 we see that—if investors were able to detect the top-performing models for the prediction of risk premia—most strategies and, as a result, also the composite HFR index, would outperform a classical asset menu on a risk-adjusted basis. In fact, even taking the resulting sample uncertainty into account, the only exceptions are the equity long-short, equity market neutral, and the fund-of-funds strategies. The equity market neutral and relative value strategies give evidence of a positive CER differential vs. the traditional asset menu, while for most strategies the 90% confidence band appears to be approximately centered on the CER estimate from Table 5. Interestingly, most of the realized economic value fails to result from a mean-variance improvement: in fact, when combined with classical assets, most (all) HF strategies yield realized Sharpe ratios that never exceed and are often inferior (this is the case of FFP, EQH, MEA, DSE, and COA) vs. the benchmark. In the case of most strategies, this is a result of higher realized volatility because Figure 2 shows that the realized mean portfolio returns are generally similar (occasionally higher) than those obtained in the absence of HF in the investment opportunity set.[38] For instance, while traditional assets only lead to an annualized mean return of 12% per month and a monthly Sharpe ratio of approximately 0.41, when the RVR strategy becomes available, the realized annual mean return increases by 60 bps and the Sharpe ratio is again 0.41. However, HF strategies grossly improve the skewness properties of the optimal portfolio from -0.6 to +0.4. It turns out that a long-run investor with $\gamma = 5$ cares enough for the shape of the entire density of realized performances to considerably tilt her allocation towards RVR because these buy positive skew and hence chances of high, right-tail performances without inflating the overall thickness of the tails of the distribution. Mechanically, this is possible only because the resulting portfolio weights in Table 12 become sufficiently extreme and time-varying to increase at the same time the resulting portfolio variance, which explains why the Sharpe ratio does not improve even though the realized mean is higher than from the best strategies in Table 7.

One tricky aspect of our story is the difference between ex-ante moments (more generally, the predictive density of realized consumption flows from the cumulative wealth process) and realized, ex-post moments from OOS backtesting. Although separate calculations confirm that a $\gamma = 5$ tilts her portfolio selection away from classical fixed income securities and towards hedge strategies in the way described to trade-off less mean, more variance, a constant or lower Sharpe ratio, in exchange for higher skewness and not higher kurtosis on an ex-ante basis, this shift appears to actually occur in terms of ex-post realized performance. In other words, an investor appears to achieve the desired positive skewness, but she also pays a price in terms of higher

[38] See also Figure A1 in the on-line Appendix for direct evidence concerning realized portfolio volatility.

realized variance for equal or slightly reduced Sharpe ratio. Yet, the decline in kurtosis that ex-ante an investor would be looking for, does not seem to fully materialize.

In unreported plots, we have found evidence similar to Figure 2 for $\gamma = 2$ and $\gamma = 10$.[39] For $\gamma = 2$ (see Tables A7 and A8) we find massively large and positive CERs for a long list of expanding VAR(1) models applied to FFP as part of the asset menu. This performance derives realized positive skewness and especially to lower realized kurtosis vs. the traditional asset menu in Table A1. As in Table A1, realized means are high as a $\gamma = 2$ investor is aggressive, but this is more than compensated by rather high realized volatilities that eventually deliver disappointing Sharpe ratios in OOS experiments (0.20 for the median model). Paradoxically, ex-post an aggressive investor who should care a lot for the Sharpe ratio ends up scoring good CERs because of the benefits of better realized higher-order moments. With a CER of 18.8%, the best-performing model is an expanding VAR(1) that includes the default spread, SMB, and the returns on a portfolio of look-back options written on commodities. The results in Table A8 concerning RVR are qualitatively similar to both Table A7 and the corresponding Table 12.

The findings on the empirical economic value of HFs turned more mixed in the case of $\gamma = 10$ (see Tables A9 and A10 in the on-line Appendix). For instance, as far the FFP is concerned, under an expanding window VAR(1) that bases its risk premia forecasts on the default spread, SMB, and the returns on a portfolio of look-back options written on commodities, a long-term investor would have achieved a realized OOS CER of 9.1% which is below the 15.5% achieved investing in stocks, bonds, and real estate only. In fact, in the case of $\gamma = 10$, FFP allows the investor to gain additional risk-adjusted returns, relative to the baseline asset menu, under none of the VARs used to capture time-varying investment opportunities. It is revealing that this occurs when there are no measurable improvements in the realized higher-order moments. The same highly risk-averse investor can improve her realized utility by investing in EVD, DSE, and COA strategies, and especially RVR (see Table A10). This finding is coherent with Agarwal and Naik's (2000) result that—within a portfolio comprising of passive asset classes and investments in nondirectional

[39] These figures are available upon request from the authors.

HF strategies—the relative importance of passive and alternative portfolios changes as one shifts down the risk-return trade-off towards the minimum variance portfolio as the weight of the equity class decreases, that of bonds and HF strategies increase; within the very set of HF styles, the weight of the directional strategies falls while that of the non-directional strategies rises.

However, before rushing to a conclusion that for intermediate and low risk-averse investors HF strategies are appealing investment opportunities, Figure 3 (that refers again to a $\gamma = 5$ investor) offers a sobering view. Indeed, Figure 2 had been built under the unrealistic assumption that an investor would know in advance what the best performing model (in terms of realized CER) would turn out to be ex-post. In reality, this is hardly the case: while academics have been heatedly debating whether there is any exploitable predictability in financial returns, a fortiori we know much less about what model could represent the "right one" on an ex-ante basis. Figure 5 has the same structure as Figure 4 but it reports the realized OOS performance of the median prediction model for asset risk premia (including, as a special case, the IID no predictability model). Picking at random "some model" an investor would have not fared so well unless she had known—once more, rather unrealistically—which specific hedge strategy to pick. On the one hand, betting on the composite HFR index or on a fund-of-fund strategy leads to median realized CERs that are negative and therefore dominated by the simplest of the portfolio strategies: 100% in cash at all times; this also applies to the generality of other HF strategies, with the only exceptions of the FFP, MAC, and EMN, for which the median model deliver positive CERs of 3-8% per year and yet exceeding the small CER found for the traditional asset menu. It seems that giving up on the fine-tuning of the predictability model may particularly hurt the strategies that just trade equities, such as EQH, DSE, and RVR. Interestingly, and contrary to the best models in Figure 2, in the case of the median across predictability models, when the economic value created is positive, this comes entirely from an improvement in realized skewness—which remains negative but draws close to zero—and kurtosis, which drops to the range 3-5; in fact, all realized means decline and realized variances increase vs. the benchmark (see also Figure A1 in the on-line Appendix), which leads to considerably lower Sharpe ratios.

Figure 4 uses the same plots that we have featured in the Introduction to provide a visual summary of our key empirical findings in this section. In each of the panels, the dotted, red lines parallel to the axes mark the realized CER and Sharpe ratio obtained by the benchmark optimal allocation to traditional assets obtained from the best, Gaussian IID model. Each HF strategy is represented by a circle the center of which is positioned in correspondence to the coordinates given by realized Sharpe ratio and CER and whose radius/diameter is proportional to the Jarque-Bera statistic, which is a weighted sum of realized skewness and excess kurtosis. In each panel, the shaded area in the north-western region, collects combinations of Sharpe ratios and CERs that jointly outperform the benchmark, marked by the red, boldfaced circles. Clearly, when an investor is allowed to select the best prediction model for excess returns, three strategies follow in this golden region. As commented before, a majority of HF strategies (their numbers identifies them at the bottom of the plots) lie almost exactly on the vertical axis and are characterized by low Sharpe ratios and CERs falling below the benchmark in correspondence to poor realized skewness and kurtosis, which is visible from the large radius of the circles. In the central panel of the picture, concerning the performance of the median prediction models, the results represented get worse: now no strategies fall in the golden region, with the near-exception of fund-of-funds strategies. Especially in this case, six HF strategies visibly fall behind the benchmarks in both performance dimensions and, particularly near the origin, the circles are large and hence connect the negative CERs to abysmal skewness and kurtosis properties. Especially in the case of median performances, there seems to be a direct relationship between Sharpe ratios and higher order moments, these all improve together moving from the origin towards the right.

5.3 The effect of parameter uncertainty on the economic value of hedge funds

Tables 12-16 and Figure 5 report key results with reference to the Bayesian portfolio analysis, i.e., when parameter uncertainty is taken into account. This seems to be of particular importance in the case of HF excess returns that tend to be predictable with a degree of uncertainty that is often higher than in the case of traditional asset classes. We present these results as an additional step vs. section 5.2 because this allows us to measure the incremental (if any) realized, OOS economic value generate by HF strategies when all forms of uncertainty are taken into account.

Table 12 deals with the benchmark case and immediately shows that in our application, dealing with parameter uncertainty yields first-order effects. When HF strategies are excluded from the asset menu, all (at least, top performing) VAR-driven models are mostly of the rolling window type and yield fairly balanced asset allocations across cash and risky assets, in some ways close to some 1/N solution when H = 1. For instance, the top performing VAR(1) model that includes as predictors the default spread, the short-term rate, and the dividend yield, implies that a 5-year horizon investor ought to hold on average 50% in REITs, 16% in corporate bonds, 14% in stocks, 7% in long-term Treasuries, and the residual 13% in cash. The rolling window nature of the best performing models is sensible because Bayesian estimation makes a much more efficient use of smaller sample sizes (in this case, these are 120-observation samples rolled over time) because these are supplemented by the information in the priors. However, such a switch from expanding to rolling window estimation schemes is likely to be causing the substantial difference in portflio shares vs. Table 4. Because the 1-month horizon (average) portfolio shares are quite different (27%, 20%, 19%, 15%, and 19% listing the asset classes in the same order as above) from the long-horizon ones, a Bayesian investor would be characterized by larger hedging demands vs. Table 4.[40] In particular, real estate is characterized by positive and large hedging demands, which shows that given the estimates of the corresponding VAR models, this asset class turns out to also provide self-insurance against the complex patterns of parameter uncertainty implied by the estimated posteriors of the coefficients. Yet, taming the portfolio effects of variation in the estimated coefficients tends to shift asset shares away from real estate and towards all other asset classes. Interestingly, all other asset classes are characterized by negative and non-negligible hedging demands and there is considerable homogeneity across predictability models in terms of their implications for portfolio weights. Moreover, the middle and lower panels of Table 12 show that compared to Table 4, the Bayesian optimal weights are more stable than those in Table 4, which is sensible in the light of the stabilizing effects of that taking into account

[40] To save space, we omit the tabulation (or the plots of the posterior coefficient densites) of the posterior Bayesian estimates of any VAR models. Note that under the priors that we assumed, all posteriors will be centered around a mean that is identical to OLS estimates reported in Table 4. Yet, reporting measures concerning the posteriors of the estimated coefficients is well beyond the space available to us.

parameter uncertainty ends up to have.

Table 13 (especially the upper panel, when transaction costs are applied ex-ante) should be compared to Table 5 and displays the realized OOS perfomances when the investor accesses only stocks, bonds, and REITs. Visibly, accounting for parameter uncertainty exercises a beneficial effects on realized performances. For instance, comparing the best (median) performing rolling window VAR(1) in the table with the best (median) expanding window model in Table 5, shows that the CER increases from 9.4 to 11.8 percent (3.15 to 8.36%), the Sharpe ratio increases from 0.41 to 0.66 per year (0.36 to 0.52) as a result of the fact that mean returns remain stable at 12.1 – 11.9% (12.5% - 11.8%) but realized volatilities decline from 20.9% to 12.9% (24.8% to 16%). Even though some strategies in Table 13 do yield lower realized kurtosis and (marginally) skewness, the cause for the large increase in realized CER is easily traced back to the high realized Sharpe ratios.[41] All in all, Table 13 represents an even tougher benchmark for HF strategies to outperform: if, without HF, the traditional asset classes can be combined—exploiting predictability—to yield portfolio outcomes that are even better than in the case in which parameter uncertatinty is ignored, it will be now harder for HF strategies to find space in optimal portfolios and hence create economic value.

Table 14, with reference to the excess returns of the HFRI weighted composite index, starts out by showing that in a Bayesian set up the demand for HF strategies as a whole is actually strengthened, now that the adverse effects (to a risk-averse portfolio optimizer) of their parameter uncertainty is taken into account. Moreover, the hedging demands characterizing FWC become now positive and substantial, reflecting the fact that it is especially long-horizon investors that may benefit from the availability of HF. For instance, the best performing strategy (a rolling window VAR(1) that includes the short rate, the dividend yield, SMB, momentum, and the returns on commodity lookback option strategies as predictors), implies that a long-horizon

[41] Even though we have not performed the additional calculations to verify this conjecture, one may push the point to claim that while the results in sections 5.3 and 5.4 dependend on the assumption of power utility preferences and the fact all moments of the predictive density of wealth and consumption would matter to the investor, under Bayesian portfolio rule qualitatively similar result could have been recovered under simpler, stylized mean-variance preferences.

investor should invest 75% on average in hedge funds, 58% in REITs, 26% in corporate bonds, shorting cash (-37%) and stocks (-23%) to finance these long positions. Because a 1-month investor would hold a less extreme portfolio (e.g., 17% in FWC, 19% in REITs, 23% in corporate bonds, 20% in Treasuries, 7% in stocks, and 14% in cash), this implies large and positive hedging demands for hedge funds and real estate (which clearly can be used to also hedge parameter uncertainty), and large and negative hedging demands for stocks, Treasuries, and cash. More generally, the top ten strategies imply a FWC demand that ranges between 16% and 58% over short horizons and between 46% and 82% over long horizons, hence implying massively positive hedging demands in the interval 25-58%. This sizeable demands for HF composite strategies and corporate bonds causes the investor to express much lower portfolio shares in REITs and triggers significant borrowing at the short-term rate (also by shorting Treasuries but only at a 60-month horizon). Finally, the range of empirical variation and the recorded volatilities of the optimal portfolio weights are much larger in Table 14 vs. Table 12. In particular, the 90% empirical portfolio bounds are wide as a result of the presence of a number of outlier weights recorded over 2009-2010, when the 10-year rolling sample is heavily influenced by the financial crisis.

For the HF styles FWC, FFP, and distressed/restructuring (DIS), Tables 15-16 (as well as a few tables in the on-line Appendix) perform afresh the computations in Tables 9-11 when parameter uncertainty is taken into account and HF are part of the asset menu. DIS is selected because among the individual HF strategies, it is one that leads to the best realized OOS performance. However, while Tables 9-11 concerned the expanding window VAR and Gaussian IID models, to be consistent with the main findings in Table 13, we now only focus on 10-year rolling window results, because these lead to superior performances.[42] Consistently with their non-negligible optimal weights, HF strategies—both in the aggregate as measured by FWC or through the FFP categories, and also individually as shown in the Appendix for the DISstrategy and in the on-line Appendix for many other strategies—generate positive and large economic value under a wide variety of predictive frameworks, as shown by boldfacing the related estimates in the Tables; in

[42] We have empirically checked that this property also holds in the case of asset menus that include HF strategies. Complete, tabulated results are available from the authors upon request.

all tables, the constant investment opportunity, Gaussian IID model has a hard time reaching the top of the ranking. For instance, in Table 15, FWC easily helps to generate higher CERs vs. the benchmark and in fact, in the case of the best performing rolling window VAR(1) model, the 90% confidence interval obtained by bootstrapping fails to overlap with the corresponding interval for the best performing performance in Table 13. This may be taken as evidence of a strong degree of backing of the generation of economic value from the adoption of hedge funds in the opportunity set faced by an investor. Interestingly, such a massive improvement in realized CER is entirely supported by higher (in fact, positive) realized skewness and lower realized kurtosis, i.e., by a higher-moment effect. This is evident from the fact that in Table 15 the realized Sharpe ratios are instead systematically lower than those reported in Table 13, although many strategies are able to direct FWC investments to enhance realized mean returns. Stated in a slightly different manner, while a simple mean-variance investor, even when parameter uncertainty is taken into account, would be unlikely to give much play to HFs as an alternative asset class, a power utility investor that also attaches importance to higher-order moments in assessing her expected utility will do so and on a rather large scale, as shown in Table 16. However, this conclusion stops short from characterizing the ranking of the performance of the median models, as the corresponding CER in Table 15 is vastly inferior to the corresponding statistic in Table 13: this means that a few of the predictive models occasionally produce very poor performances when hedge funds are included in the analysis so that an investor blindly selecting "some" model to perform asset allocation when HFs belong to the asset menu may be occasionally worse off.

The evidence in the Appendix on the economic value of DISHF strategies is qualitatively identical to the one uncovered in Table 15 for the composite of all HF styles: rolling VAR-driven asset allocation dominates; it is easy to uncover large CERs that far exceed the benchmark (e.g., 15.8% under the best performing model); this finding is entirely driven by the stronger higher-order moment properties of HF excess returns as the realized Sharpe ratios (e.g., 0.36 for the best performing CER model) are below the benchmark. It remains the case that some blind choice of the median model to support asset allocation in the presence of DISstrategies would make the investor worse off. In Table 16, concerning FFP strategies, results are instead less clear-cut and, therefore, more similar to those in Table 10: although the best rolling VAR model delivers a realized CER that outperforms that obtainable without HF, for most models (as well as for the median one), it is not easy to generate economic value. As mentioned in section 5.4, there is a small literature that has indeed questioned the net contribution that funds of HFs may give to applied portfolio management and this seems to apply also in normative terms, when transaction costs and parameter uncertainty are both taken into account. However, similarly to FWC and RVR, also in the case of FFP strategies, any evidence of net positive economic value entirely derives from the superior, realized higher-moment properties of strategies that include HFs, as shown by the fact that for top models ranked by CER, the realized Sharpe ratios are generally disappointing in comparative terms (between 0.31 and 0.37). Visibly, the issue is that FFP strategies imply an increase in skewness (often positive), a reduction in the realized portfolio kurtosis but also relatively high volatility. An on-line Appendix provides further evidence on the economic value of HFs with references to additional strategies: Table A5 concerns RVR and therefore is directly comparable to Table 12. Similarly to DIS, RVR leads to a marked improvement in realized CERs. However, differently from DIS, this case appears to be less clear-cut as the strong CERs are also supported by relatively high Sharpe ratios, i.e., the good realized performance seems to occur across the board of the properties of potential interest. Table A6 concerns instead merger and acquisition specialized HF strategies that also deliver relatively good economic value whose origin is however harder to interprete as the result seems to be supported both by relatively high Sharpe ratios and (we must infer) by realized portfolio return/wealth moments of order higher than skewness or kurtosis.[43]

Figure 5 summarizes the results in this section by displaying best model-realized mean, skewness, kurtosis, Sharpe ratios, and especially CER for the benchmark in Table 14, FWC, FFP, and all other HF strategies obtained from the Bayesian portfolio OOS exercises, similarly to Figure 2. As already commented, essentially with the only exception of COA (that marks a consistent

[43] This claim exploits the fact that an investor maximizing power expected utility will take into account the entire predictive distribution of future wealth (portfolio returns) and such all of its moments and not only the first four. Complete tables concerning all HF strategies under Bayesian portfolio methods are available from the authors upon request.

reduction in Sharpe ratio), all strategies generate CERs that outperform the benchmark, even though their 90% confidence intervals always overlap to some extent and include the best CER for the traditional asset menu. Such incremental performance fails to originate from mean-variance improvements revealed by the realized Sharpe ratios which are always approximately identical to the 0.41 ratio obtained from a traditional asset menu; to the contrary, the evidence of economic value stems from an increase in skewness (which turns in a few cases positive, like for FWC, PPF, and COA) and some lower realized kurtosis. All in all, also because power utility is more strongly affected by skewness than by kurtosis, it turns out that the improvements in realized skewness (along with some minor increases in Sharpe ratios stemming from higher realized means but curbed by higher realized volatilities, see Figure A1 in the on-line Appendix) more than compensate cases in which kurtosis climbs up and this delivers the CER gains shown in the top panel.[44] These features of our empirical findings also emerge from the rightmost picture in Figure 4, which clearly shows that a few (4) HF strategies bring the optimal model performances within the north-western, golden region in which both the Sharpe ratio and the CER are no less than under the benchmark, with a few additional styles delivering either the same Sharpe ratio as the traditional assets but a higher CER, or the same CER but slightly higher Sharpe ratios. All in all, this is evidence hard to dismiss that—especially when parameter uncertainty is taken into account—HF strategies may significantly enrich classical stock-bond-real estate asset menus and generate economic value.

5.5 *The effect of transaction costs*

To save space and also because the presence of transaction costs represents the (only) realistic case, in this section we summarize a few remarks concerning the role played by transaction costs in our analysis but direct the interested Reader to the on-line Appendix for the tabulated results. For the case of $\gamma = 5$, Table A11 reports on the same OOS portfolio benchmark results as Table 4 when transaction costs are ignored. It is clear that the absence of transaction costs makes the

[44] Using Taylor, polynomial expansions of expected power utility, it is easy to show that skewness will always dominate over kurtosis, see, e.g., Guidolin and Timmermann (2008).

positions in the different asset classes—even their simple averages over time—more extreme and at the same time more volatile, as one would expect. REITs and stocks remain dominant in the best performing predictive models, now joined by corporate bonds. Yet, an investor would have been advised, over time, to take massive short positions in cash and Treasury notes, to finance long positions in stocks and real estate. With reference to realized OOS performances, Table A12 reveals that in the absence of transaction costs, realized performances under a traditional asset menu are only slightly superior (e.g., the CER of the best performing model is 9.99% vs. 9.41% and Sharpe ratio is 0.44 vs. 0.41 when transaction costs are accounted for). However, these results turn strongly in favor of accounting for transaction costs and benefitting from their stabilizing effects when one focusses on the performances of the median model.

Finally, in Tables A13-A15, results turn decidedly in favor of the economic value of HF strategies—even the parameter uncertainty that characterizes them is ignored—when transaction costs go unaccounted for. For instance, at least eight predictive models may generate positive economic value in terms of additional, positive realized CER when the composite, value-weighted HFRI index is considered. Such incremental value estimates are even larger in the case of FFP (see Table A14) and extend to a variety of HF styles, including RVR (see Table A15).

6 Discussion and conclusions

We report systematic, out-of-sample evidence of the potential economic benefits of diversifying into various HF strategies accruing to a long-term, risk-averse investor, who is already well-diversified across stocks, REITs, and government and corporate debt. We have obtained the optimal weights and consumption rules using the dynamic programming, simulation-based methods in Barberis (2000) while adjusting the underlying VAR estimates for small-sample biases following Engsted and Pedersen (2012). In a range of recursive OOS experiments, we have estimated the CERs for a number of (simple but widely used) statistical models that can capture predictability in the risk premia of the asset classes under investigation to assess which HF strategies, if any, lead to an improved realized OOS utility relative to a baseline asset menu. To guard against the ill-effects of parameter uncertainty, we have also computed optimal Bayesian portfolios and for added realism, our baseline exercises have been carried out imposing transaction costs on an ex-ante basis, as integral part of our backward solution algorithm.

Especially with reference to intermediate and high risk aversion (γ = 5, 10) and to aggregate indices of HF strategies (FWC) and fund-of-funds excess returns, we find that on average the optimal portfolios tend to be skewed towards public real estate and, to a lesser extent, stocks. When HF are part of the asset menu, their demand is modest but never zero, even though this grows considerably for more aggressive investors (γ = 2) when parameter uncertainty is taken into account. Probably because we explicitly account for transaction costs (as well as the fact that trading HF strategies may be costly because they are hard to replicate and this occurs over-the-counter, see the discussion in Hamza et al., 2006), leverage seldom appears and portfolio turnover tends to be moderate. The small-sample bias correction has a sizable effect on total and hedging demands as well as on the speed of mean-reversion implied by the estimated VAR models. Hedging demands for HFs tend to be positive due to their negative positive first-order serial correlation in returns. This effect is strengthened when hedging parameter uncertainty is allowed, in the case of Bayesian optimal portfolios.

Our OOS experiments within the baseline asset space show that an investor who chooses to follow a simple buy-and-hold strategy achieves, on average, lower CERs than an investor who rebalances on a monthly basis. In the case of the extended menu, not all HF strategies have the potential to benefit long-term investors. Only strategies whose payoffs are highly nonlinear (relative value, merger arbitrage, and distressed restructuring), and therefore not easily replicable (by going long or short in the original asset classes) yield the highest utility gains. Our key results are robust across different values of the coefficient of relative risk aversion, although the largest benefits from diversifying into HFs accrue to medium and low risk-aversion investors, as one may expect when dealing with HF. Interestingly, the key findings on the value of HFs stem *not* from the ability of hedge strategies to increase realized mean returns, lower volatility, and therefore improve realized Sharpe ratios (as often debated by financial commentators), but from the ability of hedge strategies—when combined within well-diversified portfolios of stocks, bonds, and REITs—to improve the higher-order moments of optimal portfolios (i.e., higher skewness and lower excess kurtosis). However, a portion of the findings that turn out to be encouraging for an assessment of HF performance critically hinges on the assumption that investors can accurately detect the best performing model for predictable risk premia. Our own assessment of the state-of-the-art in the empirical finance literature tends to be lukewarm at best with regard to such a discovery process occurring effectively (see the review in Rapach and Zhou, 2013).[45] When we assess the OOS performance of the median model of predictable returns (or lack thereof), we find that only specific HF strategies may still generate economic value, while composite value-weighted portfolio or strategies (as well as funds-of-funds) fail to do so.

Our findings should encourage further analyses along several dimensions. First, distinguishing between bull and bear regimes may generate optimal portfolios which yield superior performances relative to simple VARs (see, e.g., Guidolin and Hyde, 2012; Tu, 2010). For instance, Avramov et al. (2011) note that in times of crisis, some HF strategies (e.g., global macro) perform better than others (e.g., equity long/short). The widespread evidence of regimes in investment opportunities may affect our results. Second, the precision of our estimates, and therefore the quality of our forecasts, might be further improved if the exposures to the state variables were allowed to be time-varying in the spirit of Bollen and Whaley (2009). Finally, along the lines of recent work by Panopoulou and Vrontos (2015), given the long set of candidate predictors suggested by the literature, we could construct improved HF fund return predictions by carefully integrating the information content through combinations of forecasts.

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[45] Recent empirical work by Joenvaara, Kosowski and Tolonen (2014) induces us to maintain a healthy degree of skepticism: when they account for the investment constraints faced by real-world HF investors, they report a reduction in average performance and in performance persistence (see also Kumar, 2015).

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Table 1

Summary Statistics for Asset Returns and Predictor Variables of the Dataset

The table presents summary statistics for monthly returns on stocks, bonds, publicly traded real estated, and HFR hedge fund strategy returns. The sample is January 1994 - December 2019. We use four predictors to model the time variation in investment opportunities, the dividend yield, the short-term riskless interest rate proxied by the 3-month Treasury constant maturity rate, the term spread calculated as the difference between the 10-year Treasury constant maturity rate and the corresponding 3-month rate, and the default spread computed as the yield differential between Moody's seasoned Baa and Aaa corporate bond portfolio rates. In the case of hedge funds, we use HFRI style indices distributed by Hedge Fund Research (HFR).

	Mean	Median	Std.Dev.	Uncond. Sharpe ratio	Min,	Max.	Skewness	Kurtosis	JB Test
Panel A: Initial Asset Menu									
30-day T-bill	0.223	0.210	0.183	0.000	0.000	0.560	0.041	1.393	27.20**
Excess stock return	0.532	1.320	4.496	0.118	-18.894	10.751	-0.932	4.728	67.83**
Excess long-term govt. bond returns	0.278	0.286	2.031	0.137	-6.752	8.508	0.062	4.116	13.23**
Excess long-term corporate bonds returns	0.385	0.554	2.621	0.147	-11.974	13.206	-0.270	7.422	208.39**
Excess real estate returns	0.827	1.215	5.721	0.145	-31.748	31.010	-0.757	11.157	722.76**
Default_spread (Baa-Aaa rate)	0.964	0.860	0.442	-	0.550	3.380	2.979	14.158	1680.11**
Riskless Term spread (10y-3m)	1.743	1.845	1.168	-	-0.700	3.690	-0.191	1.901	14.21**
Short-term nominal rate (3m)	2.763	2.810	2.236	-	0.010	6.360	0.013	1.334	29.14**
Div_yield	2.005	2.013	0.458	-	1.002	3.161	-0.255	2.459	5.8
Panel B: Hedge Funds									
HFRI Fund Weighted Hedge Fund excess return	0.460	0.608	1.966	0.234	-9.532	6.931	-0.823	6.314	143.77**
HFRI Fund Of Funds Composite excess return	0.209	0.397	1.655	0.126	-8.194	6.186	-0.945	7.526	252.61**
HFRI Equity Hedge excess return	0.561	0.686	2.574	0.218	-10.016	9.888	-0.413	5.263	60.93**
HFRI Event- Driven excess return	0.562	0.888	1.902	0.296	-9.751	4.634	-1.427	8.174	366.62**
HFRI Macro excess return	0.379	0.342	1.806	0.210	-6.824	6.158	0.072	4.223	15.93**
HFRI Relative Value excess return	0.433	0.574	1.203	0.360	-8.452	3.858	-3.015	21.163	3845.63**
HFRI EH Equity Market Neutral excess return	0.229	0.275	0.850	0.269	-3.020	3.210	-0.461	5.179	58.78**
HFRI ED Merger Arbitrage excess return	0.381	0.539	0.972	0.392	-6.288	2.718	-1.955	12.070	1024.40**
HFRI ED Distressed/Restructuring excess return	0.527	0.710	1.773	0.297	-8.930	5.550	-1.437	8.526	407.45**
HFRI RV Fixed Inc.- Conv.Arb. excess return	0.401	0.630	1.982	0.202	-16.090	9.740	-2.720	28.745	7270.36**
CBOE S&P 500 Buywrite Index	0.679	1.181	3.147	-	-15.131	10.015	-1.244	7.327	261.52**
Ptfs Currency Lookback Straddle	-0.848	-5.220	19.252	-	-30.130	90.270	1.331	5.508	140.50**
Ptfs Commodity Lookback Straddle	-0.237	-2.895	14.210	-	-24.650	64.750	1.093	4.728	81.50**
Ptfs Bond Lookback Straddle	-1.559	-3.900	15.258	-	-26.630	68.860	1.361	5.477	142.21**
Ptfs Short Term Interest Rate Lookback Straddle	-0.386	-5.615	26.022	-	-34.640	221.920	4.274	30.425	8664.24**
SMB	0.120	-0.100	3.402	-	-18.272	20.147	0.388	10.422	584.74**
HML	0.156	0.120	3.229	-	-14.053	13.024	-0.228	6.311	117.32**
Momentum	0.300	0.553	5.460	-	-42.434	16.873	-2.562	20.564	3514.80**

* Significance at 5%

** Significance at 1%

Table 2

Correlation Matrix

The table presents estimated correlations and reported significance levels for monthly returns on stocks, bonds, publicly traded real estated, and HFR hedge fund strategy returns. The sample is January 1994 - December 2019. The estimated pairwise correlations also involve the four predictors described in Table 1. The acronyms used in the tables are listed at the bottom of three panels.

Panel A: Initial Asset Menu

	1-month bill	Stocks	Gov.	Corp.	REITS	DY	Def.	Term.	Short
Stocks	-	1	-0.175**	0.242**	0.572**	0.155*	-0.119	0.001	-0.023
Gov.	-	-	1	0.673**	-0.024	-0.013	0.061	-0.002	-0.044
Corp.	-	-	-	1	0.335**	0.012	0.101	0.07	-0.103
REITS	-	-	-	-	1	0.028	-0.092	0.051	-0.062
DY	-	-	-	-	-	1	-0.101	0.254**	-0.244**
Def.	-	-	-	-	-	-	1	0.327**	-0.487**
Term.	-	-	-	-	-	-	-	1	-0.769**
Short	-	-	-	-	-	-	-	-	1

* Significance at 5%

** Significance at 1%

Panel B: Initial Asset Menu vs. Hedge Funds

	FWC	FOF	EQH	EVD	MAC	REL	EMN	MEA	DIS	COA	BMX	PtfsFX	PtfsCM	PtfsBD	PtfsIR	SMB	HML	Mom.
Stocks	0.823**	0.667**	0.831**	0.780**	0.359**	0.612**	0.323**	0.608**	0.631**	0.503**	0.870**	-0.198**	-0.175**	-0.253**	-0.285**	0.248**	-0.222**	-0.267**
Gov.	-0.194**	-0.154**	-0.207**	-0.237**	0.152**	-0.147**	-0.075	-0.147**	-0.246**	-0.099	-0.174**	0.118*	0.081	0.233**	0.049	-0.195**	0.043	0.158**
Corp.	0.248**	0.244**	0.216**	0.248**	0.252**	0.358**	0.055	0.222**	0.221**	0.408**	0.254**	-0.091	-0.04	0.049	-0.229**	0	0.028	-0.128*
REITS	0.445**	0.339**	0.450**	0.491**	0.128*	0.485**	0.160**	0.397**	0.454**	0.420**	0.555**	-0.143*	-0.152**	-0.146*	-0.176**	0.257**	0.275**	-0.345**
DY	0.082	0.052	0.068	0.114*	-0.004	0.083	0.056	0.098	0.111*	0.003	0.066	-0.079	-0.006	-0.069	-0.103	-0.044	-0.072	-0.025
Def.	-0.093	-0.135*	-0.124*	-0.153**	-0.006	-0.075	-0.234**	-0.082	-0.160**	0.062	-0.131*	0.046	-0.035	0.028	0.121*	0.064	-0.096	-0.227**
Term.	-0.023	-0.04	-0.082	-0.015	-0.035	-0.014	-0.147**	-0.149**	0.103	-0.019	-0.074	-0.028	-0.018	-0.019	-0.099	0.118*	-0.036	-0.07
Short	0.047	0.034	0.1	0.045	0.043	-0.002	0.173**	0.175**	-0.046	-0.009	0.074	0.048	-0.017	0.034	0.133*	-0.088	0.044	0.116*

* Significance at 5%

** Significance at 1%

Table 2 (continued)

Correlation Matrix

Panel C: Hedge Funds

	FWC	FOF	EQH	EVD	MAC	REL	EMN	MEA	DIS	COA	BMX	PtfsFX	PtfsCM	PtfsBD	PtfsIR	SMB	HML	Mom.
FWC	1	0.925**	0.965**	0.916**	0.621**	0.759**	0.476**	0.703**	0.804**	0.634**	0.676**	-0.133*	-0.141*	-0.261**	-0.354**	0.463**	-0.327**	-0.110*
FOF	-	1	0.867**	0.851**	0.696**	0.753**	0.501**	0.629**	0.801**	0.633**	0.529**	-0.094	-0.085	-0.275**	-0.404**	0.396**	-0.261**	0.037
EQH	-	-	1	0.875**	0.537**	0.724**	0.505**	0.675**	0.750**	0.625**	0.694**	-0.152**	-0.153**	-0.240**	-0.354**	0.480**	-0.327**	-0.08
EVD	-	-	-	1	0.494**	0.817**	0.468**	0.772**	0.885**	0.685**	0.684**	-0.195**	-0.215**	-0.329**	-0.386**	0.429**	-0.145*	-0.202**
MAC	-	-	-	-	1	0.301**	0.323**	0.332**	0.405**	0.233**	0.229**	0.196**	0.181**	-0.033	-0.093	0.245**	-0.207**	0.120*
REL	-	-	-	-	-	1	0.436**	0.673**	0.829**	0.875**	0.590**	-0.291**	-0.253**	-0.356**	-0.448**	0.239**	-0.038	-0.205**
EMN	-	-	-	-	-	-	1	0.415**	0.442**	0.347**	0.256**	0.007	-0.079	-0.235**	-0.205**	0.144*	0.035	0.349**
MEA	-	-	-	-	-	-	-	1	0.600**	0.534**	0.620**	-0.107*	-0.179**	-0.226**	-0.318**	0.290**	-0.075	-0.143*
DIS	-	-	-	-	-	-	-	-	1	0.710**	0.548**	-0.211**	-0.222**	-0.416**	-0.382**	0.358**	-0.039	-0.160**
COA	-	-	-	-	-	-	-	-	-	1	0.486**	-0.251**	-0.233**	-0.245**	-0.413**	0.175**	-0.01	-0.265**
BMX	-	-	-	-	-	-	-	-	-	-	1	-0.198**	-0.176**	-0.235**	-0.306**	0.126*	-0.085	-0.273**
PtfsFX	-	-	-	-	-	-	-	-	-	-	-	1	0.353**	0.270**	0.256**	-0.017	0.008	0.119*
PtfsCM	-	-	-	-	-	-	-	-	-	-	-	-	1	0.190**	0.228**	-0.071	-0.032	0.189**
PtfsBD	-	-	-	-	-	-	-	-	-	-	-	-	-	1	0.211**	-0.077	-0.075	0.019
PtfsIR	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	-0.107*	-0.005	0
SMB	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	-0.360**	0.054
HML	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	-0.153**
Mom.	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1

Legend: DY = dividend yield; Def. = default spread; Term = riskless term spread; Short = 3-month nominal interest rate; FWC = returns on the HFRI Fund Weighted Composite Index; FFP = returns on the HFRI Fund of Funds Composite Index; EQH = returns on the HFRI Equity Hedge Index; EVD = returns on the HFRI Event Driven Index; MAC = returns on the HFRI Macro Index; RVR = returns on the HFRI Relative Value Index; EMN = returns on the HFRI Equity Market Neutral Index; MEA = returns on the HFRI Merger Arbitrage Index; DSE = returns on the HFRI Distressed/Restructuring Index; COA = returns on the HFRI RV Fixed Income Convertible Arbitrage; BMX = returns on the HFRI CBOE S&P 500 BuyWrite Index; PtfsFX = returns on a portfolio of lookback straddles on currency; PtfsCom = returns on a portfolio of lookback straddles on commodities; Ptfs BD = returns on a portfolio of lookback straddles on bonds; PtfsIR = returns on a portfolio of lookback straddles on interest rates; SMB = = returns on the Fama-French small-minus-big long-short portfolio, HML = returns on the Fama-Frenc high-minus-low book-to-market portfolio; Mom. = = returns on a past stock winners-losers momentum strategy.

Table 3

Full Sample (1994:01–2019:12) Estimates of Full VAR(1): Baseline Asset Menu

The table presents full-sample OLS estimate of a VAR(1) model

$$\boldsymbol{z}_{t+1} = \boldsymbol{\Phi}_0 + \boldsymbol{\Phi}_1 \boldsymbol{z}_t + \boldsymbol{v}_{t+1},$$

where $\boldsymbol{z}_{t+1}$ collects the short rate, benchmark excess returns, and the predictors, $\boldsymbol{\Phi}_0$ is the $(n+m)$ vector of intercepts, $\boldsymbol{\Phi}_1$ is the $(n+m)$x$(n+m)$ coefficient matrix, and $\boldsymbol{v}_{t+1} \overset{i.i.d.}{\sim} \mathcal{N}(\boldsymbol{0}, \sum_{\boldsymbol{v}})$. We take into account the instability in parameters and adjust the estimates for small-sample bias as in Engsted and Pedersen (2012), by using the formula $\widehat{\boldsymbol{\Phi}}_1$ as $\boldsymbol{Bias}_T = -\frac{\boldsymbol{b}}{T} + O\left(T^{-\frac{3}{2}}\right)$.

Dependent variable	Rf_t	$Stocks_t$	Gov $Treas_t$	$Corp_t$	$REITs_t$	Def_t	$Term_t$	Short $Rate_t$	DY_t	R^2
Rf_{t+1} - adj.	-0.068	0.000	-0.001	0.001	0.000	0.000	-0.005	0.083	-0.003	
Not adjusted	-0.082	0.000	-0.001	0.001	0.000	0.000	-0.005	0.084	-0.003	0.988
t-stat	(-1.29)	(-0.84)	(-1.04)	(0.67)	(0.93)	(0.04)	(-3.49)	(15.80)	(-0.75)	
$Stocks_{t+1}$ - adj.	-3.122	0.087	-0.033	0.191	-0.026	-1.905	-0.558	-0.203	0.917	
Not adjusted	-2.379	0.085	-0.036	0.189	-0.025	-1.987	-0.664	-0.215	1.890	0.057
t-stat	(-0.18)	(0.87)	(-0.11)	(0.60)	(-0.30)	(-1.80)	(-2.01)	(-0.20)	(2.77)	
Gov $Treas_{t+1}$ - adj.	-1.427	-0.079	-0.013	0.078	-0.063	-0.169	0.196	0.070	-0.210	
Not adjusted	-1.538	-0.080	-0.019	0.077	-0.062	-0.075	0.354	0.215	-0.262	0.096
t-stat	(-0.35)	(-2.20)	(-0.20)	(0.83)	(-1.75)	(-0.17)	(2.07)	(0.61)	(-0.83)	
$Corp_{t+1}$ - adj.	0.740	0.035	0.021	0.139	-0.079	0.432	0.319	-0.070	-0.317	
Not adjusted	0.653	0.037	0.030	0.121	-0.077	0.666	0.497	0.092	-0.151	0.064
t-stat	(0.12)	(0.80)	(0.16)	(0.63)	(-1.26)	(0.89)	(2.21)	(0.22)	(-0.41)	
$REITs_{t+1}$ - adj.	-12.826	0.257	0.186	0.339	-0.096	-1.496	0.407	0.847	-0.979	
Not adjusted	-12.550	0.270	0.185	0.343	-0.111	-1.432	0.365	0.904	-0.181	0.094
t-stat	(-0.93)	(1.87)	(0.33)	(0.61)	(-0.78)	(-0.66)	(0.87)	(0.83)	(-0.25)	
Def_{t+1} - adj.	0.389	-0.001	0.035	-0.031	-0.002	0.975	0.002	-0.033	-0.002	
Not adjusted	0.393	-0.001	0.035	-0.031	-0.002	0.961	0.001	-0.034	-0.004	0.959
t-stat	(1.66)	(-0.79)	(3.98)	(-3.30)	(-0.77)	(39.76)	(0.10)	(-1.92)	(-0.29)	
$Term_{t+1}$ - adj.	2.227	0.000	-0.024	-0.025	0.006	0.064	0.965	-0.190	0.005	
Not adjusted	2.186	0.000	-0.024	-0.025	0.006	0.063	0.945	-0.196	0.009	0.968
t-stat	(3.97)	(0.05)	(-2.04)	(-2.92)	(1.58)	(1.54)	(52.78)	(-4.51)	(0.29)	
Short $Rate_{t+1}$ - adj.	-2.128	0.001	-0.035	0.016	0.001	-0.058	0.021	1.179	0.036	
Not adjusted	-2.076	0.001	-0.034	0.016	0.001	-0.064	0.027	1.172	0.039	0.994
t-stat	(-4.08)	(0.32)	(-3.59)	(2.24)	(0.54)	(-2.38)	(1.94)	(29.82)	(1.74)	
DY_{t+1} - adj.	0.077	-0.002	-0.001	-0.004	0.000	0.017	0.003	-0.005	0.984	
Not adjusted	0.061	-0.002	-0.001	-0.004	0.000	0.018	0.005	-0.005	0.962	0.962
t-stat	(0.28)	(-1.04)	(-0.09)	(-0.51)	(0.07)	(0.55)	(0.80)	(-0.28)	(64.24)	

Correlation of residuals (bias-adjusted coefficients)

	Rf	Stock	Gov. Treas	Corp.	REITS	Def.	Term.	Short Rate	DY
Rf	1	-0.021	0.042	0.019	0.065	0.072	0.014	-0.017	0.013
Stock	-	1	-0.161	0.261	0.585	-0.222	0.062	0.072	-0.922
Gov. Treas	-	-	1	0.686	0.016	0.079	-0.547	-0.169	0.091
Corp.	-	-	-	1	0.353	-0.171	-0.432	0.034	-0.320
REITS	-	-	-	-	1	-0.233	-0.076	0.028	-0.670
Def.	-	-	-	-	-	1	-0.062	-0.075	0.245
Term.	-	-	-	-	-	-	1	-0.560	-0.054
Short Rate	-	-	-	-	-	-	-	1	-0.047
DY	-	-	-	-	-	-	-	-	1

Table 4

Summary Statistics for Monthly Realized, Recursively Rebalanced Optimal Portfolio Weights: Baseline Asset Menu (γ = 5)

The table shows sample means, standard deviations, and lower and upper bounds of the 90% sample range of recursive portfolio weights computed from the ten best performing among all VAR models for risk premia and all constant investment opportunities (IID) models. The table presents statistics for 1-month vs. long-term weights, and for their differences, the hedging demands. Transactions costs are accounted for.

CER rank	Model	Lags	Predictors included				Cash			Stocks			US Long-Term Treasuries			US Corporate Bonds			REITs		
			Default	Term	Short	DY	T=1	Long	Hedging	T=1	Long	Hedging	T=1	Long	Hedging	T=1	Long	Hedging	T=1	Long	Hedging
													Sample mean of portofolio weights								
1	Exp. Gaussian IID	_	_	_	_	_	0.000	0.011	0.011	0.033	0.028	-0.006	0.000	0.011	0.011	0.017	0.019	0.003	0.950	0.931	-0.019
2	Rolling Gaussian IID	_	_	_	_	_	0.000	0.010	0.010	0.075	0.102	0.027	0.025	0.018	-0.007	0.000	0.010	0.010	0.900	0.860	-0.040
3	Rolling VAR	1	Y	Y	Y	N	0.000	0.009	0.009	0.017	0.018	0.001	0.000	0.009	0.009	0.017	0.017	0.000	0.966	0.948	-0.018
4	Rolling VAR	1	Y	N	N	N	0.000	0.010	0.010	0.017	0.010	-0.007	0.000	0.010	0.010	0.017	0.018	0.002	0.967	0.952	-0.015
5	Rolling VAR	1	Y	Y	Y	Y	0.000	0.009	0.009	0.033	0.026	-0.007	0.000	0.009	0.009	0.017	0.017	0.001	0.950	0.939	-0.012
6	Rolling VAR	1	Y	Y	N	Y	-1.245	-1.645	-0.400	0.980	1.689	0.709	-0.111	-0.881	-0.770	-0.236	0.177	0.413	1.612	1.660	0.048
7	Rolling VAR	1	Y	N	Y	N	-1.559	-1.655	-0.096	0.778	1.389	0.611	-0.295	-1.488	-1.193	0.394	1.081	0.688	1.682	1.672	-0.010
8	Expanding VAR	1	Y	N	N	Y	0.008	0.018	0.010	0.075	0.093	0.018	0.008	0.010	0.002	0.000	0.010	0.010	0.908	0.868	-0.040
9	Rolling VAR	1	Y	N	N	N	-1.492	-1.686	-0.194	0.435	1.012	0.577	-0.023	-1.213	-1.190	0.359	1.185	0.826	1.722	1.703	-0.019
10	Expanding VAR	2	Y	N	N	Y	-1.339	-1.643	-0.304	0.612	1.679	1.067	-0.059	-1.519	-1.460	0.158	0.827	0.669	1.629	1.657	0.027
													Sample Standard Deviation of Portofolio Weights								
1	Exp. Gaussian IID	_	_	_	_	_	0.000	0.045	0.031	0.180	0.135	0.050	0.000	0.045	0.000	0.129	0.101	0.005	0.219	0.234	0.087
2	Rolling Gaussian IID	_	_	_	_	_	0.000	0.044	0.012	0.264	0.290	0.112	0.156	0.100	0.041	0.001	0.044	0.019	0.301	0.337	0.003
3	Rolling VAR	1	Y	Y	Y	N	0.000	0.039	0.039	0.129	0.099	0.142	0.000	0.039	0.039	0.129	0.098	0.100	0.180	0.200	0.179
4	Rolling VAR	1	Y	N	N	N	0.000	0.044	0.044	0.129	0.044	0.110	0.000	0.044	0.044	0.129	0.100	0.102	0.180	0.196	0.174
5	Rolling VAR	1	Y	Y	Y	Y	0.000	0.040	0.040	0.177	0.134	0.175	0.000	0.040	0.040	0.129	0.099	0.100	0.216	0.220	0.233
6	Rolling VAR	1	Y	Y	N	Y	0.769	0.594	0.486	0.683	0.475	0.810	0.544	1.187	1.274	0.596	1.122	0.966	0.635	0.572	0.589
7	Rolling VAR	1	Y	N	Y	N	0.496	0.526	0.526	0.846	0.845	0.821	0.655	0.763	1.006	0.602	0.443	0.518	0.559	0.489	0.594
8	Expanding VAR	1	Y	N	N	Y	0.091	0.100	0.044	0.264	0.278	0.164	0.091	0.044	0.102	0.000	0.044	0.044	0.290	0.327	0.253
9	Rolling VAR	1	Y	N	N	N	0.465	0.456	0.572	0.729	1.264	1.068	0.422	1.071	1.097	0.599	0.511	0.423	0.363	0.394	0.421
10	Expanding VAR	2	Y	N	N	Y	0.639	0.661	0.580	0.643	0.543	0.647	0.454	0.819	0.823	0.403	0.587	0.741	0.577	0.626	0.347
													Empirical 90% Range								
1	Exp. Gaussian IID	_	_	_	_	_	0.000	0.164	0.164	0.000	0.200	0.200	0.000	0.164	0.164	0.000	0.195	0.164	0.505	0.800	0.002
2	Rolling Gaussian IID	_	_	_	_	_	0.000	0.100	0.100	1.000	1.000	0.000	0.000	0.200	0.100	0.000	0.100	0.100	1.000	1.000	0.000
3	Rolling VAR	1	Y	Y	Y	N	0.000	0.058	0.058	0.000	0.150	0.000	0.000	0.058	0.058	0.000	0.117	0.057	0.000	0.660	0.000
4	Rolling VAR	1	Y	N	N	N	0.000	0.100	0.100	0.000	0.101	0.000	0.000	0.100	0.100	0.000	0.200	0.100	0.000	0.800	0.001
5	Rolling VAR	1	Y	Y	Y	Y	0.000	0.068	0.068	0.001	0.177	0.070	0.000	0.068	0.068	0.000	0.144	0.068	0.468	0.800	0.274
6	Rolling VAR	1	Y	Y	N	Y	2.793	2.000	1.195	1.800	1.600	2.601	1.298	2.800	3.660	2.040	2.800	2.754	1.360	1.600	1.091
7	Rolling VAR	1	Y	N	Y	N	1.029	2.000	1.002	2.067	2.373	3.399	1.800	2.000	2.805	1.784	1.596	1.069	0.044	1.600	1.446
8	Expanding VAR	1	Y	N	N	Y	0.000	0.200	0.100	1.000	1.000	0.200	0.000	0.100	0.100	0.000	0.100	0.100	1.000	1.000	0.800
9	Rolling VAR	1	Y	N	N	N	1.018	1.767	1.030	2.423	3.600	3.571	0.571	2.800	2.800	1.572	1.600	1.140	0.073	1.600	0.873
10	Expanding VAR	2	Y	N	N	Y	1.225	2.000	1.046	1.588	1.200	1.681	0.642	2.399	2.298	1.000	1.800	2.314	0.259	1.600	0.199

Table 5

Top 10 Models Ranked According to Realized CER for Buy-and-Hold and Monthly Rebalancing Strategies: Baseline Asset Menu (γ = 5)

The tables shows annualized sample means, standard deviations, the Sharpe ratio, and the Certainty Equivalent Return computed from the recursive portfolio weights under the ten top performing (in terms of CER) models for (predictable) risk premia, as defined by the VAR order (where a VAR(0) is equivalent to IID, constant investment opportunities), the selection of predictors, and whether the models are estimated either on an expanding or on a rolling 10-year window of data. The calculations are performed assuming the investor assess realized performance with a 5-year horizon (H = 60 months). Transactions costs are accounted for on an ex-ante basis.

CER rank	Model	Lags	Predictors included				H	Annualized mean (%)			Annualized volatility (%)			Annualized Sharpe ratio			Annualized CER (%)			Skewness	Kurtosis
			Default	Term.	Short	DY		Mean returns	90% Conf. Int. - LB	90% Conf. Int. - UB	Volatility	90% Conf. Int. - LB	90% Conf. Int. - UB	Sharpe ratio	90% Conf. Int. - LB	90% Conf. Int. - UB	CER (% Ann.)	90% Conf. Int. - LB	90% Conf. Int. - UB		
Buy-and-hold																					
1	Expanding VAR	1	N	N	Y	Y	60	12.068	12.039	12.096	30.203	29.243	31.183	0.300	0.105	0.496	0.478	-0.534	1.250	-0.585	6.671
2	Expanding VAR	1	Y	N	Y	Y	60	11.848	11.671	11.727	29.631	28.668	30.678	0.299	0.106	0.492	0.475	-0.533	1.246	-0.616	6.904
3	Exp. Gaussian IID	_	_	_	_	_	60	12.340	12.265	12.323	30.221	29.247	31.236	0.309	0.116	0.502	0.472	-0.509	1.254	-0.608	6.748
4	Rolling Gaussian IID	_	_	_	_	_	60	12.555	12.121	12.178	29.665	28.690	30.658	0.322	0.128	0.516	0.446	-0.553	1.229	-0.603	6.827
5	Expanding VAR	1	Y	Y	Y	Y	60	11.229	11.612	11.668	29.633	28.674	30.647	0.278	0.085	0.470	0.433	-0.565	1.183	-0.614	6.897
6	Expanding VAR	2	Y	Y	Y	Y	60	12.369	12.410	12.468	30.656	29.716	31.694	0.306	0.110	0.501	0.425	-0.573	1.197	-0.595	6.650
7	Rolling VAR	2	N	N	Y	Y	60	12.088	12.372	12.431	30.469	29.517	31.498	0.298	0.107	0.489	0.403	-0.581	1.197	-0.570	6.610
8	Rolling VAR	2	N	N	N	Y	60	12.683	12.404	12.462	30.469	29.506	31.474	0.318	0.127	0.509	0.403	-0.551	1.201	-0.570	6.610
9	Rolling VAR	1	Y	N	Y	N	60	11.738	12.149	12.207	30.629	29.654	31.627	0.285	0.097	0.474	0.380	-0.877	1.180	-0.609	6.639
10	Expanding VAR	2	N	Y	N	Y	60	12.106	11.427	11.485	30.092	29.156	31.127	0.303	0.111	0.494	0.374	-0.615	1.145	-0.572	6.698
		Median Expanding VAR performance					60	12.481	11.947	12.007	31.363	30.405	32.390	0.302	0.110	0.495	0.245	-1.196	1.165	-0.623	6.399
		Median Rolling VAR performance					60	13.444	12.826	12.884	30.708	29.722	31.703	0.340	0.148	0.532	0.268	-1.024	1.172	-0.613	6.589
Montlhy rebalancing																					
1	Exp. Gaussian IID	_	_	_	_	_	60	12.132	12.113	12.632	20.908	20.438	21.379	0.413	0.302	0.523	9.413	8.271	10.295	-0.658	3.714
2	Rolling Gaussian IID	_	_	_	_	_	60	12.247	12.055	12.725	20.835	20.372	21.298	0.420	0.311	0.529	6.885	6.062	7.632	-0.549	3.620
3	Rolling VAR	1	Y	Y	Y	N	60	12.039	11.744	12.268	24.760	23.842	25.678	0.345	0.227	0.462	6.564	5.283	7.489	-1.375	8.353
4	Rolling VAR	1	Y	N	N	N	60	11.732	11.176	11.984	23.819	22.882	24.755	0.346	0.228	0.464	6.380	5.442	7.154	-1.706	9.198
5	Rolling VAR	1	Y	Y	Y	Y	60	11.348	11.216	11.551	24.288	23.369	25.207	0.323	0.207	0.439	5.862	4.881	6.649	-1.531	8.640
6	Rolling VAR	1	Y	Y	N	Y	60	13.070	12.921	14.139	20.177	19.702	20.652	0.474	0.363	0.586	5.725	5.068	6.380	-0.616	3.973
7	Rolling VAR	1	Y	N	Y	N	60	11.888	11.386	12.317	23.847	22.920	24.774	0.352	0.235	0.469	5.716	5.058	6.378	-1.653	9.044
8	Expanding VAR	1	Y	N	N	Y	60	11.522	11.248	12.267	20.180	19.704	20.655	0.398	0.287	0.508	5.636	4.936	6.285	-0.616	3.975
9	Rolling VAR	1	Y	N	N	N	60	12.868	12.122	13.145	23.951	23.015	24.886	0.391	0.275	0.507	5.477	4.659	6.185	-1.648	8.960
10	Expanding VAR	2	Y	N	N	Y	60	13.589	13.519	13.542	23.613	22.655	24.571	0.427	0.310	0.545	5.470	4.512	6.287	-1.701	9.414
		Median Expanding VAR performance					60	12.495	11.964	12.879	24.747	23.817	25.678	0.363	0.250	0.476	3.146	2.468	3.765	-1.614	8.353
		Median Rolling VAR performance					60	11.993	11.682	12.044	23.951	23.015	24.886	0.355	0.242	0.467	2.622	1.791	3.436	-1.641	7.769

Table 6

Summary Statistics for Monthly Realized, Recursively Rebalanced Optimal Portfolio Weights: Asset Menu Including HFRI Fund Weighted Composite Index (FWC) ($\gamma = 5$)

The table shows sample means, standard deviations, and lower and upper bounds of the 90% sample range of recursive portfolio weights computed from the ten best performing among all VAR models for risk premia and all constant investment opportunities (IID) models. The table presents statistics for 1-month vs. long-term weights, and for their differences, the hedging demands. Transactions costs are accounted for.

CER rank	Model	Lags	Predictors included								Cash			Stock			US Long-Term Treasuries			US Corporate			REITs			FWC		
			Term	Short	DY	Default	SMB	PtfsBD	PtfsIR	COM	T=1	Long	Hedging	T=1	Long	Hedging	T=1	Long	Hedging	T=1	Long	Hedging	T=1	Long	Hedging	T=1	Long	Hedging
			Sample mean of portofolio weights																									
1	Expanding VAR	1	N	Y	N	Y	N	N	N	N	0.000	0.008	0.008	0.092	0.061	-0.030	0.096	0.045	-0.052	0.017	0.016	0.000	0.793	0.843	0.050	0.002	0.027	0.025
2	Expanding VAR	1	N	Y	N	Y	N	Y	N	Y	0.000	0.008	0.007	0.098	0.075	-0.023	0.047	0.028	-0.019	0.017	0.016	-0.001	0.838	0.849	0.011	0.000	0.025	0.025
3	Expanding VAR	1	N	Y	N	Y	N	Y	Y	Y	0.000	0.008	0.008	0.050	0.079	0.029	0.052	0.033	-0.019	0.017	0.008	-0.009	0.864	0.824	-0.040	0.016	0.047	0.031
4	Expanding VAR	1	N	Y	N	Y	N	N	Y	N	0.000	0.007	0.007	0.063	0.076	0.012	0.087	0.090	0.003	0.017	0.015	-0.001	0.830	0.781	-0.049	0.003	0.031	0.028
5	Expanding VAR	1	N	Y	N	Y	N	Y	N	N	0.000	0.008	0.008	0.080	0.051	-0.029	0.029	0.026	-0.002	0.017	0.008	-0.009	0.860	0.870	0.009	0.014	0.038	0.023
6	Expanding VAR	1	N	Y	N	Y	N	Y	Y	N	0.000	0.008	0.008	0.098	0.055	-0.042	0.099	0.080	-0.018	0.017	0.016	-0.001	0.772	0.806	0.034	0.015	0.035	0.020
7	Expanding VAR	1	N	Y	N	Y	N	N	N	Y	0.000	0.007	0.007	0.090	0.057	-0.034	0.073	0.035	-0.038	0.017	0.007	-0.009	0.810	0.849	0.039	0.010	0.046	0.035
8	Expanding VAR	1	N	Y	N	Y	N	N	Y	Y	0.000	0.008	0.008	0.074	0.052	-0.022	0.046	0.103	0.057	0.008	0.016	0.008	0.857	0.804	-0.053	0.015	0.018	0.003
9	Expanding VAR	1	N	Y	N	Y	Y	N	N	N	0.000	0.008	0.008	0.071	0.088	0.017	0.093	0.016	-0.077	0.017	0.016	-0.001	0.811	0.841	0.030	0.009	0.032	0.023
10	Expanding VAR	1	N	Y	N	Y	Y	Y	Y	N	0.000	0.008	0.008	0.038	0.070	0.032	0.062	0.061	-0.001	0.017	0.008	-0.008	0.874	0.800	-0.074	0.008	0.052	0.044
			Sample Standard Deviation of Portofolio Weights																									
1	Expanding VAR	1	N	Y	N	Y	N	N	N	N	0.000	0.035	0.035	0.314	0.275	0.200	0.040	0.113	0.140	0.128	0.097	0.098	0.235	0.247	0.172	0.003	0.082	0.077
2	Expanding VAR	1	N	Y	N	Y	N	Y	N	Y	0.000	0.033	0.033	0.335	0.248	0.193	0.006	0.134	0.057	0.129	0.097	0.098	0.235	0.241	0.164	0.009	0.073	0.078
3	Expanding VAR	1	N	Y	N	Y	N	Y	Y	Y	0.000	0.034	0.034	0.293	0.229	0.191	0.050	0.126	0.078	0.129	0.034	0.134	0.219	0.231	0.172	0.002	0.195	0.203
4	Expanding VAR	1	N	Y	N	Y	N	N	Y	N	0.000	0.032	0.032	0.332	0.255	0.182	0.070	0.048	0.140	0.129	0.096	0.097	0.235	0.237	0.156	0.008	0.083	0.070
5	Expanding VAR	1	N	Y	N	Y	N	Y	N	N	0.000	0.032	0.032	0.306	0.240	0.186	0.085	0.086	0.068	0.128	0.032	0.133	0.219	0.226	0.164	0.008	0.202	0.206
6	Expanding VAR	1	N	Y	N	Y	N	Y	Y	N	0.000	0.034	0.034	0.340	0.277	0.195	0.038	0.077	0.055	0.129	0.097	0.098	0.235	0.244	0.168	0.008	0.080	0.071
7	Expanding VAR	1	N	Y	N	Y	N	N	N	Y	0.000	0.032	0.032	0.306	0.258	0.199	0.088	0.065	0.127	0.129	0.032	0.133	0.235	0.239	0.159	0.004	0.204	0.203
8	Expanding VAR	1	N	Y	N	Y	N	N	Y	Y	0.000	0.035	0.035	0.292	0.248	0.246	0.036	0.057	0.151	0.091	0.097	0.035	0.201	0.247	0.171	0.008	0.072	0.091
9	Expanding VAR	1	N	Y	N	Y	Y	N	N	N	0.000	0.035	0.035	0.329	0.247	0.192	0.009	0.119	0.093	0.129	0.097	0.098	0.235	0.245	0.170	0.009	0.070	0.078
10	Expanding VAR	1	N	Y	N	Y	Y	Y	Y	N	0.000	0.035	0.035	0.237	0.239	0.236	0.023	0.128	0.116	0.128	0.035	0.134	0.201	0.233	0.196	0.001	0.202	0.213
			Empirical 90% Range																									
1	Expanding VAR	1	N	Y	N	Y	N	N	N	N	0.000	0.061	0.061	0.000	0.167	0.000	0.000	0.061	0.061	0.000	0.144	0.061	1.000	0.804	0.000	0.000	0.062	0.074
2	Expanding VAR	1	N	Y	N	Y	N	Y	N	Y	0.000	0.045	0.045	0.000	0.167	0.000	0.000	0.045	0.045	0.000	0.120	0.045	1.000	0.796	0.001	0.000	0.044	0.020
3	Expanding VAR	1	N	Y	N	Y	N	Y	Y	Y	0.000	0.075	0.075	0.000	0.167	0.000	0.000	0.075	0.075	0.000	0.074	0.074	0.500	0.883	0.001	0.000	0.123	0.097
4	Expanding VAR	1	N	Y	N	Y	N	N	Y	N	0.000	0.045	0.045	0.000	0.167	0.000	0.000	0.045	0.045	0.000	0.106	0.045	0.998	0.868	0.001	0.000	0.044	0.050
5	Expanding VAR	1	N	Y	N	Y	N	Y	N	N	0.000	0.063	0.063	0.000	0.167	0.000	0.000	0.062	0.062	0.000	0.062	0.062	0.500	0.830	0.001	0.000	0.096	0.073
6	Expanding VAR	1	N	Y	N	Y	N	Y	Y	N	0.000	0.052	0.052	0.000	0.167	0.000	0.000	0.052	0.053	0.000	0.130	0.053	0.999	0.796	0.001	0.000	0.054	0.061
7	Expanding VAR	1	N	Y	N	Y	N	N	N	Y	0.000	0.043	0.043	0.000	0.167	0.000	0.000	0.043	0.043	0.000	0.043	0.043	1.000	0.865	0.001	0.000	0.114	0.143
8	Expanding VAR	1	N	Y	N	Y	N	N	Y	Y	0.000	0.066	0.066	0.000	0.167	0.000	0.000	0.066	0.066	0.000	0.149	0.066	0.000	0.829	0.001	0.000	0.067	0.055
9	Expanding VAR	1	N	Y	N	Y	Y	N	N	N	0.000	0.053	0.053	0.000	0.167	0.000	0.000	0.053	0.053	0.000	0.136	0.053	1.000	0.844	0.000	0.000	0.054	0.025
10	Expanding VAR	1	N	Y	N	Y	Y	Y	Y	N	0.000	0.101	0.101	0.000	0.167	0.049	0.000	0.101	0.101	0.000	0.101	0.101	0.000	0.860	0.206	0.000	0.135	0.126

Table 7

Summary Statistics for Monthly Realized, Recursively Rebalanced Optimal Portfolio Weights: Asset Menu Including HFRI Fund of Funds Composite (FFP) (γ = 5)

The table shows sample means, standard deviations, and lower and upper bounds of the 90% sample range of recursive portfolio weights computed from the ten best performing among all VAR models for risk premia and all constant investment opportunities (IID) models. The table presents statistics for 1-month vs. long-term weights, and for their differences, the hedging demands. Transactions costs are accounted for.

CER rank	Model	Lags	Predictors included								Cash			Stock			US Long-Term Treasuries			US Corporate			REITs			FOF		
			Default	Term	Short	DY	SMB	BMX	Mom.	COM	T=1	Long	Hedging	T=1	Long	Hedging	T=1	Long	Hedging	T=1	Long	Hedging	T=1	Long	Hedging	T=1	Long	Hedging
			Sample mean of portofolio weights																									
1	Expanding VAR	1	Y	N	N	N	N	N	Y	N	0.008	0.015	0.006	0.103	0.086	-0.017	0.017	0.030	0.013	0.017	0.015	-0.002	0.778	0.774	-0.004	0.077	0.081	0.004
2	Expanding VAR	1	Y	N	N	N	N	Y	Y	N	0.008	0.015	0.006	0.057	0.053	-0.004	0.037	0.056	0.019	0.017	0.015	-0.002	0.785	0.742	-0.043	0.095	0.119	0.024
3	Expanding VAR	1	Y	N	Y	N	N	Y	Y	N	0.000	0.007	0.007	0.088	0.053	-0.035	0.027	0.085	0.058	0.017	0.015	-0.002	0.766	0.750	-0.016	0.101	0.090	-0.012
4	Expanding VAR	1	Y	N	Y	N	N	Y	N	N	-1.407	-1.691	-0.284	0.358	0.852	0.495	-0.034	-0.576	-0.541	0.155	0.862	0.706	2.051	1.858	-0.193	-0.123	-0.306	-0.183
5	Expanding VAR	1	Y	N	N	N	N	Y	N	N	0.000	0.007	0.007	0.109	0.051	-0.058	0.060	0.062	0.002	0.017	0.016	-0.001	0.742	0.787	0.045	0.072	0.077	0.005
6	Expanding VAR	1	Y	N	Y	N	N	Y	N	Y	0.000	0.007	0.007	0.113	0.055	-0.058	0.042	0.018	-0.023	0.017	0.015	-0.001	0.753	0.809	0.055	0.075	0.095	0.020
7	Expanding VAR	1	Y	N	Y	N	N	N	Y	N	0.000	0.006	0.006	0.079	0.072	-0.007	0.027	0.086	0.059	0.017	0.014	-0.002	0.806	0.731	-0.075	0.071	0.091	0.019
8	Expanding VAR	1	Y	N	Y	N	N	N	N	N	0.000	0.007	0.007	0.074	0.080	0.007	0.031	0.047	0.015	0.017	0.015	-0.001	0.797	0.738	-0.058	0.082	0.112	0.030
9	Expanding VAR	1	Y	N	Y	N	Y	Y	Y	N	0.000	0.008	0.008	0.085	0.082	-0.003	0.011	0.044	0.033	0.008	0.016	0.008	0.837	0.719	-0.117	0.060	0.131	0.071
10	Expanding VAR	1	Y	N	Y	N	Y	Y	N	N	-0.381	-0.643	-0.262	0.318	0.822	0.504	-0.076	-0.588	-0.512	0.174	0.846	0.672	1.080	0.304	-0.776	-0.115	0.259	0.374
			Sample Standard Deviation of Portofolio Weights																									
1	Expanding VAR	1	Y	N	N	N	N	N	Y	N	0.091	0.095	0.030	0.180	0.131	0.110	0.000	0.030	0.030	0.128	0.095	0.096	0.235	0.229	0.143	0.005	0.042	0.064
2	Expanding VAR	1	Y	N	N	N	N	Y	Y	N	0.091	0.095	0.029	0.157	0.135	0.148	0.000	0.029	0.029	0.129	0.095	0.096	0.219	0.231	0.172	0.026	0.056	0.038
3	Expanding VAR	1	Y	N	Y	N	N	Y	Y	N	0.000	0.031	0.031	0.180	0.135	0.115	0.000	0.031	0.031	0.129	0.096	0.097	0.219	0.220	0.155	0.015	0.049	0.035
4	Expanding VAR	1	Y	N	Y	N	N	Y	N	N	0.528	0.438	0.598	0.631	0.671	0.820	0.399	0.608	0.719	0.471	1.196	1.173	0.091	0.314	0.339	0.378	1.710	1.733
5	Expanding VAR	1	Y	N	N	N	N	Y	N	N	0.000	0.031	0.031	0.180	0.132	0.112	0.000	0.031	0.031	0.129	0.096	0.097	0.219	0.219	0.155	0.008	0.067	0.064
6	Expanding VAR	1	Y	N	Y	N	N	Y	N	Y	0.000	0.030	0.030	0.180	0.133	0.112	0.000	0.030	0.030	0.129	0.096	0.097	0.219	0.217	0.150	0.018	0.034	0.063
7	Expanding VAR	1	Y	N	Y	N	N	N	Y	N	0.000	0.028	0.028	0.180	0.131	0.110	0.000	0.028	0.028	0.129	0.095	0.096	0.219	0.208	0.136	0.039	0.038	0.035
8	Expanding VAR	1	Y	N	Y	N	N	N	N	N	0.000	0.031	0.031	0.180	0.133	0.112	0.000	0.031	0.031	0.129	0.096	0.097	0.219	0.221	0.156	0.018	0.048	0.041
9	Expanding VAR	1	Y	N	Y	N	Y	Y	Y	N	0.000	0.033	0.033	0.157	0.132	0.145	0.000	0.033	0.033	0.091	0.097	0.033	0.180	0.226	0.162	0.011	0.072	0.060
10	Expanding VAR	1	Y	N	Y	N	Y	Y	N	N	0.607	0.614	0.591	0.641	0.787	0.857	0.338	0.644	0.687	0.477	1.207	1.160	0.358	0.471	0.393	0.437	1.731	1.780
			Empirical 90% Range																									
1	Expanding VAR	1	Y	N	N	N	N	N	Y	N	0.000	0.054	0.021	0.000	0.113	0.000	0.000	0.020	0.020	0.000	0.054	0.021	0.998	0.833	0.000	0.000	0.020	0.020
2	Expanding VAR	1	Y	N	N	N	N	Y	Y	N	0.000	0.052	0.046	0.000	0.156	0.024	0.000	0.045	0.045	0.000	0.052	0.045	0.500	0.833	0.121	0.000	0.048	0.048
3	Expanding VAR	1	Y	N	Y	N	N	Y	Y	N	0.000	0.034	0.035	0.000	0.164	0.000	0.000	0.034	0.034	0.000	0.064	0.034	0.500	0.816	0.000	0.000	0.033	0.033
4	Expanding VAR	1	Y	N	Y	N	N	Y	N	N	0.999	1.722	1.074	1.784	1.633	3.133	0.563	1.778	1.379	0.918	2.600	2.600	0.087	0.457	0.548	0.800	3.600	3.600
5	Expanding VAR	1	Y	N	N	N	N	Y	N	N	0.000	0.062	0.062	0.000	0.149	0.003	0.000	0.063	0.063	0.000	0.090	0.066	0.497	0.768	0.003	0.000	0.059	0.059
6	Expanding VAR	1	Y	N	Y	N	N	Y	N	Y	0.000	0.052	0.052	0.000	0.155	0.000	0.000	0.050	0.051	0.000	0.076	0.052	0.500	0.745	0.000	0.000	0.053	0.053
7	Expanding VAR	1	Y	N	Y	N	N	N	Y	N	0.000	0.040	0.040	0.000	0.128	0.003	0.000	0.038	0.038	0.000	0.044	0.041	0.497	0.679	0.003	0.000	0.043	0.043
8	Expanding VAR	1	Y	N	Y	N	N	N	N	N	0.000	0.042	0.042	0.000	0.161	0.000	0.000	0.042	0.042	0.000	0.097	0.042	0.500	0.784	0.000	0.000	0.042	0.042
9	Expanding VAR	1	Y	N	Y	N	Y	Y	Y	N	0.000	0.049	0.049	0.000	0.164	0.002	0.000	0.048	0.048	0.000	0.122	0.049	0.000	0.818	0.008	0.000	0.048	0.048
10	Expanding VAR	1	Y	N	Y	N	Y	Y	N	N	1.175	1.949	1.032	1.642	1.633	3.391	0.476	1.911	1.296	0.915	2.600	2.600	0.102	1.445	0.762	0.486	3.600	3.600

Table 8

Summary Statistics for Monthly Realized, Recursively Rebalanced Optimal Portfolio Weights: Asset Menu Including HFRI Relative Value (RVR) (γ = 5)

The table shows sample means, standard deviations, and lower and upper bounds of the 90% sample range of recursive portfolio weights computed from the ten best performing among all VAR models for risk premia and all constant investment opportunities (IID) models. The table presents statistics for 1-month vs. long-term weights, and for their differences, the hedging demands. Transactions costs are accounted for.

CER rank	Model	Lags	Predictors included								Cash			Stock			US Long-Term Treasuries			US Corporate			REITs			REL		
			Default	Term	Short	DY	SMB	PtfsBD	PtfsIR	COM	T=1	Long	Hedging	T=1	Long	Hedging	T=1	Long	Hedging	T=1	Long	Hedging	T=1	Long	Hedging	T=1	Long	Hedging
											Sample mean of portofolio weights																	
1	Expanding VAR	1	Y	N	Y	N	Y	N	N	Y	0.000	0.005	0.005	0.151	0.087	-0.064	0.114	0.070	-0.044	0.017	0.013	-0.004	0.596	0.679	0.082	0.122	0.147	0.024
2	Expanding VAR	1	Y	N	Y	N	Y	Y	N	Y	0.000	0.005	0.005	0.115	0.113	-0.002	0.010	0.074	0.064	0.025	0.013	-0.012	0.785	0.684	-0.101	0.065	0.111	0.046
3	Expanding VAR	1	Y	N	Y	N	Y	Y	Y	Y	0.000	0.005	0.005	0.169	0.118	-0.052	0.145	0.104	-0.041	0.025	0.014	-0.011	0.624	0.700	0.076	0.037	0.060	0.023
4	Expanding VAR	1	Y	N	Y	N	Y	N	Y	Y	0.008	0.014	0.005	0.131	0.129	-0.001	0.036	0.064	0.028	0.025	0.014	-0.011	0.759	0.671	-0.088	0.041	0.108	0.067
5	Expanding VAR	1	Y	N	Y	N	Y	N	N	N	0.000	0.004	0.004	0.117	0.152	0.035	0.029	0.106	0.076	0.017	0.004	-0.012	0.810	0.674	-0.135	0.027	0.059	0.032
6	Expanding VAR	1	Y	N	Y	N	N	N	N	N	0.000	0.004	0.004	0.136	0.070	-0.066	0.058	0.138	0.080	0.017	0.004	-0.012	0.751	0.649	-0.102	0.038	0.134	0.096
7	Expanding VAR	1	Y	N	Y	N	N	N	N	Y	0.000	0.004	0.004	0.126	0.088	-0.038	0.052	0.145	0.092	0.017	0.004	-0.012	0.715	0.661	-0.054	0.090	0.098	0.007
8	Expanding VAR	1	Y	N	Y	N	N	N	Y	N	0.000	0.004	0.004	0.113	0.081	-0.032	0.028	0.081	0.052	0.017	0.004	-0.012	0.727	0.715	-0.013	0.115	0.114	0.000
9	Expanding VAR	1	Y	N	Y	N	N	Y	N	N	0.000	0.004	0.004	0.158	0.074	-0.085	0.015	0.028	0.013	0.017	0.004	-0.012	0.773	0.771	-0.002	0.037	0.119	0.082
10	Expanding VAR	1	Y	N	Y	N	Y	Y	N	N	0.000	0.005	0.004	0.099	0.073	-0.026	0.156	0.063	-0.093	0.017	0.004	-0.012	0.690	0.756	0.066	0.039	0.099	0.061
											Sample Standard Deviation of Portofolio Weights																	
1	Expanding VAR	1	Y	N	Y	N	Y	N	N	Y	0.000	0.027	0.027	0.374	0.313	0.204	0.138	0.154	0.081	0.129	0.095	0.095	0.762	0.732	0.418	0.225	0.211	0.092
2	Expanding VAR	1	Y	N	Y	N	Y	Y	N	Y	0.000	0.027	0.027	0.337	0.326	0.231	0.142	0.211	0.122	0.156	0.095	0.131	0.809	0.722	0.623	0.281	0.262	0.072
3	Expanding VAR	1	Y	N	Y	N	Y	Y	Y	Y	0.000	0.030	0.030	0.352	0.313	0.182	0.116	0.225	0.225	0.156	0.095	0.132	0.839	0.795	0.562	0.221	0.236	0.066
4	Expanding VAR	1	Y	N	Y	N	Y	N	Y	Y	0.091	0.095	0.028	0.353	0.286	0.258	0.098	0.109	0.138	0.157	0.095	0.132	0.798	0.806	0.576	0.062	0.131	0.139
5	Expanding VAR	1	Y	N	Y	N	Y	N	N	N	0.000	0.026	0.026	0.378	0.280	0.213	0.138	0.091	0.229	0.128	0.026	0.131	0.799	0.677	0.536	0.251	0.280	0.150
6	Expanding VAR	1	Y	N	Y	N	N	N	N	N	0.000	0.026	0.026	0.364	0.289	0.226	0.043	0.163	0.134	0.128	0.026	0.131	0.767	0.726	0.510	0.282	0.248	0.099
7	Expanding VAR	1	Y	N	Y	N	N	N	N	Y	0.000	0.026	0.026	0.362	0.267	0.238	0.073	0.224	0.094	0.128	0.026	0.132	0.788	0.756	0.468	0.262	0.209	0.116
8	Expanding VAR	1	Y	N	Y	N	N	N	Y	N	0.000	0.026	0.026	0.377	0.283	0.242	0.085	0.130	0.113	0.129	0.026	0.132	0.764	0.757	0.504	0.265	0.253	0.108
9	Expanding VAR	1	Y	N	Y	N	N	Y	N	N	0.000	0.026	0.026	0.319	0.291	0.259	0.020	0.119	0.094	0.129	0.026	0.132	0.757	0.735	0.574	0.266	0.239	0.077
10	Expanding VAR	1	Y	N	Y	N	Y	Y	N	N	0.000	0.026	0.026	0.330	0.288	0.193	0.076	0.209	0.099	0.128	0.026	0.132	0.770	0.681	0.460	0.211	0.259	0.113
											Empirical 90% Range																	
1	Expanding VAR	1	Y	N	Y	N	Y	N	N	Y	0.000	0.000	0.000	0.000	0.115	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.823	-0.012	0.000	0.092	0.077
2	Expanding VAR	1	Y	N	Y	N	Y	Y	N	Y	0.000	0.000	0.000	0.000	0.128	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.821	0.020	0.000	0.050	0.052
3	Expanding VAR	1	Y	N	Y	N	Y	Y	Y	Y	0.000	0.000	0.000	0.000	0.164	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.883	-0.009	0.000	0.086	0.008
4	Expanding VAR	1	Y	N	Y	N	Y	N	Y	Y	0.000	0.000	0.000	0.002	0.158	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.836	0.011	0.000	0.097	0.049
5	Expanding VAR	1	Y	N	Y	N	Y	N	N	N	0.000	0.000	0.000	0.000	0.088	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.786	0.022	0.000	0.108	0.006
6	Expanding VAR	1	Y	N	Y	N	N	N	N	N	0.000	0.000	0.000	0.000	0.087	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.879	0.042	0.000	0.023	0.062
7	Expanding VAR	1	Y	N	Y	N	N	N	N	Y	0.000	0.000	0.000	0.000	0.095	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.793	0.042	0.000	0.032	0.006
8	Expanding VAR	1	Y	N	Y	N	N	N	Y	N	0.000	0.000	0.000	0.000	0.096	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.822	-0.014	0.000	0.051	0.074
9	Expanding VAR	1	Y	N	Y	N	N	Y	N	N	0.000	0.000	0.000	0.000	0.103	0.000	0.000	0.000	0.001	0.000	0.000	0.001	1.000	0.837	0.006	0.000	0.084	0.081
10	Expanding VAR	1	Y	N	Y	N	Y	Y	N	N	0.000	0.001	0.001	0.000	0.104	0.000	0.000	0.001	0.001	0.000	0.001	0.001	0.999	0.875	-0.005	0.000	0.026	0.073

Table 9

Top 18 Models Ranked According to Realized CER: HFRI Fund Weighted Composite Index (FWC) (γ = 5)

The tables shows annualized sample means, standard deviations, the Sharpe ratio, and the Certainty Equivalent Return computed from the recursive portfolio weights under the ten top performing (in terms of CER) models for (predictable) risk premia, as defined by the VAR order (where a VAR(0) is equivalent to IID, constant investment opportunities), the selection of predictors, and whether the models are estimated either on an expanding or on a rolling 10-year window of data. The calculations are performed assuming the investor assess realized performance with a 5-year horizon (H = 60 months). Monthly rebalancing applies and it is taken into account by a long-horizon investor. In the table, we have boldfaced all performance statistics that turn out to be superior to those recorded in Table 5 for the baseline asset menu that excludes the hedge fund strategies.

CER rank	Model	Lags	Predictors included								H	Annualized mean (%)			Annualized volatility (%)			Annualized Sharpe ratio			Annualized CER (%)			Skewness	Kurtosis
			Term	Short	DY	Default	SMB	PtfsBD	PtfsIR	COM		Mean returns	90% Conf. Int. - LB	90% Conf. Int. - UB	Volatility	90% Conf. Int. - LB	90% Conf. Int. - UB	Sharpe ratio	90% Conf. Int. - LB	90% Conf. Int. - UB	CER (% Ann.)	90% Conf. Int. - LB	90% Conf. Int. - UB		
1	Expanding VAR	1	Y	N	Y	N	N	N	N	N	60	12.660	12.522	12.797	20.705	20.244	21.204	0.442	**0.364**	0.521	9.314	8.131	10.226	-0.663	3.803
2	Expanding VAR	1	Y	N	Y	N	N	Y	N	Y	60	11.969	11.832	12.105	20.688	20.224	21.182	0.409	0.331	0.487	9.274	8.129	10.147	-0.666	3.809
3	Expanding VAR	1	Y	N	Y	N	N	Y	Y	Y	60	12.888	12.752	13.023	20.612	20.157	21.115	0.455	**0.378**	0.533	9.110	7.963	9.958	-0.676	3.837
4	Expanding VAR	1	Y	N	Y	N	N	N	Y	N	60	11.965	11.827	12.103	20.610	20.158	21.117	0.411	0.332	0.489	9.081	7.951	9.950	-0.676	3.839
5	Expanding VAR	1	Y	N	Y	N	N	Y	N	N	60	12.843	12.705	12.981	20.609	20.158	21.096	0.453	**0.374**	0.532	9.066	7.935	9.918	-0.675	3.839
6	Expanding VAR	1	Y	N	Y	N	N	Y	Y	N	60	11.701	11.562	11.840	20.607	20.151	21.095	0.398	0.319	0.477	9.054	7.960	9.944	-0.674	3.838
7	Expanding VAR	1	Y	N	Y	N	N	N	N	Y	60	12.551	12.410	12.692	21.162	20.696	21.651	0.428	0.347	0.508	9.025	7.701	9.925	-0.626	3.663
8	Expanding VAR	1	Y	N	Y	N	N	N	Y	Y	60	12.533	12.396	12.669	20.764	20.308	21.264	0.435	0.357	0.513	8.973	7.795	9.858	-0.667	3.769
9	Expanding VAR	1	Y	N	Y	N	Y	N	N	N	60	12.053	11.913	12.192	21.005	20.531	21.481	0.407	0.328	0.487	8.657	7.581	9.519	-0.594	3.696
10	Expanding VAR	1	Y	N	Y	N	Y	Y	Y	N	60	11.634	11.498	11.771	20.552	20.110	21.045	0.396	0.318	0.474	8.408	7.411	9.175	-0.648	3.843
11	Expanding VAR	1	Y	N	Y	N	Y	Y	N	N	60	11.639	11.502	11.775	20.564	20.101	21.054	0.396	0.318	0.474	8.401	7.443	9.192	-0.649	3.836
12	Expanding VAR	1	Y	N	Y	N	Y	Y	N	Y	60	12.595	12.455	12.735	20.995	20.543	21.494	0.433	0.353	0.513	8.316	7.247	9.130	-0.617	3.686
13	Expanding VAR	1	Y	N	Y	N	Y	N	Y	N	60	10.942	10.807	11.078	20.653	20.212	21.150	0.360	0.283	0.438	8.147	7.220	8.918	-0.648	3.793
14	Expanding VAR	1	Y	N	Y	N	Y	N	N	Y	60	13.076	12.930	13.221	21.740	21.265	22.238	0.440	0.357	0.524	8.066	6.906	8.957	-0.573	3.660
15	Expanding VAR	1	Y	N	Y	N	Y	Y	Y	Y	60	10.053	9.916	10.190	20.844	20.384	21.348	0.314	0.236	0.393	7.726	6.719	8.469	-0.656	3.731
16	Expanding VAR	1	Y	N	Y	N	Y	N	Y	Y	60	10.978	10.835	11.121	21.141	20.694	21.645	0.354	0.272	0.435	7.634	6.606	8.402	-0.660	3.676
17	Exp. Gaussian IID	_	Y	N	Y	N	_	_	_	_	60	11.558	11.384	11.731	26.032	25.164	27.011	0.310	0.210	0.409	-4.974	-5.544	-4.396	-1.748	7.684
18	Expanding VAR	0	N	N	N	Y	N	Y	N	N	60	10.824	10.428	11.220	60.161	58.951	61.342	0.122	-0.105	0.348	-10.102	-12.843	-7.492	-0.565	**3.064**
	Median Expanding VAR performance										60	11.820	11.672	11.969	29.648	26.479	32.849	0.281	0.145	0.416	-10.102	-12.843	-7.492	-0.656	3.663

Table 10

Top 18 Models Ranked According to Realized CER: HFRI Fund of Funds Composite (FFP) (γ = 5)

The tables shows annualized sample means, standard deviations, the Sharpe ratio, and the Certainty Equivalent Return computed from the recursive portfolio weights under the ten top performing (in terms of CER) models for (predictable) risk premia, as defined by the VAR order (where a VAR(0) is equivalent to IID, constant investment opportunities), the selection of predictors, and whether the models are estimated either on an expanding or on a rolling 10-year window of data. The calculations are performed assuming the investor assess realized performance with a 5-year horizon (*H* = 60 months). Monthly rebalancing applies and it is taken into account by a long-horizon investor. In the table, we have boldfaced all performance statistics that turn out to be superior to those recorded in Table 5 for the baseline asset menu that excludes the hedge fund strategies.

CER rank	Model	Lags	Predictors included								*H*	Annualized mean (%)			Annualized volatility (%)			Annualized Sharpe ratio			Annualized CER (%)			Skewness	Kurtosis
			Term	Short	DY	Default	SMB	PtfsBD	PtfsIR	COM		Mean returns	90% Conf. Int. - LB	90% Conf. Int. - UB	Volatility	90% Conf. Int. - LB	90% Conf. Int. - UB	Sharpe ratio	90% Conf. Int. - LB	90% Conf. Int. - UB	CER (% Ann.)	90% Conf. Int. - LB	90% Conf. Int. - UB		
1	Expanding VAR	1	Y	N	Y	N	N	N	Y	N	60	11.785	11.363	12.208	27.889	25.398	30.380	0.297	0.173	0.421	**9.444**	6.971	**11.606**	**0.190**	**3.194**
2	Expanding VAR	1	Y	N	Y	N	N	Y	Y	N	60	11.979	11.396	12.561	27.848	25.327	30.370	0.304	0.179	0.430	9.110	6.531	**11.435**	**-0.189**	**3.218**
3	Expanding VAR	1	Y	N	Y	N	N	Y	Y	N	60	11.020	10.783	11.257	**10.333**	**9.298**	**11.368**	**0.728**	0.600	0.855	8.958	7.820	9.848	-0.675	3.811
4	Expanding VAR	1	Y	N	Y	N	N	Y	N	N	60	10.675	8.429	12.921	**10.351**	**9.309**	**11.392**	**0.693**	0.566	0.820	8.917	7.789	9.776	-0.671	3.796
5	Expanding VAR	1	Y	N	Y	N	N	Y	N	N	60	10.365	9.571	11.158	27.918	25.429	30.407	0.246	0.125	0.367	8.914	6.391	**11.115**	**-0.165**	**3.218**
6	Expanding VAR	1	Y	N	Y	N	N	Y	N	Y	60	11.923	11.850	11.995	**10.685**	**9.636**	**11.734**	**0.788**	0.659	0.917	8.806	7.502	9.723	-0.629	3.645
7	Expanding VAR	1	Y	N	Y	N	N	N	Y	N	60	10.822	10.844	10.801	**10.524**	**9.473**	**11.575**	**0.696**	0.568	0.823	8.729	7.482	9.638	-0.658	3.687
8	Expanding VAR	1	Y	N	Y	N	N	N	N	N	60	10.722	10.125	11.319	**10.715**	**9.675**	**11.754**	**0.674**	0.547	0.801	8.710	7.410	9.646	-0.632	3.644
9	Expanding VAR	1	Y	N	Y	N	Y	Y	Y	N	60	9.502	6.111	12.893	**10.385**	**9.348**	**11.422**	**0.578**	0.450	0.706	8.475	7.399	9.318	-0.680	3.770
10	Expanding VAR	1	Y	N	Y	N	Y	Y	N	N	60	10.351	7.438	13.263	**10.498**	**9.453**	**11.543**	**0.653**	0.525	0.780	8.459	7.315	9.314	-0.677	3.705
11	Expanding VAR	1	Y	N	Y	N	N	N	N	N	60	11.774	11.341	12.208	27.908	25.410	30.405	0.296	0.175	0.418	8.383	6.060	**10.416**	**-0.166**	**3.183**
12	Expanding VAR	1	Y	N	Y	N	N	Y	Y	Y	60	12.425	11.061	13.788	**10.516**	**9.471**	**11.560**	**0.849**	0.719	0.978	8.346	7.226	9.219	-0.686	3.704
13	Expanding VAR	1	Y	N	Y	N	Y	N	N	N	60	9.952	8.470	11.433	**11.019**	**9.929**	**12.109**	**0.586**	0.459	0.712	8.306	6.918	9.303	-0.635	3.730
14	Expanding VAR	1	Y	N	Y	N	N	N	N	Y	60	9.636	5.939	13.333	**11.282**	**10.103**	**12.460**	**0.544**	0.417	0.671	8.121	6.644	9.108	-0.628	3.916
15	Expanding VAR	1	Y	N	Y	N	Y	Y	N	Y	60	12.088	12.070	12.105	**10.930**	**9.840**	**12.019**	**0.786**	0.659	0.912	8.084	6.798	8.962	-0.622	3.672
16	Expanding VAR	1	Y	N	Y	N	Y	N	Y	N	60	12.811	10.805	**14.816**	**10.613**	**9.566**	**11.659**	**0.877**	0.751	1.004	8.048	6.895	8.926	-0.706	3.701
17	Expanding VAR	1	Y	N	Y	N	Y	Y	Y	Y	60	10.163	8.799	11.527	**10.597**	**9.533**	**11.661**	**0.629**	0.501	0.756	7.973	6.845	8.824	-0.710	3.711
18	Exp. Gaussian IID	_	_	_	_	_	_	_	_	_	60	11.142	8.195	14.088	27.750	25.326	30.174	0.275	0.152	0.399	7.932	5.355	10.155	-0.222	**3.112**
	Median Expanding VAR performance										60	11.157	10.113	12.201	**21.559**	**19.289**	**23.829**	**0.355**	0.240	**0.471**	7.925	6.119	9.035	**-0.614**	**3.658**

Table 11

Top 18 Models Ranked According to Realized CER: HFRI Fixed Income Relative Value/Arbitrage (RVR) (γ = 5)

The tables shows annualized sample means, standard deviations, the Sharpe ratio, and the Certainty Equivalent Return computed from the recursive portfolio weights under the ten top performing (in terms of CER) models for (predictable) risk premia, as defined by the VAR order (where a VAR(0) is equivalent to IID, constant investment opportunities), the selection of predictors, and whether the models are estimated either on an expanding or on a rolling 10-year window of data. The calculations are performed assuming the investor assess realized performance with a 5-year horizon (H = 60 months). Monthly rebalancing applies and it is taken into account by a long-horizon investor. In the table, we have boldfaced all performance statistics that turn out to be superior to those recorded in Table 5 for the baseline asset menu that excludes the hedge fund strategies.

CER rank	Model	Lags	Predictors included								*H*	Annualized mean (%)			Annualized volatility (%)			Annualized Sharpe ratio			Annualized CER (%)			Skewness	Kurtosis
			Default	Term	Short	DY	SMB	PtfsBD	PtfsIR	COM		Mean returns	90% Conf. Int. - LB	90% Conf. Int. - UB	Volatility	90% Conf. Int. - LB	90% Conf. Int. - UB	Sharpe ratio	90% Conf. Int. - LB	90% Conf. Int. - UB	CER (% Ann.)	90% Conf. Int. - LB	90% Conf. Int. - UB		
1	Expanding VAR	1	Y	N	Y	N	Y	N	N	Y	60	12.640	12.489	12.791	22.492	21.912	23.111	0.406	0.320	0.493	**10.754**	**9.325**	**11.842**	**0.394**	4.735
2	Expanding VAR	1	Y	N	Y	N	Y	Y	N	Y	60	12.574	12.429	12.719	21.875	21.346	22.425	0.415	0.332	0.498	**10.343**	**9.053**	**11.332**	**0.148**	4.128
3	Expanding VAR	1	Y	N	Y	N	Y	Y	Y	Y	60	11.432	11.295	11.569	20.637	20.193	21.143	0.384	0.306	0.463	8.921	7.987	9.704	**-0.230**	3.811
4	Expanding VAR	1	Y	N	Y	N	Y	N	Y	Y	60	12.817	12.678	12.956	20.911	20.441	21.406	0.446	**0.366**	0.525	8.880	7.937	9.652	**-0.142**	3.747
5	Expanding VAR	1	Y	N	Y	N	Y	N	N	N	60	12.502	12.329	12.676	25.802	24.988	26.655	0.349	0.250	0.448	7.939	6.222	9.083	**0.556**	6.451
6	Expanding VAR	1	Y	N	Y	N	N	N	N	N	60	11.762	11.593	11.931	25.746	24.967	26.590	0.321	0.224	0.417	7.902	6.227	9.055	**0.532**	6.374
7	Expanding VAR	1	Y	N	Y	N	N	N	N	Y	60	11.503	11.335	11.671	25.349	24.608	26.132	0.316	0.220	0.412	7.696	6.110	8.793	**0.370**	5.865
8	Expanding VAR	1	Y	N	Y	N	N	N	Y	N	60	11.930	11.763	12.098	25.300	24.598	26.091	0.333	0.238	0.429	7.675	6.000	8.727	**0.352**	5.808
9	Expanding VAR	1	Y	N	Y	N	N	Y	N	N	60	11.533	11.367	11.699	24.937	24.229	25.665	0.322	0.227	0.417	7.468	5.882	8.499	**0.203**	5.396
10	Expanding VAR	1	Y	N	Y	N	Y	Y	N	N	60	12.496	12.330	12.661	24.929	24.230	25.658	0.361	0.266	0.456	7.462	5.892	8.501	**0.202**	5.391
11	Expanding VAR	1	Y	N	Y	N	N	Y	N	Y	60	12.153	11.989	12.316	24.604	23.943	25.313	0.352	0.258	0.445	7.259	5.759	8.239	**0.069**	5.068
12	Expanding VAR	1	Y	N	Y	N	Y	N	Y	N	60	12.217	12.051	12.382	24.474	23.831	25.179	0.356	0.262	0.451	7.168	5.699	8.143	**0.020**	4.961
13	Expanding VAR	1	Y	N	Y	N	N	N	Y	Y	60	11.053	10.893	11.213	24.320	23.691	24.995	0.311	0.219	0.402	7.070	5.703	8.047	**-0.042**	4.839
14	Expanding VAR	1	Y	N	Y	N	N	Y	Y	N	60	11.099	10.939	11.259	24.323	23.674	24.988	0.312	0.221	0.404	7.066	5.644	8.023	**-0.040**	4.843
15	Expanding VAR	1	Y	N	Y	N	N	Y	Y	Y	60	12.964	12.805	13.123	23.768	23.199	24.402	0.398	0.307	0.489	6.625	5.308	7.515	**-0.250**	4.516
16	Expanding VAR	1	Y	N	Y	N	Y	Y	Y	N	60	11.801	11.643	11.960	23.678	23.092	24.289	0.351	0.260	0.441	6.545	5.261	7.399	**-0.279**	4.480
17	Expanding AR	1	Y	N	N	N	N	N	N	N	60	11.479	11.268	11.690	31.758	30.810	32.817	0.251	0.131	0.372	-0.423	-1.901	0.519	**-0.214**	6.238
18	Exp. Gaussian IID	_	_	_	_	_	_	_	_	_	60	10.489	10.080	10.898	61.000	59.870	62.134	0.115	-0.119	0.348	-1.700	-4.116	0.620	**0.251**	**2.860**
	ledian Expanding VAR performanc										60	12.168	11.969	12.368	46.205	45.165	47.310	0.225	0.089	0.362	5.569	3.634	7.390	**-0.166**	**3.970**

Table 12

Summary Statistics for Monthly Realized, Recursively Rebalanced Optimal Portfolio Weights: Bayesian Strategies Applied to the Baseline Asset Menu (γ = 5)

The tables shows sample means, standard deviations, and the lower and upper bounds of the 90% sample range of the recursive portfolio weights computed from a range of VAR models for predictable risk premia and of constant investment opportunities (IID) models. The table presents statistics for 1-m T-bill weights, long-term (infinite horizon) weights, and for their differences, the hedging demands. Weights are computed using the predictive density from a Bayesian estimate of the predictability model for excess asset returns, when the priors are non-informative

CER rank	Model	Lags	Predictors included				Cash			Stocks			US Long-Term Treasuries			US Corporate Bonds			REITs		
			Default	Term	Short	DY	T=1	Long	Hedging	T=1	Long	Hedging	T=1	Long	Hedging	T=1	Long	Hedging	T=1	Long	Hedging
							Sample Mean of Portofolio Weights														
1	Rolling VAR	1	Y	N	Y	Y	0.191	0.124	-0.067	0.194	0.142	-0.052	0.150	0.072	-0.078	0.200	0.160	-0.041	0.265	0.502	0.238
2	Rolling VAR	1	Y	Y	N	N	0.191	0.123	-0.069	0.196	0.133	-0.064	0.150	0.065	-0.085	0.198	0.134	-0.064	0.264	0.545	0.281
3	Rolling VAR	1	Y	Y	N	Y	0.194	0.130	-0.065	0.200	0.157	-0.043	0.153	0.069	-0.084	0.198	0.135	-0.063	0.255	0.509	0.254
4	Rolling VAR	1	N	Y	Y	N	0.193	0.127	-0.066	0.199	0.141	-0.058	0.153	0.063	-0.089	0.197	0.133	-0.065	0.257	0.536	0.278
5	Rolling VAR	1	N	Y	Y	Y	0.193	0.126	-0.066	0.202	0.163	-0.038	0.152	0.065	-0.087	0.197	0.130	-0.066	0.257	0.515	0.258
6	Rolling VAR	1	Y	N	Y	N	0.190	0.130	-0.061	0.197	0.141	-0.056	0.147	0.059	-0.088	0.194	0.132	-0.062	0.272	0.539	0.267
7	Rolling VAR	1	Y	Y	Y	Y	0.188	0.123	-0.065	0.198	0.144	-0.054	0.143	0.053	-0.090	0.192	0.130	-0.062	0.278	0.550	0.271
8	Expanding VAR	1	Y	N	Y	N	0.194	0.128	-0.066	0.199	0.143	-0.056	0.153	0.064	-0.089	0.197	0.133	-0.064	0.258	0.533	0.275
9	Rolling VAR	1	N	N	Y	Y	0.191	0.129	-0.061	0.196	0.140	-0.056	0.147	0.058	-0.089	0.195	0.132	-0.063	0.271	0.540	0.269
10	Rolling VAR	1	Y	N	N	N	0.193	0.127	-0.066	0.200	0.149	-0.051	0.152	0.062	-0.090	0.197	0.132	-0.065	0.258	0.530	0.272
							Sample Standard Deviation of Portofolio Weights														
1	Rolling VAR	1	Y	N	Y	Y	0.010	0.027	0.028	0.013	0.055	0.055	0.012	0.073	0.068	0.015	0.095	0.090	0.040	0.164	0.124
2	Rolling VAR	1	Y	Y	N	N	0.004	0.008	0.010	0.006	0.024	0.028	0.004	0.062	0.059	0.005	0.029	0.029	0.014	0.095	0.081
3	Rolling VAR	1	Y	Y	N	Y	0.005	0.012	0.016	0.006	0.052	0.052	0.004	0.031	0.031	0.004	0.015	0.018	0.018	0.095	0.078
4	Rolling VAR	1	N	Y	Y	N	0.004	0.013	0.016	0.005	0.030	0.032	0.004	0.029	0.029	0.003	0.016	0.018	0.015	0.080	0.066
5	Rolling VAR	1	N	Y	Y	Y	0.004	0.013	0.016	0.006	0.060	0.060	0.004	0.036	0.035	0.004	0.015	0.017	0.013	0.100	0.087
6	Rolling VAR	1	Y	N	Y	N	0.017	0.039	0.040	0.018	0.032	0.038	0.017	0.017	0.027	0.017	0.023	0.031	0.072	0.092	0.020
7	Rolling VAR	1	Y	Y	Y	Y	0.016	0.015	0.027	0.028	0.066	0.059	0.016	0.016	0.028	0.017	0.044	0.047	0.071	0.115	0.044
8	Expanding VAR	1	Y	N	Y	N	0.004	0.013	0.016	0.005	0.031	0.032	0.004	0.024	0.024	0.003	0.015	0.017	0.015	0.076	0.061
9	Rolling VAR	1	N	N	Y	Y	0.017	0.041	0.040	0.017	0.036	0.042	0.016	0.016	0.027	0.016	0.025	0.032	0.070	0.092	0.021
10	Rolling VAR	1	Y	N	N	N	0.004	0.013	0.016	0.005	0.040	0.041	0.004	0.027	0.027	0.003	0.018	0.019	0.014	0.085	0.072
							Empirical 90% Range														
1	Rolling VAR	1	Y	N	Y	Y	0.014	0.036	0.049	0.026	0.151	0.152	0.015	0.057	0.089	0.029	0.292	0.279	0.053	0.493	0.440
2	Rolling VAR	1	Y	Y	N	N	0.012	0.021	0.031	0.018	0.059	0.073	0.013	0.026	0.036	0.018	0.051	0.064	0.042	0.195	0.153
3	Rolling VAR	1	Y	Y	N	Y	0.014	0.020	0.033	0.020	0.146	0.153	0.014	0.027	0.039	0.012	0.028	0.037	0.048	0.240	0.192
4	Rolling VAR	1	N	Y	Y	N	0.010	0.020	0.029	0.014	0.072	0.083	0.009	0.033	0.043	0.009	0.031	0.039	0.038	0.192	0.154
5	Rolling VAR	1	N	Y	Y	Y	0.010	0.024	0.033	0.017	0.170	0.175	0.010	0.032	0.039	0.010	0.030	0.037	0.036	0.268	0.232
6	Rolling VAR	1	Y	N	Y	N	0.024	0.035	0.079	0.021	0.074	0.110	0.027	0.032	0.067	0.026	0.036	0.075	0.086	0.214	0.128
7	Rolling VAR	1	Y	Y	Y	Y	0.038	0.046	0.088	0.023	0.116	0.138	0.038	0.047	0.088	0.037	0.050	0.096	0.098	0.307	0.209
8	Expanding VAR	1	Y	N	Y	N	0.010	0.020	0.029	0.014	0.077	0.088	0.009	0.032	0.041	0.009	0.032	0.038	0.038	0.171	0.133
9	Rolling VAR	1	N	N	Y	Y	0.019	0.037	0.072	0.032	0.070	0.114	0.020	0.033	0.062	0.018	0.041	0.075	0.079	0.237	0.157
10	Rolling VAR	1	Y	N	N	N	0.010	0.021	0.029	0.016	0.124	0.131	0.009	0.026	0.033	0.009	0.036	0.041	0.037	0.228	0.191

Table 13

Top 10 Models Ranked According to Realized CER for Buy-and-Hold and Monthly Rebalancing Strategies: Bayesian Strategies Applied to the Baseline Asset Menu (γ = 5)

The tables shows annualized sample means, standard deviations, the Sharpe ratio, and the Certainty Equivalent Return computed from the recursive portfolio weights under the ten top performing (in terms of CER) models for (predictable) risk premia, as defined by the VAR order (where a VAR(0) is equivalent to IID, constant investment opportunities), the selection of predictors, and whether the models are estimated either on an expanding or on a rolling 10-year window of data. The calculations are performed assuming the investor assess realized performance with a 5-year horizon (H = 60 months) and using the predictive density from a Bayesian estimate of the predictability model for excess asset returns, when the priors are non-informative.

CER rank	Model	Lags	Predictors included				*H*	Annualized mean (%)			Annualized volatility (%)			Sharpe ratio			Annualized CER (%)			Skewness	Kurtosis
			Default	Term	Short	DY		Mean returns	90% Conf. Int. - LB	90% Conf. Int. - UB	Volatility	90% Conf. Int. - LB	90% Conf. Int. - UB	Sharpe ratio	90% Conf. Int. - LB	90% Conf. Int. - UB	CER (% Ann.)	90% Conf. Int. - LB	90% Conf. Int. - UB		
Transaction costs																					
1	Rolling VAR	1	N	N	Y	N	60	11.914	11.800	12.028	12.856	9.917	15.794	0.655	0.271	0.974	11.800	6.696	16.903	-1.359	5.567
2	Rolling VAR	1	N	Y	Y	N	60	12.332	12.224	12.441	12.391	9.496	15.287	0.713	0.327	1.034	10.828	5.960	15.696	-1.423	5.815
3	Rolling VAR	1	N	Y	N	N	60	11.746	11.639	11.854	12.075	9.207	14.943	0.683	0.296	1.005	10.796	5.966	15.626	-1.428	5.792
4	Rolling VAR	1	N	N	N	Y	60	11.866	11.762	11.971	11.990	9.203	14.777	0.698	0.312	1.019	10.460	5.636	15.284	-1.400	5.793
5	Rolling VAR	1	N	Y	Y	Y	60	12.352	12.249	12.455	11.850	9.216	14.483	0.747	0.362	1.068	10.448	5.584	15.312	-1.371	5.615
6	Rolling VAR	1	Y	Y	N	Y	60	11.646	11.542	11.750	11.919	9.173	14.665	0.683	0.299	1.004	10.434	5.608	15.259	-1.395	5.782
7	Rolling VAR	1	Y	N	Y	N	60	10.991	10.883	11.099	12.366	9.190	15.542	0.606	0.213	0.933	10.412	5.530	15.295	-1.540	7.146
8	Expanding VAR	1	N	Y	N	Y	60	10.613	10.509	10.716	11.899	9.167	14.632	0.598	0.212	0.920	10.399	5.617	15.182	-1.394	5.752
9	Rolling VAR	1	Y	N	N	Y	60	11.609	11.505	11.714	11.980	9.209	14.751	0.677	0.291	0.999	10.367	5.676	15.057	-1.402	5.808
10	Rolling VAR	1	N	N	Y	Y	60	12.206	12.104	12.308	11.696	9.067	14.324	0.744	0.361	1.064	10.288	5.581	14.995	-1.381	5.690
	Median Expanding VAR performance						60	11.462	11.318	11.607	17.119	13.409	20.829	0.465	0.089	0.779	8.765	1.090	16.441	-1.325	5.760
	Median Rolling VAR performance						60	11.793	11.650	11.936	16.010	12.583	19.436	0.518	0.142	0.832	8.356	1.072	15.641	-1.256	6.029
No transaction costs																					
1	Rolling VAR	1	Y	N	Y	Y	60	12.222	11.844	12.599	9.667	5.310	14.024	0.902	0.146	1.407	5.619	1.478	9.761	-1.274	8.095
2	Rolling VAR	1	Y	Y	N	N	60	11.598	10.932	12.264	16.570	3.310	29.830	0.489	-0.352	1.049	2.626	-3.181	8.432	-1.239	4.852
3	Rolling VAR	1	Y	Y	N	Y	60	11.287	10.490	12.085	18.849	2.873	34.824	0.413	-0.987	1.347	2.456	-2.330	7.242	-1.790	5.029
4	Rolling VAR	1	N	Y	Y	N	60	11.814	10.834	12.794	23.456	3.692	43.221	0.354	-0.666	1.035	1.736	-2.252	5.724	-1.891	4.638
5	Rolling VAR	1	N	Y	Y	Y	60	10.922	10.505	11.338	10.453	3.984	16.922	0.710	-0.035	1.206	1.374	-1.537	4.285	-0.591	6.833
6	Rolling VAR	1	Y	N	Y	N	60	10.844	10.410	11.278	10.861	3.396	18.326	0.676	-0.061	1.168	1.078	-1.931	4.086	-0.894	4.355
7	Rolling VAR	1	Y	Y	Y	Y	60	12.056	11.476	12.636	14.297	3.506	25.087	0.598	-0.152	1.098	0.891	-2.140	3.922	-0.870	5.154
8	Expanding VAR	1	Y	N	Y	N	60	11.561	9.810	13.312	44.353	16.497	72.209	0.182	-0.668	0.748	0.495	-6.589	7.578	-0.628	4.018
9	Rolling VAR	1	N	N	Y	Y	60	11.396	10.569	12.223	19.726	4.235	35.217	0.400	-0.640	1.094	0.074	-4.506	4.654	-1.367	4.949
10	Expanding VAR	1	Y	N	N	N	60	10.390	9.591	11.188	19.343	3.340	35.346	0.356	-0.565	0.970	-0.301	-5.043	4.442	-1.618	5.213
	Median Expanding VAR performance						60	11.257	9.989	12.525	34.245	6.384	62.106	0.227	-1.192	1.172	-6.110	-11.320	-0.901	-1.767	4.331
	Median Rolling VAR performance						60	11.392	10.094	12.689	31.565	4.335	58.795	0.250	-0.769	0.930	-11.139	-16.877	-5.402	-1.488	4.476

Table 14

Summary Statistics for Monthly Realized, Recursively Rebalanced Optimal Portfolio Weights: Bayesian Strategies Applied to theAsset Menu Including HFRI Fund Weighted Composite Index (FWC) (γ = 5)

The tables shows sample means, standard deviations, and the lower and upper bounds of the 90% sample range of the recursive portfolio weights computed from a range of VAR models for predictable risk premia and of constant investment opportunities (IID) models. The table presents statistics for 1-m T-bill weights, long-term (infinite horizon) weights, and for their differences, the hedging demands.

CER rank	Model	Lags	Predictors included								Cash			Stocks			US Long-Term Treasuries			US Corporate Bonds			REITs			FWC		
			Default	Term	Short	DY	SMB	BMX	MOM	COM	T=1	Long	Hedging	T=1	Long	Hedging	T=1	Long	Hedging	T=1	Long	Hedging	T=1	Long	Hedging	T=1	Long	Hedging
											Sample Mean of Portfolio Weights																	
1	Rolling VAR	1	N	N	Y	Y	Y	N	Y	Y	0.149	-0.374	-0.523	0.071	-0.226	-0.297	0.198	0.009	-0.189	0.226	0.263	0.037	0.187	0.580	0.393	0.170	0.748	0.578
2	Rolling VAR	1	N	N	Y	Y	Y	Y	Y	N	0.007	-0.188	-0.195	0.246	0.123	-0.124	0.002	-0.025	-0.027	0.168	0.220	0.052	0.388	0.393	0.005	0.189	0.478	0.289
3	Rolling VAR	1	N	N	Y	Y	N	N	Y	Y	-0.002	-0.254	-0.251	0.218	0.187	-0.030	0.045	-0.062	-0.107	0.206	0.246	0.040	0.377	0.375	-0.002	0.157	0.507	0.350
4	Rolling AR	1	N	N	N	N	N	N	N	N	0.012	-0.225	-0.237	0.227	0.207	-0.020	0.008	-0.085	-0.093	0.180	0.209	0.029	0.389	0.394	0.005	0.183	0.500	0.318
5	Rolling VAR	1	N	N	Y	Y	N	Y	Y	Y	-0.452	-0.533	-0.081	-0.163	-0.028	0.135	0.035	0.035	0.000	0.204	0.254	0.050	0.798	0.449	-0.349	0.579	0.824	0.245
6	Rolling VAR	1	N	N	Y	Y	N	N	N	Y	0.004	-0.224	-0.228	0.245	0.156	-0.089	-0.014	-0.043	-0.029	0.184	0.203	0.019	0.367	0.401	0.033	0.213	0.507	0.294
7	Rolling VAR	1	N	N	Y	Y	Y	N	N	Y	0.044	-0.195	-0.239	0.203	0.205	0.002	0.054	-0.056	-0.110	0.179	0.179	0.000	0.354	0.379	0.025	0.166	0.488	0.322
8	Rolling VAR	1	N	N	N	N	N	N	N	N	0.030	-0.209	-0.239	0.236	0.259	0.023	0.027	-0.087	-0.113	0.175	0.200	0.025	0.371	0.379	0.008	0.162	0.459	0.297
9	Rolling VAR	1	N	N	Y	Y	N	Y	N	N	0.032	-0.215	-0.247	0.248	0.192	-0.056	0.012	-0.077	-0.089	0.171	0.233	0.062	0.377	0.376	0.000	0.161	0.492	0.331
10	Rolling VAR	1	N	N	Y	Y	Y	Y	N	N	0.028	-0.186	-0.214	0.250	0.183	-0.066	0.015	-0.074	-0.089	0.165	0.193	0.028	0.360	0.390	0.030	0.183	0.494	0.311
											Sample Standard Deviation of Portfolio Weights																	
1	Rolling VAR	1	N	N	Y	Y	Y	N	Y	Y	0.291	0.255	-0.036	0.519	0.770	0.251	0.334	0.559	0.513	0.378	0.419	0.418	0.493	0.352	0.438	0.370	0.211	0.332
2	Rolling VAR	1	N	N	Y	Y	Y	Y	Y	N	0.497	0.521	0.025	0.905	1.075	0.171	0.759	0.894	1.112	0.595	0.794	0.917	0.998	0.966	1.317	0.799	0.580	0.494
3	Rolling VAR	1	N	N	Y	Y	N	N	Y	Y	0.494	0.530	0.036	0.887	1.086	0.199	0.718	0.874	1.091	0.566	0.782	0.891	1.003	0.994	1.318	0.803	0.596	0.496
4	Rolling AR	1	N	N	N	N	N	N	N	N	0.504	0.508	0.004	0.937	1.050	0.113	0.783	0.875	1.084	0.608	0.771	0.916	1.017	0.990	1.292	0.814	0.594	0.487
5	Rolling VAR	1	N	N	Y	Y	N	Y	Y	Y	0.453	0.453	0.006	1.063	1.063	0.086	0.817	0.817	0.000	0.737	0.737	0.385	0.827	0.827	0.777	0.661	0.496	0.038
6	Rolling VAR	1	N	N	Y	Y	N	N	N	Y	0.495	0.503	0.008	0.909	1.062	0.153	0.762	0.855	1.101	0.576	0.773	0.910	1.006	0.952	1.310	0.805	0.571	0.501
7	Rolling VAR	1	N	N	Y	Y	Y	N	N	Y	0.437	0.514	0.077	0.784	1.056	0.272	0.614	0.835	0.967	0.516	0.777	0.855	0.890	0.945	1.198	0.712	0.567	0.464
8	Rolling VAR	1	N	N	N	N	N	N	N	N	0.396	0.503	0.107	0.726	1.044	0.318	0.576	0.812	0.937	0.469	0.770	0.823	0.813	0.948	1.143	0.650	0.569	0.444
9	Rolling VAR	1	N	N	Y	Y	N	Y	N	N	0.414	0.499	0.085	0.752	1.042	0.290	0.598	0.810	0.941	0.498	0.763	0.825	0.846	0.953	1.161	0.677	0.572	0.467
10	Rolling VAR	1	N	N	Y	Y	Y	Y	N	N	0.418	0.492	0.074	0.781	1.059	0.279	0.671	0.845	1.036	0.518	0.768	0.864	0.915	0.927	1.226	0.732	0.556	0.467
											Empirical 90% Range																	
1	Rolling VAR	1	N	N	Y	Y	Y	N	Y	Y	0.506	0.461	0.690	1.010	1.267	1.291	0.558	0.898	0.871	0.604	0.676	0.690	0.616	0.305	0.735	0.285	0.826	0.834
2	Rolling VAR	1	N	N	Y	Y	Y	Y	Y	N	1.302	1.395	1.685	1.676	1.783	2.264	1.427	1.561	1.976	1.142	1.412	1.659	1.619	1.585	2.431	0.788	1.181	1.323
3	Rolling VAR	1	N	N	Y	Y	N	N	Y	Y	1.268	1.363	1.724	1.647	1.765	2.304	1.340	1.520	1.955	1.064	1.376	1.558	1.622	1.597	2.532	0.845	1.186	1.307
4	Rolling AR	1	N	N	N	N	N	N	N	N	1.327	1.306	1.695	1.723	1.765	2.240	1.454	1.550	1.977	1.172	1.399	1.658	1.627	1.612	2.475	0.873	1.168	1.299
5	Rolling VAR	1	N	N	Y	Y	N	Y	Y	Y	1.134	1.134	0.431	1.776	1.776	0.750	1.427	1.427	0.000	1.350	1.350	0.693	1.357	1.357	0.518	1.109	1.109	0.094
6	Rolling VAR	1	N	N	Y	Y	N	N	N	Y	1.297	1.307	1.783	1.695	1.763	2.280	1.441	1.494	1.928	1.091	1.389	1.624	1.702	1.566	2.478	0.824	1.164	1.323
7	Rolling VAR	1	N	N	Y	Y	Y	N	N	Y	1.095	1.352	1.658	1.508	1.730	2.077	1.156	1.454	1.711	0.968	1.362	1.520	1.495	1.545	2.100	0.710	1.179	1.277
8	Rolling VAR	1	N	N	N	N	N	N	N	N	0.981	1.321	1.561	1.339	1.715	2.040	1.070	1.423	1.679	0.849	1.355	1.470	1.337	1.537	1.883	0.591	1.150	1.214
9	Rolling VAR	1	N	N	Y	Y	N	Y	N	N	1.007	1.321	1.538	1.418	1.735	2.031	1.141	1.428	1.627	0.916	1.366	1.457	1.369	1.519	1.975	0.674	1.178	1.239
10	Rolling VAR	1	N	N	Y	Y	Y	Y	N	N	1.031	1.261	1.596	1.498	1.767	2.135	1.284	1.498	1.818	0.955	1.416	1.564	1.496	1.510	2.157	0.702	1.155	1.245

Table 15

Top 18 Models Ranked According to Realized CER: Bayesian Strategies Applied to the Asset Menu Including HFRI Fund Weighted Composite Index (FWC) ($\gamma = 5$)

The tables shows annualized sample means, standard deviations, the Sharpe ratio, and the Certainty Equivalent Return computed from the recursive portfolio weights under the ten top performing (in terms of CER) models for (predictable) risk premia, as defined by the VAR order (where a VAR(0) is equivalent to IID, constant investment opportunities), the selection of predictors, and whether the models are estimated either on an expanding or on a rolling 10-year window of data. The calculations are performed assuming the investor assess realized performance with a 5-year horizon (H = 60 months) and using the predictive density from a Bayesian estimate of the predictability model for excess asset returns, when the priors are non-informative. Monthly rebalancing applies and it is taken into account by a long-horizon investor. In the table, we have boldfaced all performance statistics that turn out to be superior to those recorded in Table 5 for the baseline asset menu that excludes the hedge fund strategies.

CER rank	Model	Lags	Predictors included								H	Annualized mean (%)			Annualized volatility (%)			Annualized Sharpe ratio			Annualized CER (%)			Skewness	Kurtosis
			Term	Short	DY	Default	SMB	PtfsBD	PtfsIR	COM		Mean returns	90% Conf. Int. - LB	90% Conf. Int. - UB	Volatility	90% Conf. Int. - LB	90% Conf. Int. - UB	Sharpe ratio	90% Conf. Int. - LB	90% Conf. Int. - UB	CER (% Ann.)	90% Conf. Int. - LB	90% Conf. Int. - UB		
1	Rolling VAR	1	N	Y	Y	N	N	Y	Y	Y	60	**13.508**	**13.327**	**13.689**	29.062	24.185	33.939	0.344	0.166	0.523	**14.878**	**11.752**	**18.004**	**0.581**	**4.256**
2	Rolling VAR	1	N	Y	Y	N	Y	Y	N	Y	60	**12.785**	**12.608**	12.962	28.586	25.109	32.063	0.325	0.158	0.492	**13.605**	**11.450**	15.760	**0.657**	**3.129**
3	Rolling VAR	1	N	Y	Y	N	N	Y	Y	Y	60	**13.134**	**12.973**	13.295	26.021	23.232	28.811	0.370	0.220	0.521	**13.500**	**10.872**	16.127	**0.324**	**3.691**
4	Rolling VAR	1	N	N	N	Y	N	N	N	Y	60	12.344	12.172	12.517	27.748	24.420	31.076	0.319	0.162	0.476	**12.091**	**8.235**	15.947	**0.701**	**3.115**
5	Rolling VAR	1	N	Y	Y	N	Y	Y	Y	N	60	**13.595**	**13.412**	**13.778**	29.705	26.452	32.958	0.340	0.193	0.487	11.159	6.375	15.942	**0.943**	**3.771**
6	Rolling VAR	1	N	Y	Y	N	Y	N	Y	N	60	12.227	12.063	12.392	27.169	23.777	30.561	0.321	0.160	0.483	10.092	**7.508**	12.675	**0.635**	**3.254**
7	Rolling VAR	1	N	Y	Y	N	N	N	Y	Y	60	**13.699**	**13.547**	**13.852**	24.714	21.576	27.853	0.413	0.256	0.570	8.770	6.332	11.207	**0.719**	**5.326**
8	Roll. Gaussian IID	0	N	N	N	N	N	N	N	N	60	**12.286**	12.106	12.465	29.297	25.441	33.153	0.300	0.151	0.449	8.767	4.670	12.864	**0.821**	**3.550**
9	Rolling VAR	1	N	Y	Y	N	Y	N	N	N	60	11.519	11.356	11.681	26.266	23.676	28.857	0.305	0.160	0.450	8.748	4.334	13.161	**0.911**	**2.470**
10	Rolling VAR	1	N	Y	Y	N	Y	N	N	N	60	11.344	11.171	11.516	27.785	23.989	31.582	0.282	0.133	0.432	6.925	5.188	8.662	**0.836**	**3.749**
11	Rolling VAR	1	N	Y	Y	N	Y	N	Y	N	60	**13.489**	**13.324**	**13.653**	26.336	22.228	30.443	0.379	0.223	0.536	6.680	3.409	9.950	**0.532**	**4.419**
12	Rolling VAR	1	N	Y	Y	Y	N	Y	N	Y	60	**12.770**	**12.592**	12.947	28.604	24.945	32.263	0.324	0.162	0.486	6.535	0.634	12.435	**0.657**	**3.437**
13	Rolling VAR	1	N	Y	Y	Y	N	Y	N	Y	60	11.495	11.362	11.628	21.724	18.632	24.815	0.368	0.227	0.509	6.053	3.902	8.204	**0.643**	**3.902**
14	Rolling VAR	1	N	Y	Y	Y	N	N	N	N	60	11.465	11.308	11.621	25.318	22.892	27.743	0.315	0.177	0.452	4.702	2.038	7.366	**0.999**	**2.388**
15	Rolling VAR	1	N	Y	Y	Y	N	N	N	N	60	12.062	**11.884**	12.240	28.446	24.701	32.191	0.301	0.157	0.445	4.243	0.616	7.870	**0.858**	**3.526**
16	Rolling VAR	1	N	Y	Y	Y	N	N	Y	Y	60	10.747	10.572	10.922	28.642	25.212	32.072	0.253	0.109	0.397	3.963	1.820	6.106	**0.844**	**3.077**
17	Rolling VAR	1	N	Y	Y	Y	Y	Y	Y	Y	60	11.346	11.164	11.527	29.256	25.112	33.399	0.268	0.120	0.416	3.695	-0.397	7.788	**0.784**	**3.836**
18	Rolling VAR	1	N	Y	Y	Y	Y	Y	N	Y	60	10.564	10.402	10.725	26.224	22.417	30.031	0.269	0.126	0.413	2.459	-1.133	6.051	**0.821**	**4.084**
	Median Rolling VAR performance										60	**12.892**	**12.548**	**13.237**	19.204	8.828	29.581	**0.489**	-0.379	**1.057**	5.345	-4.321	13.506	**0.617**	**3.976**

Table 16

Top 18 Models Ranked According to Realized CER: Bayesian Strategies Applied to the Asset Menu Including HFRI Fund of Funds Composite (FFP) (γ = 5)

The tables shows annualized sample means, standard deviations, the Sharpe ratio, and the Certainty Equivalent Return computed from the recursive portfolio weights under the ten top performing (in terms of CER) models for (predictable) risk premia, as defined by the VAR order (where a VAR(0) is equivalent to IID, constant investment opportunities), the selection of predictors, and whether the models are estimated either on an expanding or on a rolling 10-year window of data. The calculations are performed assuming the investor assess realized performance with a 5-year horizon (H = 60 months) and using the predictive density from a Bayesian estimate of the predictability model for excess asset returns, when the priors are non-informative. Monthly rebalancing applies and it is taken into account by a long-horizon investor. In the table, we have boldfaced all performance statistics that turn out to be superior to those recorded in Table 5 for the baseline asset menu that excludes the hedge fund strategies.

CER rank	Model	Lags	Predictors included								H	Annualized mean (%)			Annualized volatility (%)			Annualized Sharpe ratio			Annualized CER (%)			Skewness	Kurtosis
			Term	Short	DY	Default	SMB	PtfsBD	PtfsIR	COM		Mean returns	90% Conf. Int. - LB	90% Conf. Int. - UB	Volatility	90% Conf. Int. - LB	90% Conf. Int. - UB	Sharpe ratio	90% Conf. Int. - LB	90% Conf. Int. - UB	CER (% Ann.)	90% Conf. Int. - LB	90% Conf. Int. - UB		
1	Rolling VAR	1	N	Y	Y	N	N	Y	Y	Y	60	**12.597**	**12.419**	**12.775**	26.823	22.795	30.850	0.339	0.169	0.509	**12.016**	**7.221**	16.810	**0.119**	**4.317**
2	Rolling AR	1	N	N	N	Y	N	N	N	N	60	**12.936**	**12.746**	**13.125**	28.088	24.311	31.865	0.336	0.177	0.494	10.191	6.161	14.220	**-0.186**	**3.617**
3	Rolling VAR	1	N	Y	Y	Y	Y	Y	Y	Y	60	**12.625**	**12.452**	**12.798**	26.227	22.845	29.610	0.348	0.189	0.506	9.082	5.290	12.875	**0.111**	**3.426**
4	Rolling VAR	1	N	Y	Y	Y	Y	N	N	Y	60	**13.871**	**13.670**	**14.073**	30.016	26.416	33.616	0.346	0.194	0.497	8.365	6.027	10.702	**0.068**	**3.056**
5	Rolling VAR	1	Y	Y	Y	Y	Y	Y	N	Y	60	12.136	**11.950**	12.322	27.841	25.137	30.545	0.310	0.161	0.459	7.628	4.559	10.698	**0.336**	**2.403**
6	Rolling VAR	1	N	Y	Y	Y	Y	Y	N	N	60	11.840	11.659	12.021	26.702	22.861	30.543	0.312	0.163	0.462	7.459	5.121	9.797	**0.249**	**3.865**
7	Rolling VAR	1	N	Y	Y	Y	Y	N	N	N	60	**12.650**	**12.456**	**12.844**	29.371	24.669	34.072	0.312	0.149	0.474	7.316	1.978	12.654	**-0.297**	**4.625**
8	Rolling VAR	1	N	Y	Y	N	N	N	Y	Y	60	11.849	11.671	12.028	26.633	22.699	30.566	0.314	0.151	0.476	6.193	2.574	9.811	**-0.327**	**4.299**
9	Rolling VAR	1	N	Y	Y	N	N	N	Y	N	60	12.082	11.908	12.256	26.566	23.200	29.931	0.323	0.180	0.466	5.709	2.898	8.519	**0.639**	**3.349**
10	Roll. Gaussian IID	0	N	N	N	N	N	N	N	N	60	**13.088**	**12.917**	**13.259**	25.952	22.722	29.182	0.369	0.217	0.522	5.701	2.681	8.721	**0.096**	**4.311**
11	Rolling VAR	1	N	Y	Y	Y	Y	Y	Y	N	60	**13.251**	**13.053**	**13.450**	30.067	25.806	34.329	0.324	0.170	0.479	5.655	1.515	9.796	**0.058**	**3.889**
12	Rolling VAR	1	N	Y	Y	N	N	N	N	Y	60	**12.881**	**12.728**	**13.034**	23.091	20.015	26.166	0.406	0.250	0.562	4.960	1.852	8.067	**-0.266**	5.833
13	Rolling VAR	1	N	Y	Y	Y	Y	N	Y	Y	60	**12.666**	**12.490**	**12.842**	26.388	22.202	30.574	0.347	0.192	0.502	3.964	0.203	7.724	**-0.180**	**4.607**
14	Rolling VAR	1	N	Y	Y	N	N	Y	N	N	60	**13.099**	**12.904**	**13.294**	28.889	25.230	32.547	0.332	0.183	0.482	3.926	0.182	7.670	**0.046**	**3.345**
15	Rolling VAR	1	N	Y	Y	N	N	Y	N	Y	60	**13.678**	**13.497**	**13.860**	27.015	23.381	30.649	0.377	0.232	0.522	3.325	1.480	5.170	**0.186**	**3.616**
16	Rolling VAR	1	N	N	Y	Y	Y	N	Y	Y	60	10.851	10.355	11.348	**9.338**	**5.728**	16.948	**0.894**	-1.665	3.454	3.026	-10.388	16.441	**-0.657**	9.173
17	Rolling VAR	1	Y	Y	Y	N	N	Y	Y	N	60	**12.781**	**12.561**	**13.002**	32.708	29.014	36.401	0.284	0.140	0.427	2.653	-0.192	5.498	**0.395**	**2.833**
18	Rolling VAR	1	N	Y	Y	N	N	N	N	N	60	**13.864**	**13.674**	**14.053**	28.977	25.627	32.327	0.358	0.209	0.506	2.385	-2.208	6.979	**-0.031**	**3.939**
	Median Rolling VAR performance										60	**12.436**	**12.077**	**12.794**	19.263	10.459	**28.067**	0.464	-0.480	**1.408**	2.258	-5.665	10.181	**-0.183**	**3.001**

Figure 1

Possible Realized Performance Indicators under Alternative Model Assumptions

The plots represent the annualized Sharpe ratio (on the horizontal axis), the annualized percentage CER (vertical axis), and the Jarque-Bera statistic for non-normalities (a composite of realized skewness and kurtosis, the size of the circles) derived from three alternative theoretical models. The latter model in fact represents the case in which the empirical properties of the data prevent from establishing any specific links between the realized Sharpe ratio, skewness, and kurtosis of the optimal portfolio weights.

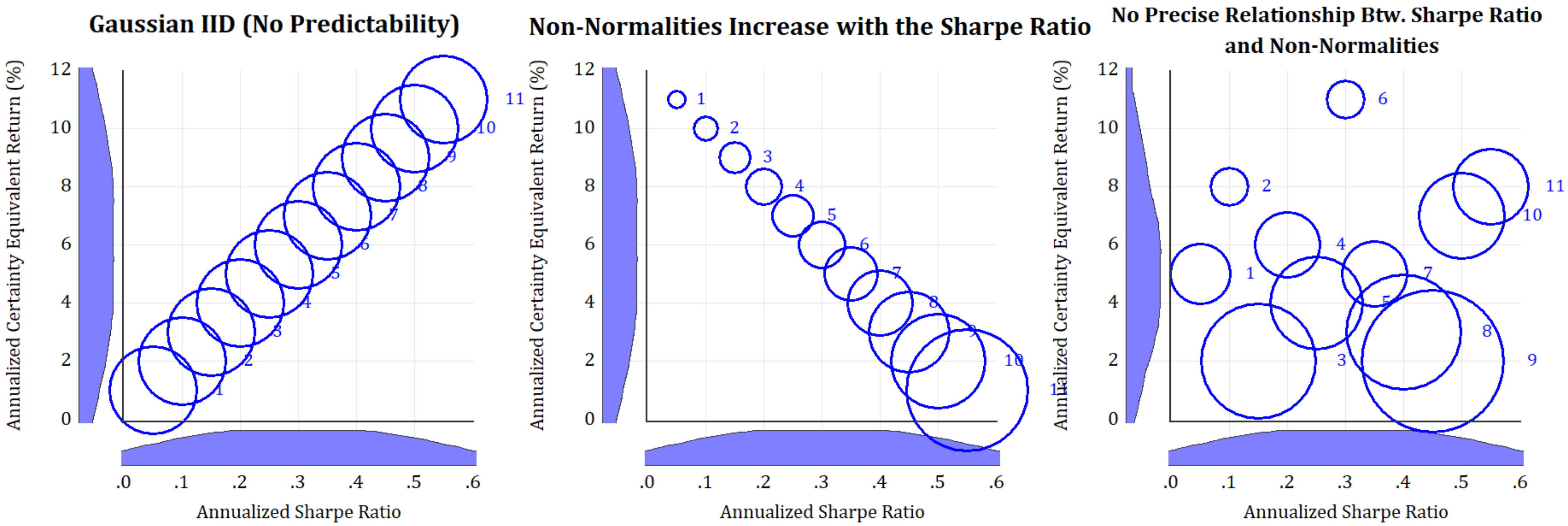


Figure 2

Comparisons of Realized Performance Indicators for the Top Models

The plots represent the mean (as a solid horizontal line), median (as a cross), and realized 90% range (as a bin) of OOS performance measures obtained with references to a recursive portfolio exercises for the sample 2004:01 – 2014:12. Each measure refers to either a benchmark asset menu that excludes HF strategies or to extended menus that include either a composite value-weighted index of all HF strategies or to one strategy at the time. In the case of skewness and kurtosis, we report 90% confidence intervals based on a delta-method approximation of their standard error.

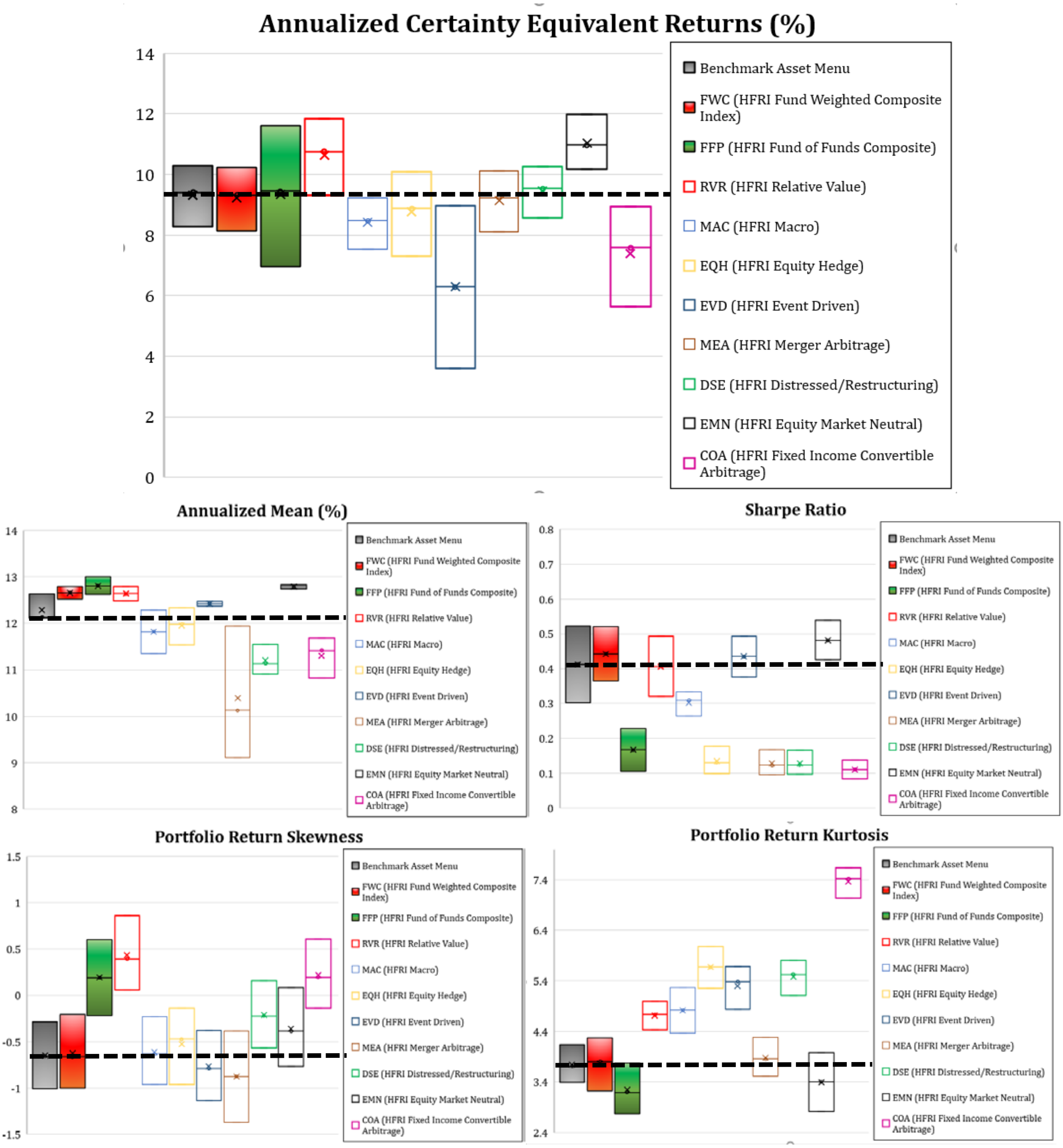


Figure 3

Comparisons of Realized Performance Indicators for the Median Expanding VAR Models

The plots represent the mean (as a solid horizontal line), median (as a cross), and realized 90% range (as a bin) of OOS performance measures obtained with references to a recursive, portfolio exercises for the sample 2004:01 – 2014:12. Each measure refers to either a benchmark asset menu that excludes HF strategies or to extended menus that include either a composite value-weighted index of all HF strategies or to one strategy at the time. In the case of skewness and kurtosis, we report 90% confidence intervals based on a delta-method approximation of their standard error.

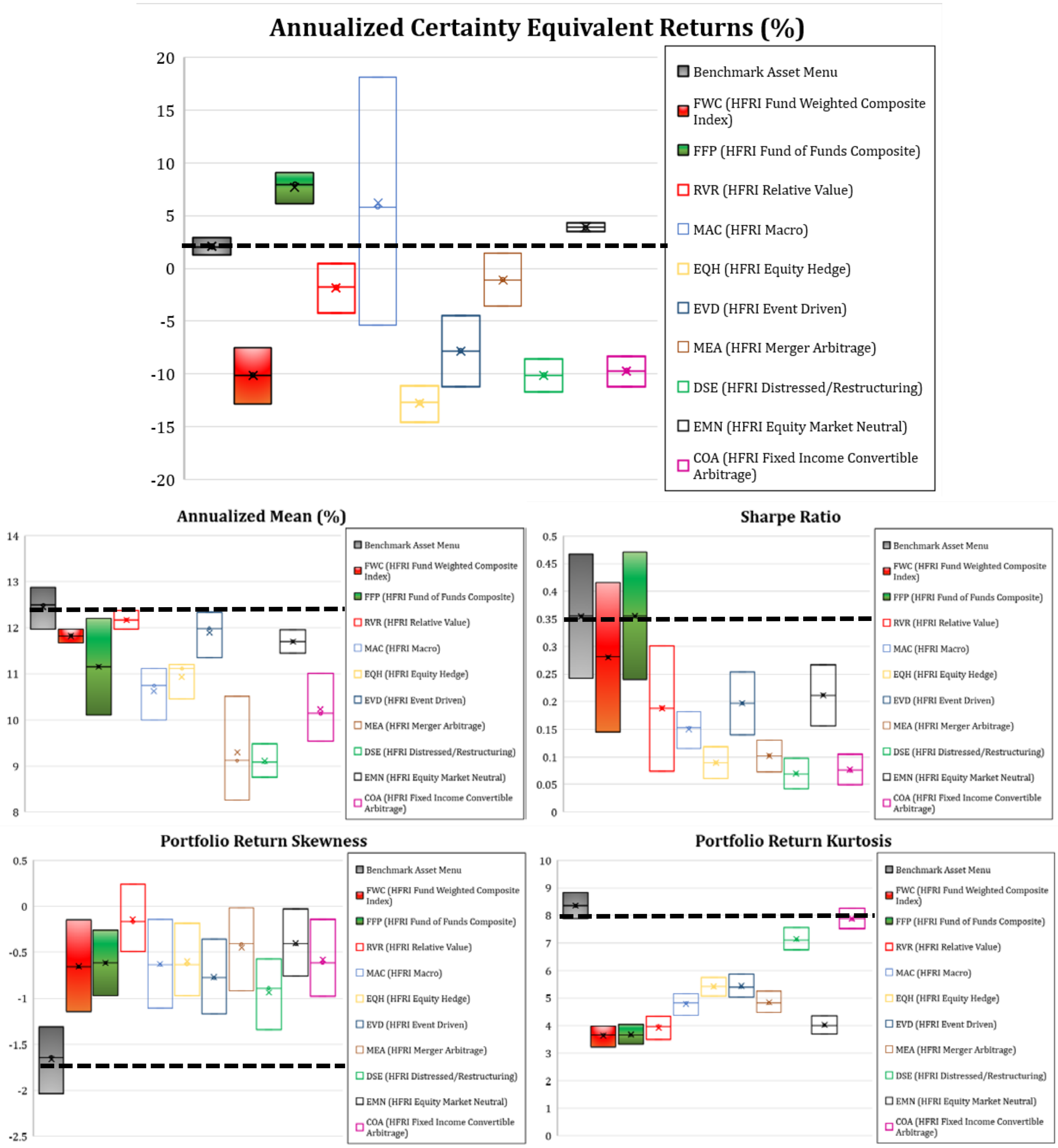
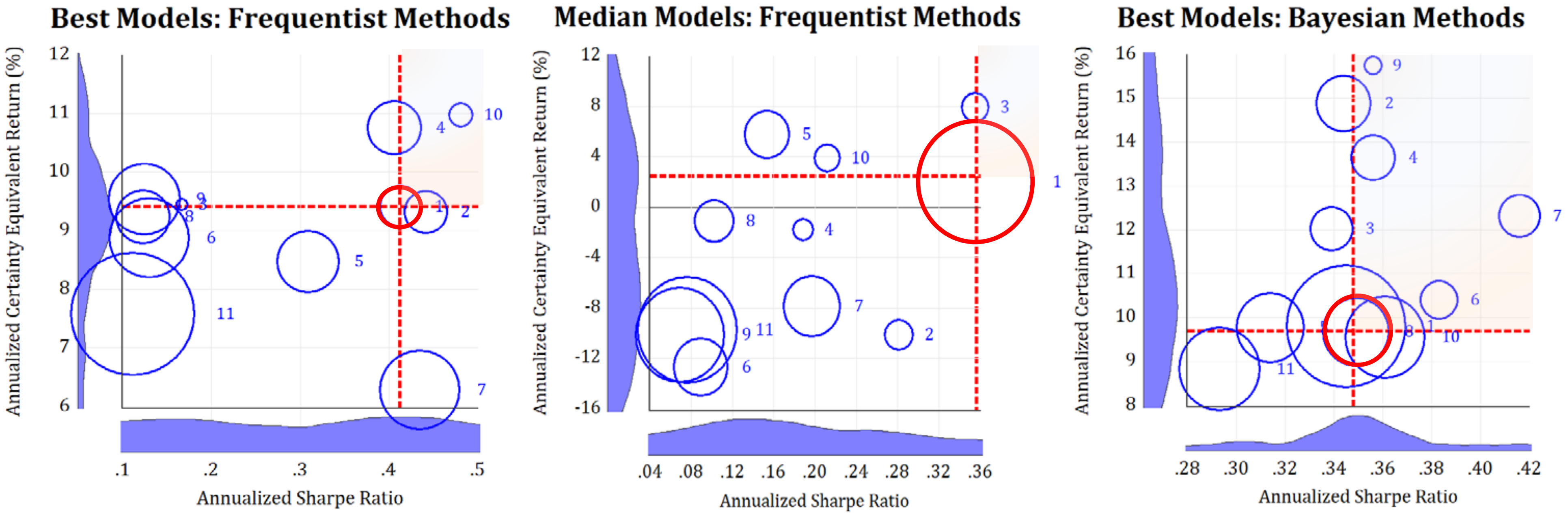


Figure 4

Empirically Realized Performance Indicators

The plots represent the annualized Sharpe ratio (on the horizontal axis), the annualized percentage CER (vertical axis), and the Jarque-Bera statistic for non-normalities (a composite of realized skewness and kurtosis, the size of the circles) derived from OOS performance measures obtained with references to recursive, portfolio exercises for the sample 2004:01 – 2019:12. The calculations are performed assuming the investor assesses realized performance with a 5-year horizon. Monthly rebalancing applies. In the plot, larger circles indicate increasing non-normalities and the red circle refers to the benchmark allocation excluding hedge fund strategies. The legends to the different portfolio/hedge fund strategies are the bottom of the plots.



Legend: 1 = Benchmark, 5-year optimal portfolio with monthly rebalancing; 2= FWC, HFRI Fund Weighted Composite Index; 3 = FFP, HFRI Fund of Funds Composite Index; 4 = RVR, HFRI Relative Value Index; 5 = MAC, HFRI Macro Index; 6 = EQH, HFRI Equity Hedge Index; 7 = EVD, HFRI Event Driven Index; 8= MEA, HFRI Merger Arbitrage Index; 9 = DSE = HFRI Distressed/Restructuring Index; 10 = EMN, HFRI Equity Market Neutral Index; 11= COA, HFRI RV Fixed Income Convertible Arbitrage.

Figure 5

Comparisons of Realized Performance Indicators for the Median Expanding VAR Models – Bayesian Optima Portfolio Strategies

The plots represent the mean (as a solid horizontal line), median (as a cross), and realized 90% range (as a bin) of OOS performance measures obtained with references to a recursive, portfolio exercises for the sample 2004:01 – 2014:12. The calculations are performed assuming the investor assess realized performance with a 5-year horizon (H = 60 months) and using the predictive density from a Bayesian estimate of the predictability model for excess asset returns, when the priors are non-informative. Monthly rebalancing applies and it is taken into account by a long-horizon investor.

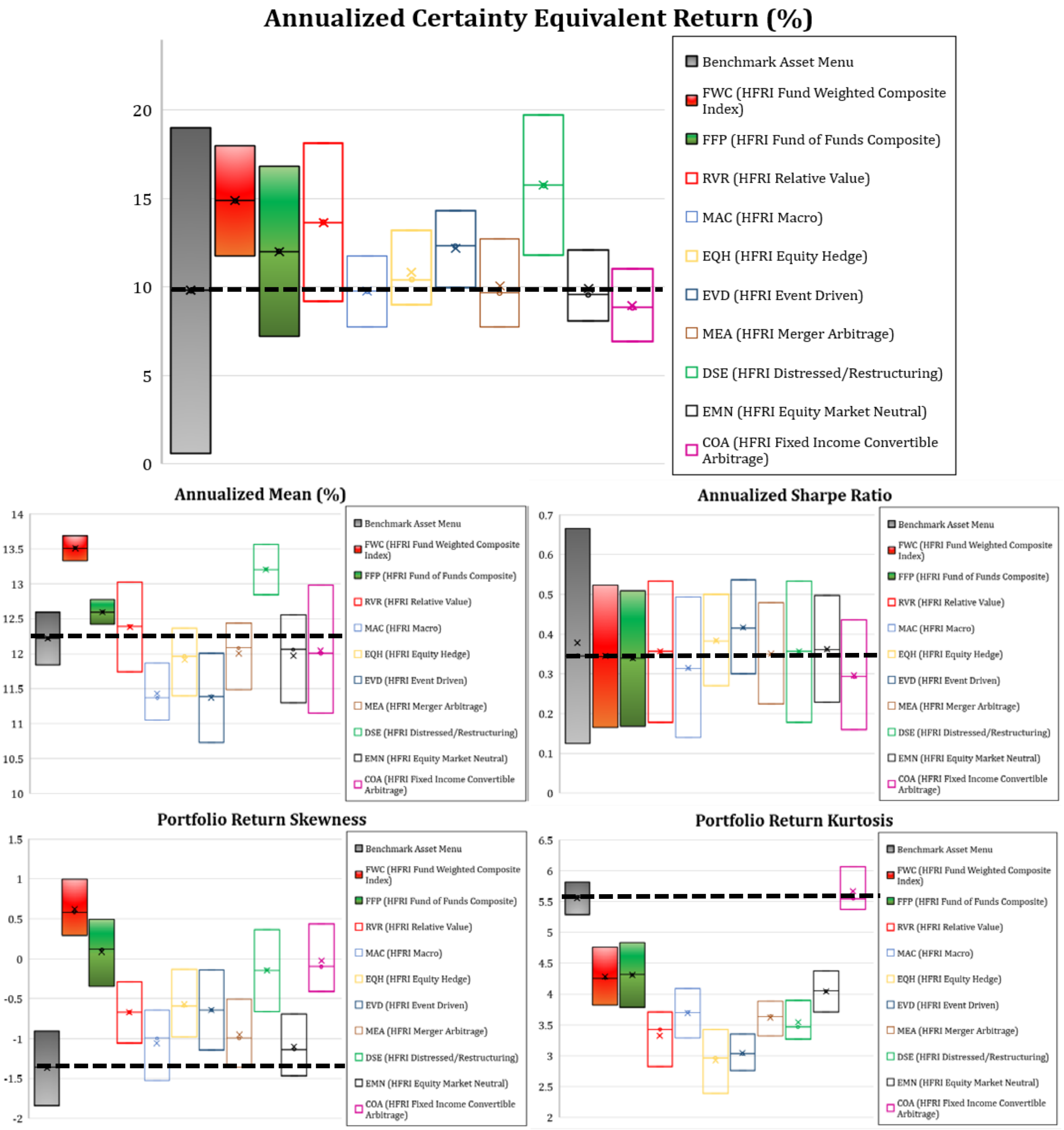


Table A1

Top 10 Models Ranked According to Realized CER for Buy-and-Hold and Monthly Rebalancing Strategies: Baseline Asset Menu (γ = 2)

The tables shows annualized sample means, standard deviations, the Sharpe ratio, and the Certainty Equivalent Return computed from the recursive portfolio weights under the ten top performing (in terms of CER) models for (predictable) risk premia, as defined by the VAR order (where a VAR(0) is equivalent to IID, constant investment opportunities), the selection of predictors, and whether the models are estimated either on an expanding or on a rolling 10-year window of data. The calculations are performed assuming the investor assess realized performance with a 5-year horizon (H = 60 months).

CER rank	Model	Lags	Predictors included				*H*	Annualized mean (%)			Annualized volatility (%)			Annualized Sharpe ratio			Annualized CER (%)			Skewness	Kurtosis
			Def.	Term	Short	DY		Mean returns	90% Conf. Int. - LB	90% Conf. Int. - UB	Volatility	90% Conf. Int. - LB	90% Conf. Int. - UB	Sharpe ratio	90% Conf. Int. - LB	90% Conf. Int. - UB	CER (% Ann.)	90% Conf. Int. - LB	90% Conf. Int. - UB		
Buy-and-hold																					
1	Rolling VAR	1	N	N	N	Y	60	14.292	14.219	14.364	30.254	27.871	32.638	0.357	0.301	0.412	1.718	-1.735	5.170	-0.616	9.173
2	Rolling VAR	1	N	Y	N	Y	60	13.802	13.731	13.873	32.869	30.483	35.256	0.313	0.258	0.369	1.717	-1.757	5.192	-0.873	9.374
3	Rolling VAR	1	N	N	Y	Y	60	13.516	13.448	13.585	30.981	28.604	33.359	0.323	0.268	0.379	1.710	-1.618	5.038	-0.806	9.934
4	Exp. Gaussian IID	0	N	N	N	N	60	14.076	14.005	14.147	32.045	29.650	34.440	0.330	0.274	0.386	1.708	-1.859	5.275	-0.872	9.685
5	Rolling Gaussian IID	0	N	N	N	N	60	13.862	13.789	13.935	30.999	28.625	33.372	0.334	0.278	0.390	1.708	-1.817	5.233	-0.815	9.146
6	Rolling VAR	2	N	Y	N	N	60	12.573	12.501	12.644	30.422	28.059	32.785	0.298	0.242	0.355	1.708	-1.842	5.258	-0.918	9.660
7	Expanding VAR	1	N	N	Y	Y	60	13.317	13.244	13.389	32.098	29.729	34.468	0.306	0.251	0.361	1.708	-1.845	5.260	-0.802	9.120
8	Expanding VAR	1	N	Y	N	Y	60	12.349	12.276	12.422	31.215	28.803	33.627	0.283	0.228	0.339	1.708	-1.793	5.209	-0.848	8.910
9	Expanding VAR	1	N	N	N	Y	60	14.319	14.247	14.391	33.793	31.405	36.182	0.320	0.265	0.376	1.708	-1.790	5.205	-0.913	10.035
10	Expanding VAR	1	N	Y	Y	Y	60	12.544	12.471	12.617	33.354	30.935	35.773	0.271	0.216	0.326	1.708	-1.757	5.172	-0.908	9.747
	Median Expanding VAR performance						60	13.766	13.643	13.889	58.022	53.567	62.478	0.177	0.121	0.233	-4.667	-12.571	3.237	-1.085	9.235
	Median Rolling VAR performance						60	13.442	13.314	13.569	57.240	52.724	61.756	0.174	0.119	0.229	-4.646	-12.662	3.369	-1.014	9.430
Montlhy rebalancing																					
1	Expanding VAR	1	Y	N	Y	N	60	13.827	13.768	13.887	26.233	24.208	28.258	0.394	0.340	0.448	12.304	8.256	16.352	0.395	3.913
2	Rolling VAR	1	Y	N	Y	Y	60	13.505	13.448	13.562	24.988	23.051	26.924	0.400	0.348	0.453	11.300	7.694	14.907	0.440	4.984
3	Rolling VAR	1	Y	N	N	Y	60	14.631	14.577	14.685	22.416	20.486	24.347	0.497	0.444	0.549	11.073	7.779	14.367	0.990	7.534
4	Rolling VAR	1	Y	Y	N	Y	60	14.725	14.672	14.777	21.562	17.363	50.487	0.521	0.467	0.574	10.879	7.576	14.183	0.605	6.303
5	Expanding VAR	2	Y	N	N	Y	60	13.693	13.631	13.754	25.190	22.738	27.641	0.405	0.347	0.462	10.335	6.283	14.386	-0.265	8.277
6	Expanding VAR	2	Y	N	Y	Y	60	14.812	14.752	14.873	25.728	23.267	28.190	0.440	0.382	0.498	9.929	5.945	13.912	-0.168	8.768
7	Rolling VAR	1	Y	Y	N	N	60	12.957	12.895	13.019	27.796	25.311	30.281	0.340	0.284	0.397	9.678	5.755	13.602	-0.104	8.298
8	Expanding VAR	2	Y	Y	N	N	60	12.558	12.495	12.620	27.727	25.199	30.256	0.327	0.270	0.383	9.548	5.619	13.477	0.064	9.036
9	Rolling VAR	1	Y	N	N	N	60	13.203	13.140	13.266	27.175	24.711	29.639	0.357	0.300	0.414	9.536	5.940	13.133	-0.018	9.002
10	Expanding VAR	2	Y	N	Y	N	60	13.579	13.518	13.640	27.513	24.990	30.037	0.366	0.310	0.423	9.412	5.558	13.265	-0.025	8.945
	Median Expanding VAR performance						60	14.014	13.932	14.095	48.676	45.523	51.828	0.216	0.161	0.271	-5.339	-8.197	-2.482	-0.492	6.176
	Median Rolling VAR performance						60	13.720	13.642	13.797	42.866	40.075	45.657	0.238	0.183	0.294	-5.932	-9.716	-2.147	-0.926	6.185

Table A2

Top 10 Models Ranked According to Realized CER for Buy-and-Hold and Monthly Rebalancing Strategies: Baseline Asset Menu (γ = 10)

The tables shows annualized sample means, standard deviations, the Sharpe ratio, and the Certainty Equivalent Return computed from the recursive portfolio weights under the ten top performing (in terms of CER) models for (predictable) risk premia, as defined by the VAR order (where a VAR(0) is equivalent to IID, constant investment opportunities), the selection of predictors, and whether the models are estimated either on an expanding or on a rolling 10-year window of data. The calculations are performed assuming the investor assess realized performance with a 5-year horizon (H = 60 months).

CER rank	Model	Lags	Predictors included				H	Annualized mean (%)			Annualized volatility (%)			Sharpe ratio			Annualized CER (%)			Skewness	Kurtosis
			Def.	Term	Short	DY		Mean returns	90% Conf. Int. - LB	90% Conf. Int. - UB	Volatility	90% Conf. Int. - LB	90% Conf. Int. - UB	Sharpe ratio	90% Conf. Int. - LB	90% Conf. Int. - UB	CER (% Ann.)	90% Conf. Int. - LB	90% Conf. Int. - UB		
Buy-and-hold																					
1	Rolling VAR	1	Y	N	Y	N	60	10.257	10.188	10.326	33.243	32.209	34.384	0.203	0.068	0.338	3.626	-2.443	9.695	-0.608	7.355
2	Rolling VAR	1	Y	Y	Y	Y	60	10.253	10.188	10.319	32.252	31.204	33.358	0.209	0.075	0.343	3.362	-3.009	9.732	-0.622	7.611
3	Rolling VAR	1	N	Y	Y	Y	60	10.255	10.188	10.321	32.207	31.134	33.278	0.210	0.075	0.344	3.271	-3.185	9.727	-0.612	7.567
4	Rolling VAR	2	Y	Y	Y	Y	60	10.249	10.184	10.314	31.328	30.293	32.399	0.215	0.082	0.348	3.245	-2.643	9.133	-0.615	7.876
5	Rolling VAR	1	Y	N	Y	Y	60	10.255	10.187	10.322	32.470	31.406	33.610	0.208	0.074	0.342	3.227	-3.464	9.917	-0.620	7.580
6	Rolling VAR	1	N	N	Y	Y	60	10.258	10.190	10.325	33.040	32.008	34.183	0.205	0.070	0.339	3.196	-3.500	9.893	-0.589	7.332
7	Rolling VAR	2	Y	N	N	Y	60	10.256	10.188	10.324	33.072	32.009	34.164	0.204	0.070	0.339	3.174	-3.443	9.791	-0.611	7.422
8	Rolling VAR	2	N	Y	Y	Y	60	10.251	10.185	10.317	31.749	30.738	32.863	0.213	0.079	0.346	3.162	-3.106	9.430	-0.611	7.716
9	Expanding VAR	2	Y	Y	Y	Y	60	10.257	10.188	10.326	33.658	32.589	34.762	0.201	0.067	0.335	3.097	-3.528	9.721	-0.596	7.314
10	Rolling VAR	2	Y	N	Y	Y	60	10.254	10.186	10.321	32.574	31.511	33.677	0.207	0.074	0.340	3.092	-3.329	9.512	-0.602	7.571
	Median Expanding VAR performance						60	10.260	10.188	10.331	34.443	33.371	35.550	0.196	0.061	0.331	1.363	-9.265	11.990	-0.624	7.040
	Median Rolling VAR performance						60	10.258	10.188	10.327	33.673	32.621	34.795	0.201	0.066	0.335	2.346	-6.740	11.433	-0.617	7.247
Montlhy rebalancing																					
1	Expanding VAR	1	Y	N	Y	N	60	10.257	10.211	10.304	22.611	22.155	23.113	0.299	-0.008	0.606	15.528	8.472	22.583	0.542	3.082
2	Rolling VAR	1	Y	N	Y	N	60	10.253	10.203	10.304	24.228	23.460	25.083	0.279	-0.041	0.599	12.869	9.153	16.584	-0.014	6.273
3	Rolling VAR	1	Y	N	Y	Y	60	10.255	10.207	10.302	22.727	22.231	23.237	0.297	-0.004	0.598	12.378	8.008	16.747	0.558	4.328
4	Expanding VAR	1	Y	N	N	N	60	10.249	10.197	10.301	25.495	24.623	26.473	0.265	-0.055	0.584	12.228	6.182	18.275	-0.203	4.258
5	Rolling VAR	1	Y	Y	N	Y	60	10.255	10.209	10.301	22.154	21.649	22.698	0.305	-0.002	0.612	11.197	7.623	14.770	0.508	4.622
6	Rolling VAR	1	Y	N	N	Y	60	10.258	10.212	10.303	22.084	21.562	22.620	0.306	0.005	0.607	10.904	7.132	14.676	0.503	4.646
7	Rolling VAR	1	Y	Y	Y	N	60	10.256	10.204	10.308	24.861	24.003	25.794	0.272	-0.049	0.592	10.715	4.860	16.569	-0.161	4.167
8	Rolling VAR	1	Y	N	N	N	60	10.251	10.197	10.305	25.947	25.001	27.010	0.260	-0.060	0.581	10.332	5.353	15.310	-0.323	5.196
9	Expanding VAR	1	Y	Y	N	N	60	10.257	10.203	10.311	26.128	25.158	27.186	0.259	-0.055	0.572	10.227	5.972	14.481	-0.306	5.116
10	Expanding VAR	2	Y	N	N	Y	60	10.254	10.200	10.307	25.881	24.924	26.995	0.261	-0.055	0.577	9.763	4.222	15.303	-0.367	5.616
	Median Expanding VAR performance						60	10.260	10.204	10.315	26.514	25.582	27.562	0.255	-0.055	0.564	7.663	2.907	12.419	-0.302	7.656
	Median Rolling VAR performance						60	10.258	10.199	10.317	27.544	26.581	28.613	0.245	-0.052	0.543	3.093	-2.077	8.263	-0.305	7.158

Table A3

Top 18 Models Ranked According to Realized CER: HFRI Equity Market Neutral (EMN) (γ = 5)

The tables shows annualized sample means, standard deviations, the Sharpe ratio, and the Certainty Equivalent Return computed from the recursive portfolio weights under the ten top performing (in terms of CER) models for (predictable) risk premia, as defined by the VAR order (where a VAR(0) is equivalent to IID, constant investment opportunities), the selection of predictors, and whether the models are estimated either on an expanding or on a rolling 10-year window of data. The calculations are performed assuming the investor assess realized performance with a 5-year horizon (H = 60 months). Monthly rebalancing applies and it is taken into account by a long-horizon investor. In the table, we have boldfaced all performance statistics that turn out to be superior to those recorded in Table 5 for the baseline asset menu that excludes the hedge fund strategies.

CER rank	Model	Lags	Predictors included								H	Annualized mean (%)			Annualized volatility (%)			Annualized Sharpe ratio			Annualized CER (%)			Skewness	Kurtosis
			Term	Short	DY	Def.	SMB	PtfsBD	PtfsIR	COM		Mean returns	90% Conf. Int. - LB	90% Conf. Int. - UB	Volatility	90% Conf. Int. - LB	90% Conf. Int. - UB	Sharpe ratio	90% Conf. Int. - LB	90% Conf. Int. - UB	CER (% Ann.)	90% Conf. Int. - LB	90% Conf. Int. - UB		
1	Expanding VAR	1	N	Y	N	Y	N	N	Y	N	60	12.791	12.747	12.834	**19.308**	**18.506**	**20.110**	**0.481**	**0.425**	0.538	**10.971**	**10.951**	**10.991**	**-0.386**	**3.402**
2	Expanding VAR	1	N	Y	N	Y	Y	N	Y	N	60	12.409	12.365	12.452	**19.301**	**18.494**	**20.108**	**0.462**	**0.404**	0.519	**10.943**	**10.924**	**10.963**	**-0.385**	**3.405**
3	Expanding VAR	1	N	Y	N	Y	N	Y	Y	N	60	12.637	12.593	12.681	**19.263**	**18.444**	**20.081**	**0.474**	**0.417**	0.532	**10.911**	**10.891**	**10.930**	**-0.391**	**3.415**
4	Expanding VAR	1	N	Y	N	Y	Y	Y	Y	N	60	12.192	12.148	12.236	**19.264**	**18.455**	**20.074**	**0.451**	**0.395**	0.507	**10.835**	**10.816**	**10.855**	**-0.388**	**3.417**
5	Expanding VAR	1	N	Y	N	Y	N	Y	Y	Y	60	12.524	12.480	12.568	**19.345**	**18.549**	**20.140**	**0.466**	**0.410**	0.523	**10.683**	**10.664**	**10.702**	**-0.396**	**3.396**
6	Expanding VAR	1	N	Y	N	Y	N	N	Y	Y	60	11.673	11.629	11.717	**19.500**	**18.679**	**20.321**	**0.419**	0.362	0.476	**10.338**	**10.319**	**10.357**	**-0.417**	**3.387**
7	Expanding VAR	1	N	Y	N	Y	N	N	N	N	60	11.720	11.674	11.766	20.589	**19.436**	21.742	0.399	0.341	0.457	9.159	**9.140**	9.179	-0.879	5.015
8	Expanding VAR	1	N	Y	N	Y	Y	N	N	N	60	13.050	13.004	13.097	20.577	**19.412**	21.743	**0.464**	**0.406**	0.522	9.067	**9.047**	9.087	-0.877	5.020
9	Expanding VAR	1	N	Y	N	Y	N	Y	N	N	60	12.014	11.967	12.061	20.526	**19.378**	21.674	**0.415**	0.357	0.473	8.954	**8.934**	8.973	-0.885	5.048
10	Expanding VAR	1	N	Y	N	Y	Y	Y	N	N	60	11.174	11.128	11.221	20.526	**19.382**	21.670	0.374	0.315	0.433	8.937	**8.917**	8.956	-0.884	5.048
11	Expanding VAR	1	N	Y	N	Y	N	Y	N	Y	60	13.044	12.998	13.091	20.596	**19.459**	21.734	**0.463**	**0.405**	0.521	8.768	**8.748**	8.787	-0.884	4.999
12	Expanding VAR	1	N	Y	N	Y	N	N	N	Y	60	12.401	12.354	12.449	20.776	**19.625**	21.927	**0.428**	**0.371**	0.486	8.378	**8.359**	8.397	-0.890	4.910
13	Expanding VAR	1	N	Y	N	Y	Y	Y	Y	Y	60	10.482	10.432	10.532	21.742	20.498	22.986	0.321	0.263	0.379	6.743	6.724	6.762	-1.087	5.204
14	Expanding VAR	1	N	Y	N	Y	Y	Y	N	Y	60	12.004	11.954	12.054	21.833	20.564	23.102	0.389	0.331	0.448	6.558	6.539	6.578	-1.320	5.283
15	Expanding VAR	1	N	Y	N	Y	Y	N	N	Y	60	10.182	10.132	10.232	21.941	20.668	23.214	0.305	0.246	0.363	6.404	6.384	6.423	-1.145	5.391
16	Expanding VAR	1	N	Y	N	Y	Y	N	Y	Y	60	10.567	10.516	10.617	22.114	20.788	23.439	0.320	0.262	0.377	6.149	6.129	6.169	-1.198	5.582
17	Expanding VAR	1	N	N	N	Y	N	N	N	Y	60	12.085	11.958	12.213	56.658	53.937	59.379	0.152	0.097	0.206	5.103	5.054	5.151	-0.149	4.036
18	Expanding VAR	1	N	N	N	Y	N	N	Y	N	60	10.278	10.153	10.402	55.205	52.653	57.757	0.123	0.067	0.179	3.963	3.922	4.003	**-0.328**	3.825
							Median Expanding VAR performance				60	**12.664**	**12.620**	12.708	43.318	40.852	45.784	0.212	0.156	0.267	**3.880**	**3.837**	**3.922**	**-0.408**	**4.013**

Table A4

Top 18 Models Ranked According to Realized CER: HFRI Event-Driven (EVD) (γ = 5)

The tables shows annualized sample means, standard deviations, the Sharpe ratio, and the Certainty Equivalent Return computed from the recursive portfolio weights under the ten top performing (in terms of CER) models for (predictable) risk premia, as defined by the VAR order (where a VAR(0) is equivalent to IID, constant investment opportunities), the selection of predictors, and whether the models are estimated either on an expanding or on a rolling 10-year window of data. The calculations are performed assuming the investor assess realized performance with a 5-year horizon (H = 60 months). Monthly rebalancing applies and it is taken into account by a long-horizon investor. In the table, we have boldfaced all performance statistics that turn out to be superior to those recorded in Table A3 for the baseline asset menu that excludes the hedge fund strategies.

CER rank	Model	Lags	Predictors included								H	Annualized mean (%)			Annualized volatility (%)			Annualized Sharpe ratio			Annualized CER (%)			Skewness	Kurtosis
			Term	Short	DY	Default	SMB	PtfsBD	PtfsIR	COM		Mean returns	90% Conf. Int. - LB	90% Conf. Int. - UB	Volatility	90% Conf. Int. - LB	90% Conf. Int. - UB	Sharpe ratio	90% Conf. Int. - LB	90% Conf. Int. - UB	CER (% Ann.)	90% Conf. Int. - LB	90% Conf. Int. - UB		
1	Expanding VAR	1	N	Y	N	Y	N	Y	N	Y	60	12.425	12.378	12.471	20.534	**19.398**	21.671	0.435	**0.377**	0.492	6.286	3.597	8.975	-0.788	5.380
2	Expanding VAR	1	N	Y	N	Y	N	N	N	Y	60	11.871	11.823	11.918	20.718	**19.583**	21.852	0.404	0.346	0.462	6.210	3.471	8.950	-0.764	5.282
3	Expanding VAR	1	N	Y	N	Y	N	Y	N	N	60	12.195	12.147	12.242	20.540	**19.406**	21.675	0.423	**0.366**	0.481	5.891	3.192	8.591	-0.777	5.354
4	Expanding VAR	1	N	Y	N	Y	N	N	N	N	60	11.946	11.898	11.994	20.819	**19.685**	21.954	0.406	0.349	0.463	5.873	3.118	8.629	-0.776	5.223
5	Expanding VAR	1	N	Y	N	Y	Y	N	N	N	60	11.220	11.171	11.268	21.408	20.229	22.586	0.361	0.304	0.417	5.861	3.048	8.674	-0.575	5.260
6	Expanding VAR	1	N	Y	N	Y	Y	N	N	Y	60	13.002	12.954	13.051	21.437	20.264	22.610	0.443	**0.387**	0.499	5.697	2.940	8.453	-0.597	5.236
7	Expanding VAR	1	N	Y	N	Y	Y	Y	N	N	60	12.656	12.609	12.704	20.969	19.847	22.091	0.437	**0.380**	0.493	5.679	2.983	8.375	-0.776	5.179
8	Expanding VAR	1	N	Y	N	Y	Y	Y	N	Y	60	12.513	12.465	12.561	21.022	19.883	22.161	0.429	**0.372**	0.486	5.423	2.739	8.108	-0.687	5.206
9	Expanding VAR	1	N	Y	N	Y	N	N	Y	N	60	11.154	11.107	11.202	20.952	19.789	22.115	0.365	0.309	0.422	5.244	2.626	7.862	-0.822	5.206
10	Expanding VAR	1	N	Y	N	Y	N	N	Y	Y	60	12.542	12.495	12.590	20.912	19.759	22.065	0.432	**0.376**	0.489	5.213	2.635	7.791	-0.828	5.224
11	Expanding VAR	1	N	Y	N	Y	N	Y	Y	N	60	11.570	11.523	11.617	20.703	**19.568**	21.837	0.390	0.334	0.446	5.115	2.392	7.838	-0.800	5.253
12	Expanding VAR	1	N	Y	N	Y	N	Y	Y	Y	60	12.045	11.998	12.092	20.706	**19.574**	21.837	0.413	0.355	0.470	4.997	2.284	7.710	-0.799	5.251
13	Expanding VAR	1	N	Y	N	Y	Y	Y	Y	Y	60	12.438	12.387	12.490	22.429	21.027	23.830	0.399	0.341	0.456	3.061	0.707	5.416	-1.249	6.490
14	Expanding VAR	1	N	Y	N	Y	Y	N	Y	Y	60	13.238	13.186	13.290	22.412	21.003	23.821	0.434	**0.378**	0.491	2.289	-0.236	4.814	-1.225	6.482
15	Expanding VAR	1	N	Y	N	Y	Y	Y	Y	N	60	10.859	10.809	10.910	22.334	20.913	23.755	0.330	0.273	0.386	2.115	-0.343	4.573	-1.241	6.535
16	Expanding VAR	1	N	Y	N	Y	Y	N	Y	N	60	10.558	10.507	10.609	22.332	20.885	23.779	0.316	0.259	0.373	2.096	-0.344	4.537	-1.240	6.532
17	Expanding AR	1	N	N	N	N	N	N	N	N	60	11.238	11.179	11.297	25.922	23.696	28.148	0.299	0.245	0.352	-6.473	-8.778	-4.168	-2.289	10.887
18	Expanding N. IID	0	N	N	N	N	N	N	N	N	60	12.467	12.410	12.525	25.269	23.010	27.527	0.355	0.301	0.408	-6.732	-8.157	-5.308	-2.478	11.580
						Median Expanding VAR performance					60	11.984	11.935	12.034	43.092	40.517	45.667	0.197	0.140	0.254	-7.829	-11.170	-4.488	**-0.777**	**5.206**

Table A5

Top 18 Models Ranked According to Realized CER: Bayesian Optimal Allocation Applied to the Asset Menu Including HFRI Fixed Income Relative Value/Arbitrage Strategies (RVR) (γ = 5)

The tables shows annualized sample means, standard deviations, the Sharpe ratio, and the Certainty Equivalent Return computed from the recursive portfolio weights under the ten top performing (in terms of CER) models for (predictable) risk premia, as defined by the VAR order (where a VAR(0) is equivalent to IID, constant investment opportunities), the selection of predictors, and whether the models are estimated either on an expanding or on a rolling 10-year window of data. The calculations are performed assuming the investor assess realized performance with a 5-year horizon (H = 60 months) and using the predictive density from a Bayesian estimate of the predictability model for excess asset returns, when the priors are non-informative. Monthly rebalancing applies and it is taken into account by a long-horizon investor. In the table, we have boldfaced all performance statistics that turn out to be superior to those recorded in Table 5 for the baseline asset menu that excludes the hedge fund strategies.

CER rank	Model	Lags	Predictors included								H	Annualized mean (%)			Annualized volatility (%)			Annualized Sharpe ratio			Annualized CER (%)			Skewness	Kurtosis
			Term	Short	DY	Default	SMB	PtfsBD	PtfsIR	COM		Mean returns	90% Conf. Int. - LB	90% Conf. Int. - UB	Volatility	90% Conf. Int. - LB	90% Conf. Int. - UB	Sharpe ratio	90% Conf. Int. - LB	90% Conf. Int. - UB	CER (% Ann.)	90% Conf. Int. - LB	90% Conf. Int. - UB		
1	Rolling AR	1	N	N	N	Y	N	N	N	N	60	12.213	**12.027**	12.400	27.766	24.349	31.183	0.314	**0.235**	0.392	**13.652**	**9.181**	**18.123**	-0.699	3.927
2	Rolling VAR	1	N	Y	Y	N	N	Y	Y	N	60	11.602	11.442	11.763	24.226	21.144	27.307	0.334	**0.256**	0.413	**12.574**	**9.140**	**16.008**	-0.417	4.042
3	Rolling VAR	1	N	Y	Y	N	N	N	Y	N	60	11.389	11.225	11.553	24.729	22.772	26.687	0.319	**0.247**	0.391	**12.469**	**10.938**	**14.000**	**-0.219**	**2.821**
4	Rolling VAR	1	N	Y	Y	N	N	Y	N	Y	60	12.903	**12.730**	13.077	26.046	22.333	29.759	0.361	**0.282**	0.440	**11.642**	**8.384**	**14.899**	-0.761	4.709
5	Rolling VAR	1	Y	N	Y	Y	N	Y	Y	Y	60	11.352	10.943	11.761	30.729	20.578	40.881	0.256	0.093	0.419	**11.613**	**6.219**	**17.006**	-1.148	5.533
6	Rolling VAR	1	N	Y	Y	N	Y	N	N	N	60	11.936	11.784	12.089	22.870	20.448	25.293	0.369	**0.291**	0.447	**10.544**	**4.710**	**16.377**	-0.410	4.326
7	Rolling VAR	1	N	Y	Y	N	N	N	Y	Y	60	11.730	11.552	11.907	26.451	22.799	30.103	0.311	**0.234**	0.388	**9.710**	**7.303**	**12.117**	-0.528	4.467
8	Rolling VAR	1	N	Y	Y	N	Y	Y	N	N	60	**12.593**	**12.403**	12.783	28.957	26.128	31.785	0.314	**0.242**	0.386	**9.114**	**3.553**	**14.676**	**-0.181**	3.089
9	Rolling VAR	1	Y	N	Y	Y	Y	N	Y	Y	60	11.343	10.969	11.716	28.291	18.963	37.620	0.277	0.121	0.434	**8.296**	0.717	**15.876**	-0.742	5.543
10	Rolling VAR	1	Y	N	Y	Y	Y	Y	Y	Y	60	10.720	10.345	11.095	28.434	20.769	36.099	0.254	0.100	0.408	**7.928**	-0.315	**16.171**	-0.743	4.331
11	Rolling VAR	1	N	Y	Y	N	N	N	N	N	60	10.662	10.468	10.856	28.939	25.682	32.195	0.247	**0.176**	0.319	**7.430**	0.422	**14.439**	**-0.207**	3.603
12	Rolling VAR	1	N	Y	Y	N	N	N	N	Y	60	10.558	10.386	10.730	26.001	22.964	29.039	0.271	**0.199**	0.343	**7.145**	**4.410**	**9.880**	**-0.177**	3.736
13	Rolling VAR	1	N	Y	Y	N	Y	N	N	Y	60	**13.216**	**13.052**	**13.381**	24.797	22.067	27.526	0.392	**0.322**	0.462	**6.788**	**4.670**	8.906	**-0.092**	4.524
14	Rolling VAR	1	N	Y	Y	N	Y	Y	N	Y	60	**12.865**	**12.691**	13.039	26.372	23.255	29.489	0.355	**0.279**	0.431	**6.435**	0.977	**11.894**	-0.630	3.705
15	Rolling VAR	1	N	Y	Y	N	N	Y	N	N	60	10.545	10.357	10.733	28.316	24.686	31.945	0.249	**0.175**	0.322	**5.756**	**3.176**	8.336	-0.710	4.165
16	Roll. Gaussian IID	0	N	N	N	N	N	N	N	N	60	10.257	10.069	10.444	28.149	24.979	31.320	0.240	**0.169**	0.311	5.426	-0.535	**11.387**	**-0.056**	3.526
17	Rolling VAR	1	N	N	Y	N	Y	Y	N	N	60	10.247	9.728	10.765	9.684	**1.829**	17.538	0.697	-0.692	**2.086**	2.414	-10.003	**14.830**	-0.784	7.818
18	Rolling VAR	1	N	N	Y	N	Y	N	Y	N	60	9.755	9.005	10.505	13.555	7.131	28.241	0.461	-0.995	**1.918**	1.738	-12.743	**16.220**	-1.261	4.987
	Median Rolling VAR performance										60	**12.799**	**12.354**	**13.244**	**20.819**	**9.074**	**32.565**	**0.447**	**-0.239**	1.132	**1.452**	-11.694	**14.598**	**-0.788**	4.655

Table A6

Top 18 Models Ranked According to Realized CER: Bayesian Optimal Allocation Applied to the Asset Menu Including HFRI Mergers and Acquisition Strategies (MEA) (γ = 5)

The tables shows annualized sample means, standard deviations, the Sharpe ratio, and the Certainty Equivalent Return computed from the recursive portfolio weights under the ten top performing (in terms of CER) models for (predictable) risk premia, as defined by the VAR order (where a VAR(0) is equivalent to IID, constant investment opportunities), the selection of predictors, and whether the models are estimated either on an expanding or on a rolling 10-year window of data. The calculations are performed assuming the investor assess realized performance with a 5-year horizon (H = 60 months) and using the predictive density from a Bayesian estimate of the predictability model for excess asset returns, when the priors are non-informative. Monthly rebalancing applies and it is taken into account by a long-horizon investor. In the table, we have boldfaced all performance statistics that turn out to be superior to those recorded in Table 5 for the baseline asset menu that excludes the hedge fund strategies.

CER rank	Model	Lags	Predictors included								H	Annualized mean (%)			Annualized volatility (%)			Annualized Sharpe ratio			Annualized CER (%)			Skewness	Kurtosis
			Term	Short	DY	Default	SMB	PtfsBD	PtfsIR	COM		Mean returns	90% Conf. Int. - LB	90% Conf. Int. - UB	Volatility	90% Conf. Int. - LB	90% Conf. Int. - UB	Sharpe ratio	90% Conf. Int. - LB	90% Conf. Int. - UB	CER (% Ann.)	90% Conf. Int. - LB	90% Conf. Int. - UB		
1	Rolling VAR	0	N	Y	Y	N	N	N	Y	Y	60	11.897	11.755	12.040	21.571	19.249	23.894	0.389	**0.310**	0.468	**9.988**	**6.471**	**13.505**	**-0.144**	4.105
2	Rolling AR	1	N	N	N	Y	N	N	N	N	60	**12.374**	**12.194**	12.555	26.965	23.376	30.555	0.329	**0.249**	0.409	**9.089**	**4.990**	**13.189**	-0.804	5.338
3	Rolling VAR	0	N	Y	Y	N	Y	Y	Y	N	60	**12.223**	**12.036**	12.410	28.112	24.670	31.554	0.310	**0.233**	0.387	**6.495**	**4.071**	8.919	-0.682	4.842
4	Rolling VAR	0	N	Y	Y	N	Y	N	N	N	60	12.106	**11.948**	12.265	24.054	20.854	27.255	0.358	**0.278**	0.438	**6.433**	**2.682**	**10.185**	-0.758	5.398
5	Rolling VAR	0	N	Y	Y	N	N	Y	N	N	60	11.426	11.259	11.593	25.258	22.191	28.325	0.314	**0.238**	0.389	**6.407**	**2.261**	**10.552**	-0.349	4.772
6	Rolling VAR	0	Y	Y	Y	N	N	N	Y	N	60	**12.390**	**12.222**	12.559	25.012	21.751	28.273	0.355	**0.281**	0.429	**5.830**	**2.702**	8.958	-0.274	5.309
7	Rolling VAR	0	N	Y	Y	N	N	N	N	Y	60	10.986	10.792	11.181	29.384	24.969	33.799	0.255	**0.176**	0.334	**5.819**	1.348	**10.290**	-0.843	6.330
8	Rolling VAR	0	N	Y	Y	N	Y	Y	Y	Y	60	**12.822**	**12.657**	12.988	24.981	22.089	27.874	0.373	**0.300**	0.446	5.307	-0.283	**10.897**	**-0.117**	6.861
9	Rolling VAR	0	N	Y	Y	N	N	Y	N	Y	60	**12.690**	**12.513**	12.867	26.453	22.217	30.688	0.347	**0.272**	0.423	4.974	1.409	8.539	-0.465	6.889
10	Roll. Gaussian IID	0	N	N	N	N	N	N	N	N	60	11.735	11.565	11.905	25.509	21.671	29.347	0.323	**0.247**	0.399	4.760	0.900	8.621	-0.658	6.336
11	Rolling VAR	0	Y	Y	Y	N	Y	Y	N	N	60	11.331	11.170	11.491	24.591	21.612	27.570	0.318	**0.241**	0.396	4.558	0.486	8.630	-0.642	4.795
12	Rolling VAR	0	N	Y	Y	N	N	Y	Y	Y	60	12.069	**11.890**	12.248	26.959	24.137	29.780	0.318	**0.248**	0.388	4.544	**1.674**	7.415	**-0.058**	4.076
13	Rolling VAR	0	N	Y	Y	N	N	Y	Y	N	60	11.813	11.650	11.976	24.463	21.130	27.796	0.340	**0.263**	0.417	4.250	**1.602**	6.898	-0.794	5.457
14	Rolling VAR	0	N	Y	Y	N	Y	N	Y	Y	60	**12.967**	**12.794**	13.140	25.801	22.325	29.277	0.367	**0.292**	0.442	3.751	-0.093	7.594	-0.716	5.546
15	Rolling VAR	0	Y	Y	Y	N	N	N	N	N	60	11.912	11.748	12.077	25.004	21.729	28.279	0.336	**0.261**	0.412	3.720	0.139	7.300	-0.664	5.250
16	Rolling VAR	0	N	Y	Y	N	Y	Y	N	Y	60	**12.432**	**12.258**	12.605	26.327	23.663	28.992	0.339	**0.267**	0.411	3.415	-2.001	8.832	-0.300	3.956
17	Rolling VAR	0	N	Y	Y	Y	Y	N	N	Y	60	**12.632**	**12.450**	12.814	27.616	24.458	30.773	0.331	**0.260**	0.401	2.600	0.110	5.090	-0.300	4.565
18	Rolling VAR	0	N	N	Y	N	Y	Y	Y	Y	60	**13.108**	**12.367**	**13.850**	13.831	**1.548**	27.114	0.695	-0.162	**1.551**	2.227	-11.579	**16.034**	-1.029	7.938
	Median Rolling VAR performance										60	**11.908**	**11.516**	12.300	**19.966**	10.206	**29.726**	**0.421**	**-0.113**	0.955	**2.174**	**-8.378**	**12.726**	**-0.866**	4.460

Table A7

Top 18 Models Ranked According to Realized CER: HFRI Fund of Funds Composite (FFP) (γ = 2)

The tables shows annualized sample means, standard deviations, the Sharpe ratio, and the Certainty Equivalent Return computed from the recursive portfolio weights under the ten top performing (in terms of CER) models for (predictable) risk premia, as defined by the VAR order (where a VAR(0) is equivalent to IID, constant investment opportunities), the selection of predictors, and whether the models are estimated either on an expanding or on a rolling 10-year window of data. The calculations are performed assuming the investor assess realized performance with a 5-year horizon (H = 60 months). Monthly rebalancing applies and it is taken into account by a long-horizon investor. In the table, we have boldfaced all performance statistics that turn out to be superior to those recorded in Table A3 for the baseline asset menu that excludes the hedge fund strategies.

CER rank	Model	Lags	Predictors included								H	Annualized mean (%)			Annualized volatility (%)			Annualized Sharpe ratio			Annualized CER (%)			Skewness	Kurtosis
			Term	Short	DY	Def.	SMB	PtfsBD	PtfsIR	COM		Mean returns	90% Conf. Int. - LB	90% Conf. Int. - UB	Volatility	90% Conf. Int. - LB	90% Conf. Int. - UB	Sharpe ratio	90% Conf. Int. - LB	90% Conf. Int. - UB	CER (% Ann.)	90% Conf. Int. - LB	90% Conf. Int. - UB		
1	Expanding VAR	1	N	Y	N	Y	N	Y	N	Y	60	13.867	12.502	14.023	60.683	49.545	71.821	0.171	0.063	0.278	**14.836**	7.268	**21.144**	0.403	**5.158**
2	Expanding VAR	1	N	Y	N	Y	N	N	N	Y	60	13.787	12.500	13.943	63.709	52.619	74.799	0.161	0.053	0.270	**14.306**	6.569	**20.753**	0.404	**5.227**
3	Expanding VAR	1	N	Y	N	Y	Y	N	N	Y	60	**15.158**	12.491	**15.314**	56.468	45.317	67.619	0.206	0.099	0.314	**13.713**	6.281	**19.906**	0.395	**6.146**
4	Expanding VAR	1	N	Y	N	Y	Y	Y	Y	Y	60	13.691	12.491	13.849	55.691	44.420	66.961	0.183	0.074	0.292	**13.575**	5.974	**19.910**	0.438	**5.273**
5	Expanding VAR	1	N	Y	N	Y	Y	Y	N	Y	60	13.676	12.486	13.833	63.163	52.034	74.291	0.161	0.053	0.269	**13.225**	6.167	**19.106**	0.442	**5.301**
6	Expanding VAR	1	N	Y	N	Y	N	N	Y	Y	60	14.211	12.483	14.366	55.851	44.555	67.147	0.192	0.084	0.300	**13.155**	5.650	**19.410**	0.412	**5.238**
7	Expanding VAR	1	N	Y	N	Y	Y	N	Y	Y	60	14.527	12.485	14.684	62.948	51.644	74.251	0.175	0.068	0.282	**13.050**	5.796	**19.095**	0.424	**5.287**
8	Expanding VAR	1	N	Y	N	Y	N	N	N	N	60	13.058	12.481	13.214	60.791	49.623	71.960	0.157	0.050	0.265	**12.924**	5.837	**18.829**	0.429	**5.266**
9	Expanding VAR	1	N	Y	N	Y	N	Y	N	N	60	**16.102**	12.480	**16.259**	59.726	48.258	71.193	0.211	0.102	0.320	**12.857**	5.742	**18.786**	0.131	**5.462**
10	Expanding VAR	1	N	Y	N	Y	Y	N	Y	N	60	13.387	12.477	13.543	62.957	51.849	74.064	0.157	0.049	0.265	**12.702**	5.481	**18.719**	0.417	**5.360**
11	Expanding VAR	1	N	Y	N	Y	N	Y	Y	Y	60	14.688	12.477	14.844	61.726	50.421	73.031	0.181	0.072	0.290	**12.673**	5.554	**18.606**	0.292	**5.420**
12	Expanding VAR	1	N	Y	N	Y	Y	Y	Y	N	60	13.123	12.476	13.279	58.964	48.516	69.412	0.163	0.053	0.273	**12.559**	5.240	**18.658**	0.300	**4.944**
13	Expanding VAR	1	N	Y	N	Y	N	N	Y	N	60	12.151	12.308	12.473	56.571	45.140	68.002	0.153	0.044	0.262	**12.557**	5.607	**18.348**	0.427	**5.443**
14	Expanding VAR	1	N	Y	N	Y	Y	Y	N	N	60	13.941	12.477	14.098	63.320	52.484	74.155	0.165	0.057	0.272	**12.403**	5.386	**18.250**	0.333	**5.028**
15	Expanding VAR	1	N	Y	N	Y	N	Y	Y	N	60	14.282	12.471	14.438	59.568	48.118	71.018	0.181	0.072	0.290	11.966	4.741	**17.987**	0.415	**5.477**
16	Expanding VAR	1	N	Y	N	Y	Y	N	N	N	60	11.895	12.052	12.466	61.947	50.808	73.087	0.136	0.030	0.241	11.529	4.929	**17.028**	0.414	**5.160**
17	Expanding VAR	1	N	N	N	Y	Y	N	N	N	60	12.468	12.170	12.527	23.999	19.029	28.970	0.374	0.263	0.485	10.603	8.185	**12.619**	-0.098	**6.736**
18	Expanding VAR	1	N	N	N	Y	Y	N	Y	N	60	14.712	12.170	14.770	21.487	**16.494**	26.481	**0.522**	0.412	0.632	10.597	8.151	**12.635**	-0.098	**8.736**
							Median Expanding VAR performance				60	12.868	12.171	12.927	**46.619**	**38.506**	54.733	0.201	0.091	**0.311**	**10.595**	**8.102**	**12.672**	**0.198**	**5.858**

Table A8

Top 18 Models Ranked According to Realized CER: HFRI Fixed Income Relative Value/Arbitrage (RVR) (γ = 2)

The tables shows annualized sample means, standard deviations, the Sharpe ratio, and the Certainty Equivalent Return computed from the recursive portfolio weights under the ten top performing (in terms of CER) models for (predictable) risk premia, as defined by the VAR order (where a VAR(0) is equivalent to IID, constant investment opportunities), the selection of predictors, and whether the models are estimated either on an expanding or on a rolling 10-year window of data. The calculations are performed assuming the investor assess realized performance with a 5-year horizon (H = 60 months). Monthly rebalancing applies and it is taken into account by a long-horizon investor. In the table, we have boldfaced all performance statistics that turn out to be superior to those recorded in Table A3 for the baseline asset menu that excludes the hedge fund strategies.

CER rank	Model	Lags	Predictors included								H	Annualized mean (%)			Annualized volatility (%)			Annualized Sharpe ratio			Annualized CER (%)			Skewness	Kurtosis
			Term	Short	DY	Def.	SMB	PtfsBD	PtfsIR	COM		Mean returns	90% Conf. Int. - LB	90% Conf. Int. - UB	Volatility	90% Conf. Int. - LB	90% Conf. Int. - UB	Sharpe ratio	90% Conf. Int. - LB	90% Conf. Int. - UB	CER (% Ann.)	90% Conf. Int. - LB	90% Conf. Int. - UB		
1	Expanding VAR	1	N	N	N	Y	Y	N	N	Y	60	**14.218**	**14.064**	**14.372**	60.727	51.125	70.329	0.177	-0.030	0.383	**18.752**	**9.653**	**27.456**	**0.697**	**3.036**
2	Expanding VAR	1	N	N	N	Y	Y	Y	Y	Y	60	**14.599**	**14.446**	**14.753**	61.194	51.368	71.021	0.181	-0.035	0.398	**13.872**	3.706	**23.596**	**0.669**	**3.066**
3	Expanding VAR	1	N	Y	N	Y	Y	Y	N	Y	60	13.646	13.592	13.701	**21.930**	**17.179**	**26.681**	**0.463**	0.253	**0.672**	**13.336**	**9.694**	**16.820**	**0.893**	6.363
4	Expanding VAR	1	N	Y	N	Y	Y	Y	Y	Y	60	13.251	13.195	13.306	**21.991**	**17.303**	**26.679**	**0.443**	0.235	**0.652**	**13.170**	**9.541**	**16.640**	-0.564	4.931
5	Expanding VAR	1	N	Y	N	Y	Y	N	N	Y	60	13.089	13.033	13.144	**22.086**	**17.385**	**26.787**	**0.434**	0.225	**0.644**	**12.845**	**9.218**	16.314	**0.649**	5.870
6	Expanding VAR	1	N	Y	N	Y	Y	N	Y	Y	60	13.349	13.293	13.405	**22.108**	**17.383**	**26.834**	**0.445**	0.232	**0.659**	12.108	**8.732**	15.337	**0.669**	5.173
7	Expanding VAR	1	N	Y	N	Y	Y	Y	N	N	60	12.704	12.645	12.764	**23.599**	**18.738**	28.460	0.390	0.173	**0.607**	9.628	6.322	12.792	**0.397**	4.627
8	Expanding VAR	1	N	Y	N	Y	N	Y	N	N	60	13.323	13.264	13.382	**23.604**	**18.729**	28.479	**0.416**	0.198	**0.635**	9.627	6.280	12.829	**0.396**	4.625
9	Expanding VAR	1	N	Y	N	Y	N	Y	Y	N	60	**14.864**	**14.805**	**14.923**	**23.606**	**18.687**	28.525	**0.481**	0.264	**0.699**	9.585	6.287	12.740	-0.287	6.511
10	Expanding VAR	1	N	Y	N	Y	Y	Y	Y	N	60	13.079	13.020	13.138	**23.601**	**18.732**	28.471	**0.406**	0.189	**0.622**	9.569	6.226	12.766	0.067	4.626
11	Expanding VAR	1	N	Y	N	Y	N	Y	N	Y	60	13.035	12.975	13.094	**23.626**	**18.742**	28.511	**0.404**	0.185	**0.622**	9.519	6.162	12.730	0.067	4.610
12	Expanding VAR	1	N	Y	N	Y	Y	N	N	N	60	13.897	**13.838**	**13.956**	**23.663**	**18.852**	28.473	**0.439**	0.220	**0.659**	9.380	6.066	12.549	0.035	5.281
13	Expanding VAR	1	N	Y	N	Y	N	Y	Y	Y	60	12.049	11.990	12.109	**23.679**	**18.813**	28.545	0.361	0.141	**0.581**	9.277	5.944	12.464	0.285	4.578
14	Expanding VAR	1	N	Y	N	Y	N	N	N	N	60	12.138	12.078	12.198	**23.692**	**18.811**	28.573	0.365	0.145	**0.584**	9.225	5.893	12.412	0.284	4.570
15	Expanding VAR	1	N	Y	N	Y	Y	N	Y	N	60	**14.149**	**14.090**	**14.208**	**23.713**	**18.895**	28.532	**0.449**	0.230	**0.668**	9.187	5.847	12.383	0.028	5.251
16	Expanding VAR	1	N	Y	N	Y	N	N	N	Y	60	13.656	13.597	13.716	**23.730**	**18.889**	28.571	**0.428**	0.211	**0.645**	9.141	5.815	12.322	-0.481	4.547
17	Expanding VAR	1	N	Y	N	Y	N	N	Y	N	60	**14.327**	**14.267**	**14.386**	**23.744**	**18.830**	28.658	**0.456**	0.236	**0.676**	9.047	5.701	12.247	-0.391	5.234
18	Expanding VAR	1	N	Y	N	Y	N	N	Y	Y	60	**14.634**	**14.574**	**14.694**	**23.769**	**18.886**	28.651	**0.468**	0.248	**0.689**	9.019	5.695	12.197	-0.377	5.220
	Median Expanding VAR performance										60	13.324	13.269	13.380	**44.736**	**37.069**	52.404	**0.220**	0.001	**0.438**	**8.671**	**3.096**	**14.004**	**0.281**	**4.118**

Table A9

Top 18 Models Ranked According to Realized CER: HFRI Fund of Funds Composite (FFP) (γ = 10)

The tables shows annualized sample means, standard deviations, the Sharpe ratio, and the Certainty Equivalent Return computed from the recursive portfolio weights under the ten top performing (in terms of CER) models for (predictable) risk premia, as defined by the VAR order (where a VAR(0) is equivalent to IID, constant investment opportunities), the selection of predictors, and whether the models are estimated either on an expanding or on a rolling 10-year window of data. The calculations are performed assuming the investor assess realized performance with a 5-year horizon (H = 60 months). Monthly rebalancing applies and it is taken into account by a long-horizon investor. In the table, we have boldfaced all performance statistics that turn out to be superior to those recorded in Table A3 for the baseline asset menu that excludes the hedge fund strategies.

CER rank	Model	Lags	Predictors included								H	Annualized mean (%)			Annualized volatility (%)			Annualized Sharpe ratio			Annualized CER (%)			Skewness	Kurtosis
			Term	Short	DY	Def.	SMB	PtfsBD	PtfsIR	COM		Mean returns	90% Conf. Int. - LB	90% Conf. Int. - UB	Volatility	90% Conf. Int. - LB	90% Conf. Int. - UB	Sharpe ratio	90% Conf. Int. - LB	90% Conf. Int. - UB	CER (% Ann.)	90% Conf. Int. - LB	90% Conf. Int. - UB		
1	Expanding VAR	1	N	Y	N	Y	N	N	Y	N	60	8.607	8.497	8.717	**20.194**	**19.153**	**21.235**	0.253	**0.195**	0.311	9.078	4.029	14.127	-0.581	3.507
2	Expanding VAR	1	N	Y	N	Y	N	N	Y	Y	60	8.787	8.675	8.898	**20.208**	**19.162**	**21.254**	0.262	**0.204**	0.319	9.061	4.002	14.120	-0.585	3.521
3	Expanding VAR	1	N	Y	N	Y	Y	N	Y	N	60	9.582	9.472	9.693	**20.201**	**19.147**	**21.254**	0.301	**0.243**	0.359	9.060	4.020	14.100	-0.883	3.516
4	Expanding VAR	1	N	Y	N	Y	N	Y	Y	Y	60	8.409	8.298	8.520	**20.209**	**19.163**	**21.255**	0.243	**0.185**	0.301	9.047	3.929	14.166	-0.585	3.522
5	Expanding VAR	1	N	Y	N	Y	N	N	N	Y	60	8.762	8.651	8.874	**20.215**	**19.157**	**21.272**	0.260	**0.202**	0.319	9.038	3.947	14.130	-0.587	3.529
6	Expanding VAR	1	N	Y	N	Y	N	Y	Y	N	60	9.415	9.305	9.525	**20.217**	**19.170**	**21.264**	0.293	**0.234**	0.351	9.025	3.980	14.070	-0.590	3.539
7	Expanding VAR	1	N	Y	N	Y	Y	N	N	N	60	8.167	8.057	8.278	**20.222**	**19.146**	**21.297**	0.231	**0.173**	0.289	9.024	4.009	14.039	-0.590	3.538
8	Expanding VAR	1	N	Y	N	Y	N	N	N	N	60	7.273	7.163	7.383	**20.228**	**19.161**	**21.296**	0.187	**0.128**	0.245	9.021	3.961	14.081	-0.193	3.549
9	Expanding VAR	1	N	Y	N	Y	Y	Y	Y	N	60	8.200	8.089	8.310	**20.218**	**19.151**	**21.285**	0.232	**0.175**	0.290	9.015	3.926	14.104	-0.590	3.541
10	Expanding VAR	1	N	Y	N	Y	Y	Y	Y	Y	60	8.381	8.270	8.491	**20.229**	**19.170**	**21.288**	0.241	**0.183**	0.299	9.007	3.943	14.071	-0.591	3.547
11	Expanding VAR	1	N	Y	N	Y	N	Y	N	N	60	7.213	7.101	7.325	**20.233**	**19.171**	**21.296**	0.184	**0.125**	0.242	8.992	3.970	14.015	-0.495	3.176
12	Expanding VAR	1	N	Y	N	Y	N	Y	N	Y	60	9.563	9.451	9.675	**20.244**	**19.173**	**21.316**	0.300	**0.242**	0.357	8.979	3.921	14.036	-0.597	3.868
13	Expanding VAR	1	N	Y	N	Y	Y	Y	N	N	60	9.922	9.812	10.031	**20.246**	**19.171**	**21.321**	**0.317**	**0.260**	0.375	8.964	3.844	14.084	-0.460	3.577
14	Expanding VAR	1	N	Y	N	Y	Y	N	N	Y	60	7.960	7.850	8.070	**20.230**	**19.183**	**21.278**	0.220	**0.163**	0.278	8.825	4.025	13.626	-0.575	3.531
15	Expanding VAR	1	N	Y	N	Y	Y	Y	N	Y	60	7.624	7.515	7.734	**20.259**	**19.202**	**21.316**	0.204	**0.145**	0.262	8.769	4.007	13.531	-0.583	3.563
16	Expanding VAR	1	N	Y	N	Y	Y	N	Y	Y	60	6.700	6.589	6.810	**20.270**	**19.223**	**21.316**	0.158	**0.100**	0.215	8.275	4.531	12.019	-0.521	3.940
17	Expanding VAR	1	N	Y	N	Y	Y	N	Y	Y	60	9.837	9.614	10.059	41.442	39.411	43.472	0.153	**0.096**	0.210	0.486	-3.612	4.583	-0.975	3.923
18	Expanding VAR	1	N	Y	N	Y	Y	N	N	Y	60	8.492	8.266	8.719	41.607	39.566	43.648	0.120	**0.063**	0.177	0.124	-3.724	3.971	-0.867	3.822
						Median Expanding VAR performance					60	8.366	8.255	8.477	36.466	34.253	38.680	0.133	**0.076**	0.190	0.096	-4.281	4.474	-0.780	**5.511**

Table A10

Top 18 Models Ranked According to Realized CER: HFRI Fixed Income Relative Value/Arbitrage (RVR) (γ = 10)

The tables shows annualized sample means, standard deviations, the Sharpe ratio, and the Certainty Equivalent Return computed from the recursive portfolio weights under the ten top performing (in terms of CER) models for (predictable) risk premia, as defined by the VAR order (where a VAR(0) is equivalent to IID, constant investment opportunities), the selection of predictors, and whether the models are estimated either on an expanding or on a rolling 10-year window of data. The calculations are performed assuming the investor assess realized performance with a 5-year horizon (H = 60 months). Monthly rebalancing applies and it is taken into account by a long-horizon investor. In the table, we have boldfaced all performance statistics that turn out to be superior to those recorded in Table A3 for the baseline asset menu that excludes the hedge fund strategies.

CER rank	Model	Lags	Predictors included								H	Annualized mean (%)			Annualized volatility (%)			Annualized Sharpe ratio			Annualized CER (%)			Skewness	Kurtosis
			Term	Short	DY	Def.	SMB	PtfsBD	PtfsIR	COM		Mean returns	90% Conf. Int. - LB	90% Conf. Int. - UB	Volatility	90% Conf. Int. - LB	90% Conf. Int. - UB	Sharpe ratio	90% Conf. Int. - LB	90% Conf. Int. - UB	CER (% Ann.)	90% Conf. Int. - LB	90% Conf. Int. - UB		
1	Expanding VAR	1	N	Y	N	Y	Y	Y	N	Y	60	**10.875**	**10.778**	**10.972**	42.427	39.731	45.122	0.174	**0.118**	0.229	11.556	6.674	16.438	0.253	4.765
2	Expanding VAR	1	N	Y	N	Y	Y	N	N	Y	60	9.975	9.878	10.073	42.736	39.985	45.488	0.152	**0.097**	0.206	11.211	5.571	16.851	0.140	4.726
3	Expanding VAR	1	N	Y	N	Y	Y	N	Y	Y	60	**10.536**	**10.437**	**10.636**	43.565	40.789	46.341	0.162	**0.107**	0.216	10.295	4.433	16.157	0.119	4.781
4	Expanding VAR	1	N	Y	N	Y	Y	Y	Y	Y	60	**10.235**	10.138	**10.332**	43.220	40.513	45.926	0.156	**0.101**	0.211	10.065	4.237	15.893	0.105	4.659
5	Expanding VAR	1	N	Y	N	Y	Y	Y	N	Y	60	9.514	9.468	9.560	**20.102**	**19.080**	**21.124**	0.299	**0.241**	0.357	8.478	6.028	10.928	-0.708	3.365
6	Expanding VAR	1	N	Y	N	Y	Y	N	N	Y	60	10.036	9.990	10.082	**20.126**	**19.092**	**21.160**	**0.325**	**0.267**	0.383	8.455	6.042	10.869	-0.516	3.990
7	Expanding VAR	1	N	Y	N	Y	Y	Y	Y	Y	60	9.317	9.271	9.363	**20.124**	**19.100**	**21.148**	0.289	**0.231**	0.347	8.451	6.012	10.889	-0.513	3.384
8	Expanding VAR	1	N	Y	N	Y	Y	N	Y	Y	60	9.905	9.859	9.950	**20.071**	**19.038**	**21.104**	**0.319**	**0.262**	0.376	8.130	6.023	10.237	-0.477	3.342
9	Expanding VAR	1	N	Y	N	Y	N	Y	Y	N	60	9.292	9.244	9.341	**21.243**	**20.148**	**22.338**	0.273	**0.216**	0.329	5.975	3.441	8.509	-0.638	3.449
10	Expanding VAR	1	N	Y	N	Y	Y	Y	Y	N	60	**10.906**	**10.858**	**10.954**	**21.283**	**20.170**	**22.396**	**0.348**	**0.291**	0.405	5.957	3.496	8.418	-0.947	3.465
11	Expanding VAR	1	N	Y	N	Y	N	Y	N	N	60	9.814	9.765	9.863	**21.366**	**20.260**	**22.473**	0.296	**0.239**	0.352	5.948	3.346	8.550	-0.649	3.478
12	Expanding VAR	1	N	Y	N	Y	N	N	Y	N	60	**10.749**	**10.700**	**10.798**	**21.541**	**20.406**	22.676	**0.337**	**0.279**	0.394	5.905	3.236	8.574	-0.663	3.514
13	Expanding VAR	1	N	Y	N	Y	Y	Y	N	N	60	**10.906**	**10.857**	**10.955**	**21.546**	**20.396**	22.695	**0.344**	**0.286**	0.401	5.651	3.040	8.262	-0.690	4.194
14	Expanding VAR	1	N	Y	N	Y	N	N	N	N	60	**10.319**	**10.268**	**10.369**	**22.041**	**20.843**	23.239	**0.309**	**0.251**	0.368	5.628	2.669	8.588	-0.706	4.704
15	Expanding VAR	1	N	Y	N	Y	Y	N	N	N	60	**10.921**	**10.871**	**10.971**	**22.185**	**20.962**	23.408	**0.335**	**0.276**	0.393	5.548	2.428	8.669	-0.717	3.814
16	Expanding VAR	1	N	Y	N	Y	Y	N	Y	N	60	**10.074**	**10.024**	10.123	**21.688**	**20.481**	22.895	0.303	**0.246**	0.360	5.302	2.826	7.778	-0.744	3.940
17	Expanding VAR	1	N	Y	N	Y	N	N	N	Y	60	**11.056**	**11.006**	**11.106**	**22.320**	**21.019**	23.622	**0.339**	**0.281**	0.397	4.800	1.864	7.735	-1.052	4.202
18	Expanding VAR	1	N	Y	N	Y	N	Y	N	Y	60	8.951	8.900	9.001	**22.007**	**20.733**	23.281	0.248	**0.189**	0.306	4.756	2.119	7.394	-0.825	4.041
										Median Expanding VAR performance	60	9.484	9.411	9.557	35.641	33.504	37.777	0.168	**0.110**	0.226	**4.751**	**2.025**	7.477	-0.648	**3.516**

Table A11

Summary Statistics for Monthly Recursive Optimally Rebalanced Portfolios: Baseline Asset Menu, No Transaction Costs (γ = 5)

The tables shows sample means, standard deviations, and the lower and upper bounds of the 90% sample range of the recursive portfolio weights computed from a range of VAR models for predictable risk premia and of constant investment opportunities (IID) models. The table presents statistics for 1-m T-bill weights, long-term (infinite horizon) weights, and for their differences, the hedging demands.

CER rank	Model	Lags	Predictors included				Cash			Stocks			US Long-Term Treasuries			US Corporate Bonds			REITs		
			Default	Term	Short	DY	T=1	Long	Hedging	T=1	Long	Hedging	T=1	Long	Hedging	T=1	Long	Hedging	T=1	Long	Hedging
											Sample mean of portofolio weights										
1	Expanding VAR	1	Y	N	Y	N	0.000	0.011	0.011	0.033	0.028	-0.006	0.000	0.011	0.011	0.017	0.019	0.003	0.950	0.931	-0.019
2	Expanding VAR	1	Y	N	N	N	-1.723	-1.663	0.060	1.286	1.546	0.260	-1.466	-1.597	-0.131	1.133	0.995	-0.139	1.770	1.720	-0.050
3	Expanding VAR	2	Y	N	Y	N	0.000	0.009	0.009	0.033	0.042	0.009	0.017	0.017	0.000	0.000	0.009	0.009	0.950	0.923	-0.027
4	Rolling VAR	1	Y	N	Y	Y	0.000	0.010	0.010	0.067	0.102	0.035	0.025	0.018	-0.007	0.000	0.010	0.010	0.908	0.860	-0.048
5	Expanding VAR	1	Y	Y	Y	N	0.000	0.009	0.009	0.025	0.018	-0.008	0.000	0.009	0.009	0.017	0.017	0.000	0.958	0.948	-0.010
6	Expanding VAR	1	Y	N	N	N	0.000	0.010	0.010	0.017	0.010	-0.007	0.000	0.010	0.010	0.017	0.018	0.002	0.967	0.952	-0.015
7	Rolling VAR	1	Y	Y	N	Y	-1.583	-1.646	-0.062	1.603	1.689	0.086	-0.966	-0.891	0.075	0.296	0.187	-0.109	1.650	1.660	0.010
8	Rolling VAR	1	Y	N	N	Y	-1.700	-1.646	0.054	1.343	1.620	0.276	-1.073	-1.102	-0.029	0.756	0.498	-0.258	1.673	1.630	-0.043
9	Expanding VAR	1	Y	Y	Y	Y	-1.660	-1.678	-0.018	1.720	1.688	-0.032	-1.670	-1.624	0.046	0.870	0.918	0.048	1.740	1.696	-0.044
10	Rolling VAR	1	Y	N	Y	N	-1.730	-1.683	0.046	1.003	1.318	0.315	-1.233	-1.468	-0.235	1.197	1.136	-0.060	1.763	1.697	-0.066
											Sample Standard Deviation of Portofolio Weights										
1	Expanding VAR	1	Y	N	Y	N	0.000	0.045	0.045	0.180	0.135	0.110	0.000	0.045	0.045	0.129	0.101	0.103	0.219	0.234	0.180
2	Expanding VAR	1	Y	N	N	N	0.484	0.541	0.492	1.079	0.736	0.887	0.924	0.659	0.845	0.654	0.428	0.448	0.329	0.350	0.485
3	Expanding VAR	2	Y	N	Y	N	0.000	0.040	0.040	0.180	0.183	0.122	0.129	0.099	0.101	0.000	0.040	0.040	0.219	0.252	0.195
4	Rolling VAR	1	Y	N	Y	Y	0.000	0.044	0.044	0.250	0.290	0.161	0.156	0.100	0.136	0.001	0.044	0.044	0.290	0.337	0.268
5	Expanding VAR	1	Y	Y	Y	N	0.000	0.039	0.039	0.157	0.099	0.108	0.000	0.039	0.039	0.129	0.098	0.100	0.201	0.200	0.155
6	Expanding VAR	1	Y	N	N	N	0.000	0.044	0.044	0.129	0.044	0.110	0.000	0.044	0.044	0.129	0.100	0.102	0.180	0.196	0.174
7	Rolling VAR	1	Y	Y	N	Y	0.767	0.594	0.307	0.737	0.475	0.771	1.341	1.266	0.881	1.314	1.204	0.915	0.722	0.572	0.675
8	Rolling VAR	1	Y	N	N	Y	0.544	0.594	0.486	1.043	0.584	0.848	1.271	1.170	0.851	1.162	1.069	0.630	0.652	0.653	0.491
9	Expanding VAR	1	Y	Y	Y	Y	0.613	0.497	0.250	0.477	0.480	0.508	0.631	0.636	0.607	0.686	0.406	0.432	0.463	0.458	0.468
10	Rolling VAR	1	Y	N	Y	N	0.439	0.467	0.494	1.397	0.952	1.046	1.140	0.863	1.033	0.611	0.491	0.389	0.336	0.375	0.450
											Empirical 90% Range										
1	Expanding VAR	1	Y	N	Y	N	0.000	0.164	0.164	0.000	0.200	0.000	0.000	0.164	0.164	0.000	0.195	0.164	0.500	0.800	0.000
2	Expanding VAR	1	Y	N	N	N	0.000	2.000	0.430	3.600	1.600	3.630	2.800	2.000	2.810	0.800	1.600	0.800	0.000	0.814	0.814
3	Expanding VAR	2	Y	N	Y	N	0.000	0.027	0.027	0.000	0.200	0.037	0.000	0.126	0.026	0.000	0.025	0.025	0.506	0.800	0.115
4	Rolling VAR	1	Y	N	Y	Y	0.000	0.100	0.100	1.000	1.000	0.200	0.000	0.200	0.100	0.000	0.100	0.100	1.000	1.000	0.800
5	Expanding VAR	1	Y	Y	Y	N	0.000	0.058	0.058	0.000	0.150	0.000	0.000	0.058	0.058	0.000	0.117	0.057	0.000	0.660	0.000
6	Expanding VAR	1	Y	N	N	N	0.000	0.100	0.100	0.000	0.101	0.000	0.000	0.100	0.100	0.000	0.200	0.100	0.000	0.800	0.000
7	Rolling VAR	1	Y	Y	N	Y	2.800	2.000	0.808	0.800	1.600	1.604	3.220	2.800	2.800	3.600	2.800	4.058	0.000	1.600	0.801
8	Rolling VAR	1	Y	N	N	Y	0.000	2.000	0.012	3.600	1.600	2.811	2.800	2.800	4.160	3.600	3.330	1.600	0.000	1.600	0.801
9	Expanding VAR	1	Y	Y	Y	Y	1.400	1.910	0.001	0.001	1.600	0.801	0.006	2.000	0.955	2.801	0.800	0.020	0.000	1.243	0.594
10	Rolling VAR	1	Y	N	Y	N	0.000	1.951	0.016	3.600	3.592	3.605	2.800	2.792	3.761	0.801	1.600	0.800	0.000	1.403	1.403

Table A12

Top 10 Models Ranked According to Realized CER: Baseline Asset Menu and No Transaction Costs ($\gamma = 5$)

The tables shows annualized sample means, standard deviations, the Sharpe ratio, and the Certainty Equivalent Return computed from the recursive portfolio weights under the ten top performing (in terms of CER) models for (predictable) risk premia, as defined by the VAR order (where a VAR(0) is equivalent to IID, constant investment opportunities), the selection of predictors, and whether the models are estimated either on an expanding or on a rolling 10-year window of data. The calculations are performed assuming the investor assess realized performance with a 5-year horizon (H = 60 months).

CER rank	Model	Lags	Predictors included				*H*	Annualized mean (%)			Annualized volatility (%)			Annualized Sharpe ratio			Annualized CER (%)			Skewness	Kurtosis
			Def.	Term	Short	DY		Mean returns	90% Conf. Int. - LB	90% Conf. Int. - UB	Volatility	90% Conf. Int. - LB	90% Conf. Int. - UB	Sharpe ratio	90% Conf. Int. - LB	90% Conf. Int. - UB	CER (% Ann.)	90% Conf. Int. - LB	90% Conf. Int. - UB		
Buy-and-hold																					
1	Rolling VAR	1	N	N	Y	Y	60	12.069	12.012	12.125	30.204	29.264	31.244	0.300	0.245	0.356	0.548	-0.478	1.324	-0.585	6.670
2	Rolling VAR	1	Y	N	Y	Y	60	12.568	12.511	12.624	29.634	28.658	30.675	0.323	0.267	0.378	0.546	-0.482	1.309	-0.617	6.903
3	Rolling VAR	2	Y	N	N	Y	60	12.751	12.694	12.808	30.223	29.252	31.222	0.323	0.267	0.378	0.542	-0.469	1.313	-0.609	6.747
4	Rolling VAR	1	Y	Y	Y	Y	60	12.244	12.188	12.299	29.729	28.765	30.741	0.311	0.255	0.366	0.518	-0.462	1.297	-0.614	6.877
5	Expanding VAR	2	Y	Y	Y	Y	60	12.607	12.549	12.665	30.658	29.684	31.665	0.313	0.258	0.369	0.495	-0.481	1.268	-0.596	6.650
6	Rolling VAR	1	N	Y	Y	Y	60	11.775	11.719	11.830	29.468	28.495	30.441	0.298	0.242	0.353	0.486	-0.537	1.257	-0.606	6.881
7	Rolling VAR	2	N	N	Y	Y	60	13.307	13.249	13.365	30.471	29.514	31.478	0.338	0.282	0.394	0.473	-0.509	1.257	-0.571	6.609
8	Rolling VAR	2	N	N	N	Y	60	12.976	12.918	13.033	30.470	29.490	31.488	0.327	0.272	0.383	0.473	-0.529	1.259	-0.571	6.609
9	Rolling VAR	2	N	Y	N	Y	60	11.875	11.818	11.932	30.275	29.293	31.292	0.293	0.237	0.349	0.451	-0.543	1.237	-0.573	6.652
10	Rolling VAR	1	Y	N	Y	N	60	13.047	12.988	13.105	30.631	29.676	31.672	0.328	0.272	0.384	0.450	-0.774	1.292	-0.610	6.638
Median Expanding VAR performance							60	11.692	11.589	11.794	60.917	59.169	62.887	0.143	0.087	0.198	-0.471	-0.489	-0.456	-0.686	6.152
Median Rolling VAR performance							60	12.389	12.282	12.497	59.460	57.637	61.397	0.158	0.103	0.213	-0.470	-0.488	-0.455	-0.676	6.238
Montlhy rebalancing																					
1	Expanding VAR	1	Y	N	Y	N	60	12.138	12.099	12.177	20.870	20.429	21.362	0.438	0.380	0.496	9.990	8.818	10.847	-0.642	3.649
2	Expanding VAR	1	Y	N	N	N	60	12.800	12.692	12.909	56.302	55.212	57.407	0.174	0.118	0.230	9.628	7.896	11.423	-0.330	3.046
3	Expanding VAR	2	Y	N	Y	N	60	12.235	12.139	12.330	50.293	49.304	51.318	0.184	0.126	0.242	7.866	6.229	9.453	-0.610	3.178
4	Rolling VAR	1	Y	N	Y	Y	60	12.701	12.661	12.740	20.771	20.327	21.229	0.467	0.411	0.523	7.519	6.729	8.258	-0.537	3.573
5	Expanding VAR	1	Y	Y	Y	N	60	11.675	11.628	11.722	24.731	23.868	25.700	0.351	0.291	0.410	7.081	5.798	8.020	-1.374	8.364
6	Expanding VAR	1	Y	N	N	N	60	11.769	11.724	11.814	23.807	22.921	24.795	0.368	0.308	0.429	6.856	5.860	7.635	-1.703	9.180
7	Rolling VAR	1	Y	Y	N	Y	60	11.311	11.272	11.349	20.171	19.716	20.657	0.412	0.354	0.470	6.612	5.947	7.244	-0.605	3.907
8	Rolling VAR	1	Y	N	N	Y	60	11.348	11.310	11.387	20.168	19.698	20.646	0.414	0.357	0.470	6.510	5.809	7.174	-0.604	3.907
9	Expanding VAR	1	Y	Y	Y	Y	60	11.812	11.766	11.859	24.140	23.266	25.111	0.365	0.306	0.424	6.212	5.225	6.987	-1.570	8.742
10	Rolling VAR	1	Y	N	Y	N	60	11.424	11.378	11.470	23.800	22.916	24.785	0.354	0.293	0.415	6.180	5.517	6.833	-1.659	9.071
Median Expanding VAR performance							60	12.378	12.317	12.439	41.035	40.066	42.075	0.229	0.173	0.285	-5.409	-6.003	-4.805	-1.099	6.011
Median Rolling VAR performance							60	11.722	11.644	11.801	40.395	39.369	41.466	0.216	0.159	0.272	-5.978	-6.634	-5.321	-1.333	6.076

Table A13

Top 18 Models Ranked According to Realized CER: HFRI Fund Weighted Composite Index (FWC) and No Transaction Costs (γ = 5)

The tables shows annualized sample means, standard deviations, the Sharpe ratio, and the Certainty Equivalent Return computed from the recursive portfolio weights under the ten top performing (in terms of CER) models for (predictable) risk premia, as defined by the VAR order (where a VAR(0) is equivalent to IID, constant investment opportunities), the selection of predictors, and whether the models are estimated either on an expanding or on a rolling 10-year window of data. The calculations are performed assuming the investor assess realized performance with a 5-year horizon (H = 60 months). Monthly rebalancing applies and it is taken into account by a long-horizon investor. In the table, we have boldfaced all performance statistics that turn out to be superior to those recorded in Table 5 for the baseline asset menu that excludes the hedge fund strategies.

CER rank	Model	Lags	Predictors included								H	Annualized mean (%)			Annualized volatility (%)			Annualized Sharpe ratio			Annualized CER (%)			Skewness	Kurtosis
			Term	Short	DY	Default	SMB	PtfsBD	PtfsIR	COM		Mean returns	90% Conf. Int. - LB	90% Conf. Int. - UB	Volatility	90% Conf. Int. - LB	90% Conf. Int. - UB	Sharpe ratio	90% Conf. Int. - LB	90% Conf. Int. - UB	CER (% Ann.)	90% Conf. Int. - LB	90% Conf. Int. - UB		
1	Expanding VAR	1	N	Y	N	Y	N	Y	N	Y	60	12.203	12.157	12.249	20.705	**19.568**	21.843	0.420	0.361	0.480	**10.350**	8.252	**12.449**	-0.649	3.735
2	Expanding VAR	1	N	Y	N	Y	N	N	N	N	60	12.323	12.276	12.370	20.671	**19.543**	21.798	0.427	0.369	0.485	**10.240**	8.220	**12.259**	-0.651	3.746
3	Expanding VAR	1	N	Y	N	Y	N	Y	Y	Y	60	11.800	11.753	11.847	20.618	**19.503**	21.734	0.403	0.343	0.462	**10.132**	8.126	**12.138**	-0.659	3.767
4	Expanding VAR	1	N	Y	N	Y	N	N	Y	N	60	12.690	12.643	12.737	20.616	**19.493**	21.739	0.446	0.388	0.504	**10.113**	8.093	**12.133**	-0.659	3.768
5	Expanding VAR	1	N	Y	N	Y	N	Y	Y	N	60	**13.046**	**12.998**	**13.094**	20.610	**19.487**	21.733	0.463	0.404	0.522	**10.073**	8.078	**12.069**	-0.657	3.768
6	Expanding VAR	1	N	Y	N	Y	N	Y	N	N	60	12.703	12.656	12.751	20.609	**19.480**	21.739	0.447	0.389	0.504	**10.062**	8.083	**12.042**	-0.657	3.769
7	Expanding VAR	1	N	Y	N	Y	N	N	N	Y	60	11.541	11.492	11.589	21.130	20.009	22.250	0.381	0.323	0.438	**10.047**	7.858	**12.236**	-0.615	3.607
8	Expanding VAR	1	N	Y	N	Y	N	N	Y	Y	60	12.496	12.449	12.544	20.757	**19.630**	21.885	0.433	0.375	0.492	**10.019**	7.948	**12.090**	-0.652	3.703
9	Expanding VAR	1	N	Y	N	Y	Y	N	N	N	60	11.076	11.028	11.123	21.097	19.959	22.236	0.359	0.302	0.417	9.477	7.514	**11.440**	-0.580	3.615
10	Expanding VAR	1	N	Y	N	Y	Y	Y	Y	N	60	12.057	12.009	12.105	20.552	**19.400**	21.703	0.416	0.359	0.474	9.468	7.716	**11.219**	-0.632	3.779
11	Expanding VAR	1	N	Y	N	Y	Y	Y	N	N	60	12.578	12.531	12.625	20.594	**19.462**	21.727	0.441	0.383	0.499	9.364	7.611	**11.117**	-0.632	3.759
12	Expanding VAR	1	N	Y	N	Y	Y	N	N	Y	60	11.078	11.028	11.127	21.747	20.586	22.907	0.348	0.291	0.406	9.135	7.076	**11.195**	-0.552	3.617
13	Expanding VAR	1	N	Y	N	Y	Y	Y	N	Y	60	11.453	11.406	11.500	20.879	19.751	22.006	0.381	0.323	0.438	9.088	7.298	**10.877**	-0.627	3.656
14	Expanding VAR	1	N	Y	N	Y	Y	N	Y	N	60	**12.986**	**12.939**	**13.033**	20.788	**19.673**	21.904	0.456	0.399	0.514	8.874	7.120	10.628	-0.636	3.686
15	Expanding VAR	1	N	Y	N	Y	Y	Y	Y	Y	60	10.529	10.481	10.576	20.838	19.719	21.957	0.337	0.279	0.395	8.802	7.064	10.541	-0.640	3.675
16	Expanding VAR	1	N	Y	N	Y	Y	N	Y	Y	60	10.178	10.130	10.226	21.171	20.042	22.300	0.315	0.258	0.373	8.528	6.747	10.310	-0.658	3.634
17	Expanding AR	1	N	N	N	N	N	N	N	N	60	12.581	12.518	12.644	27.354	25.200	29.509	0.332	0.276	0.388	0.337	-0.970	1.644	-1.471	6.951
18	Exp. Gaussian IID	0	N	N	N	N	N	N	N	N	60	11.538	11.465	11.610	31.734	29.408	34.060	0.253	0.197	0.309	-0.499	-3.391	2.394	-0.602	6.258
	Median Expanding VAR performance										60	11.924	11.876	11.971	**28.869**	**26.735**	**31.004**	**0.292**	**0.235**	**0.349**	**-4.159**	**-5.647**	**-2.670**	**-0.657**	**3.763**

Table A14

Top 18 Models Ranked According to Realized CER: HFRI Fund of Funds Composite (FFP) and No Transaction Costs (γ = 5)

The tables shows annualized sample means, standard deviations, the Sharpe ratio, and the Certainty Equivalent Return computed from the recursive portfolio weights under the ten top performing (in terms of CER) models for (predictable) risk premia, as defined by the VAR order (where a VAR(0) is equivalent to IID, constant investment opportunities), the selection of predictors, and whether the models are estimated either on an expanding or on a rolling 10-year window of data. The calculations are performed assuming the investor assess realized performance with a 5-year horizon (H = 60 months). Monthly rebalancing applies and it is taken into account by a long-horizon investor. In the table, we have boldfaced all performance statistics that turn out to be superior to those recorded in Table 5 for the baseline asset menu that excludes the hedge fund strategies.

CER rank	Model	Lags	Predictors included								H	Annualized mean (%)			Annualized volatility (%)			Annualized Sharpe ratio			Annualized CER (%)			Skewness	Kurtosis
			Term	Short	DY	Default	SMB	PtfsBD	PtfsIR	COM		Mean returns	90% Conf. Int. - LB	90% Conf. Int. - UB	Volatility	90% Conf. Int. - LB	90% Conf. Int. - UB	Sharpe ratio	90% Conf. Int. - LB	90% Conf. Int. - UB	CER (% Ann.)	90% Conf. Int. - LB	90% Conf. Int. - UB		
1	Expanding VAR	1	N	N	N	Y	N	N	Y	N	60	**13.156**	**13.028**	**13.283**	56.056	53.289	58.822	0.172	0.116	0.229	**12.473**	7.257	**17.690**	**-0.182**	3.261
2	Expanding VAR	1	N	N	N	Y	N	Y	Y	N	60	**12.899**	**12.772**	**13.027**	56.100	53.359	58.841	0.168	0.111	0.224	**12.450**	7.339	**17.562**	**-0.173**	3.267
3	Expanding VAR	1	N	N	N	Y	N	Y	N	N	60	**13.424**	**13.296**	**13.552**	55.900	53.136	58.664	0.178	0.121	0.234	**12.407**	7.686	**17.128**	**-0.169**	3.292
4	Expanding VAR	1	N	N	N	Y	N	N	N	N	60	**13.339**	**13.210**	**13.468**	55.980	53.220	58.740	0.176	0.120	0.231	**12.257**	7.120	**17.393**	**-0.184**	3.260
5	Expanding VAR	1	N	N	N	Y	N	Y	N	Y	60	12.183	12.056	12.311	56.019	53.266	58.771	0.155	0.098	0.212	**11.768**	6.837	**16.699**	**-0.178**	3.241
6	Expanding VAR	1	N	N	N	Y	N	N	N	Y	60	**12.909**	**12.781**	**13.037**	56.793	54.016	59.570	0.166	0.109	0.222	**11.591**	5.984	**17.198**	**-0.176**	3.149
7	Expanding VAR	1	N	N	N	Y	Y	Y	N	N	60	12.474	12.346	12.602	55.914	53.196	58.633	0.160	0.104	0.217	**11.526**	6.734	**16.317**	**-0.169**	3.272
8	Expanding VAR	1	N	N	N	Y	Y	Y	Y	N	60	12.609	12.481	12.737	56.129	53.396	58.862	0.162	0.106	0.219	**11.085**	5.994	**16.176**	**-0.173**	3.229
9	Expanding VAR	1	N	N	N	Y	Y	Y	N	Y	60	**13.682**	**13.552**	**13.812**	56.366	53.640	59.093	0.181	0.125	0.236	**10.292**	5.489	**15.094**	**-0.160**	3.181
10	Expanding VAR	1	N	N	N	Y	Y	N	Y	N	60	11.941	11.813	12.069	56.423	53.697	59.149	0.150	0.093	0.206	**10.203**	5.026	**15.381**	**-0.170**	3.169
11	Expanding VAR	1	N	Y	N	Y	N	Y	Y	N	60	**13.294**	**13.247**	**13.341**	20.668	**19.551**	21.784	**0.474**	**0.415**	**0.533**	**10.004**	8.000	**12.008**	-0.658	5.743
12	Expanding VAR	1	N	Y	N	Y	N	Y	N	N	60	12.033	11.985	12.080	20.700	**19.578**	21.823	0.412	0.354	0.471	9.960	7.953	**11.968**	-0.625	5.530
13	Expanding VAR	1	N	N	N	Y	N	Y	Y	Y	60	11.904	11.780	12.028	54.646	52.036	57.255	0.154	0.096	0.211	9.947	5.539	**14.355**	**-0.317**	3.122
14	Expanding VAR	1	N	Y	N	Y	N	Y	N	Y	60	**13.834**	**13.784**	**13.883**	21.370	20.233	22.508	**0.484**	**0.425**	**0.542**	9.872	7.659	**12.084**	-0.910	4.586
15	Expanding VAR	1	N	Y	N	Y	N	N	Y	N	60	11.481	11.433	11.529	21.048	19.941	22.155	0.379	0.321	0.437	9.782	7.672	**11.892**	-1.064	3.624
16	Expanding VAR	1	N	Y	N	Y	N	N	N	N	60	10.672	10.623	10.720	21.427	20.274	22.581	0.335	0.277	0.393	9.764	7.521	**12.008**	-1.061	3.587
17	Expanding VAR	1	N	Y	N	Y	Y	Y	N	N	60	**13.670**	**13.623**	**13.717**	20.994	19.864	22.123	**0.484**	**0.426**	**0.542**	9.518	7.474	**11.562**	-0.960	4.864
18	Expanding VAR	1	N	Y	N	Y	Y	Y	Y	N	60	11.794	11.746	11.841	20.768	**19.638**	21.898	0.399	0.342	0.457	9.515	7.551	**11.479**	-0.662	4.705
	Median Expanding VAR performance										60	**12.444**	**12.394**	**12.493**	31.765	29.362	34.168	**0.282**	**0.224**	**0.340**	**8.814**	**5.682**	**11.947**	**-0.611**	**3.718**

Table A15

Top 18 Models Ranked According to Realized CER: HFRI Fixed Income Relative Value/Arbitrage (RVR), No Transaction Costs (γ = 5)

The tables shows annualized sample means, standard deviations, the Sharpe ratio, and the Certainty Equivalent Return computed from the recursive portfolio weights under the ten top performing (in terms of CER) models for (predictable) risk premia, as defined by the VAR order (where a VAR(0) is equivalent to IID, constant investment opportunities), the selection of predictors, and whether the models are estimated either on an expanding or on a rolling 10-year window of data. The calculations are performed assuming the investor assess realized performance with a 5-year horizon (H = 60 months). Monthly rebalancing applies and it is taken into account by a long-horizon investor. In the table, we have boldfaced all performance statistics that turn out to be superior to those recorded in Table 5 for the baseline asset menu that excludes the hedge fund strategies.

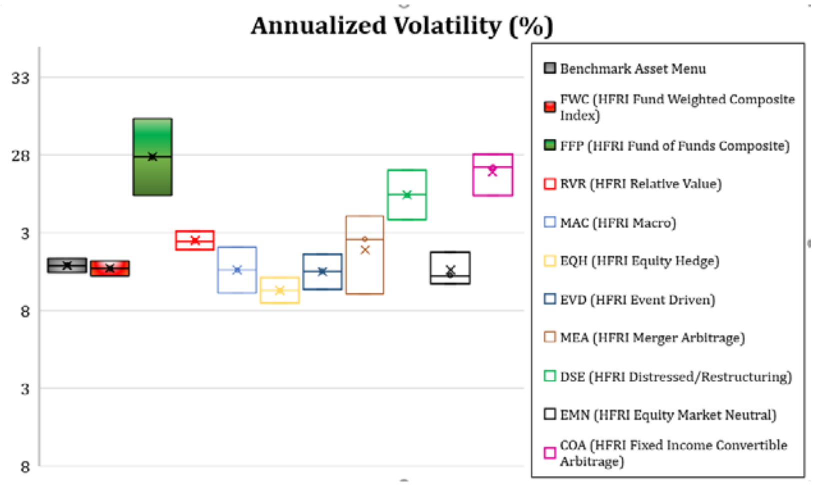
CER rank	Model	Lags	Predictors included								H	Annualized mean (%)			Annualized volatility (%)			Annualized Sharpe ratio			Annualized CER (%)			Skewness	Kurtosis
			Term	Short	DY	Default	SMB	PtfsBD	PtfsIR	COM		Mean returns	90% Conf. Int. - LB	90% Conf. Int. - UB	Volatility	90% Conf. Int. - LB	90% Conf. Int. - UB	Sharpe ratio	90% Conf. Int. - LB	90% Conf. Int. - UB	CER (% Ann.)	90% Conf. Int. - LB	90% Conf. Int. - UB		
1	Expanding VAR	1	N	Y	N	Y	Y	N	N	Y	60	**12.821**	12.766	12.875	22.312	20.911	23.713	0.418	0.362	0.474	**11.772**	**9.336**	**14.207**	**-0.051**	4.561
2	Expanding VAR	1	N	Y	N	Y	Y	Y	N	Y	60	**13.592**	13.538	**13.645**	21.554	20.348	22.760	0.468	0.411	**0.526**	**11.227**	**9.006**	**13.447**	-0.345	3.909
3	Expanding VAR	1	N	Y	N	Y	Y	Y	Y	Y	60	**12.937**	12.886	**12.987**	20.624	**19.496**	21.752	0.458	0.399	0.516	**10.104**	8.329	**11.879**	-0.617	3.754
4	Expanding VAR	1	N	Y	N	Y	Y	N	Y	Y	60	**12.937**	12.886	**12.989**	20.897	19.740	22.054	0.452	0.394	0.509	**10.023**	8.285	**11.761**	-0.523	3.709
5	Expanding VAR	1	N	Y	N	Y	Y	N	N	N	60	**13.587**	13.524	**13.650**	25.713	23.746	27.680	0.392	0.337	0.447	8.977	6.098	**11.856**	**0.165**	6.815
6	Expanding VAR	1	N	Y	N	Y	N	N	N	N	60	**13.725**	13.662	**13.788**	25.679	23.737	27.620	0.398	0.342	0.454	8.958	6.152	11.763	**0.150**	6.667
7	Expanding VAR	1	N	Y	N	Y	N	N	N	Y	60	11.827	11.765	11.889	25.391	23.541	27.241	0.328	0.273	0.383	8.811	6.018	11.604	**0.032**	5.988
8	Expanding VAR	1	N	Y	N	Y	N	Y	N	N	60	12.343	12.280	12.405	25.246	23.422	27.071	0.350	0.295	0.405	8.739	6.035	11.442	**-0.024**	5.811
9	Expanding VAR	1	N	Y	N	Y	Y	N	Y	N	60	**13.540**	**13.478**	**13.603**	25.127	23.333	26.920	0.400	0.344	0.455	8.670	6.025	11.315	**-0.074**	6.667
10	Expanding VAR	1	N	Y	N	Y	N	N	Y	N	60	11.633	11.572	11.694	24.980	23.237	26.723	0.326	0.270	0.381	8.588	5.971	11.204	**-0.133**	5.499
11	Expanding VAR	1	N	Y	N	Y	Y	Y	N	N	60	**13.401**	**13.338**	**13.463**	24.976	23.221	26.731	0.396	0.340	0.452	8.587	5.953	11.222	**-0.134**	5.495
12	Expanding VAR	1	N	Y	N	Y	N	Y	N	Y	60	**13.319**	**13.257**	**13.381**	24.874	23.154	26.594	0.395	0.339	0.451	8.523	5.898	11.149	**-0.177**	6.382
13	Expanding VAR	1	N	Y	N	Y	N	N	Y	Y	60	10.920	10.859	10.982	24.388	22.796	25.980	0.304	0.248	0.361	8.220	5.751	10.690	-0.372	4.916
14	Expanding VAR	1	N	Y	N	Y	N	Y	Y	N	60	**13.441**	**13.382**	**13.501**	24.047	22.529	25.566	0.413	0.357	0.470	7.971	5.625	10.318	-0.701	5.662
15	Expanding VAR	1	N	Y	N	Y	Y	Y	Y	N	60	11.765	11.707	11.824	23.745	22.265	25.226	0.348	0.292	0.404	7.731	5.555	9.908	-0.614	4.492
16	Expanding VAR	1	N	Y	N	Y	N	Y	Y	Y	60	**12.988**	**12.929**	**13.046**	23.639	22.170	25.109	0.401	0.344	0.459	7.626	5.418	9.834	-0.755	5.450
17	Expanding vAR	1	N	N	N	Y	Y	N	N	Y	60	11.970	11.818	12.123	62.344	59.486	65.201	0.136	0.080	0.192	2.672	-2.136	7.479	-0.143	**2.942**
18	Expanding VAR	1	N	N	N	Y	Y	Y	N	Y	60	**12.906**	**12.753**	**13.059**	61.961	59.098	64.824	0.152	0.096	0.208	0.863	-3.723	5.450	-0.171	**2.935**
	Median Expanding VAR performance										60	**13.690**	**13.631**	**13.748**	31.778	29.378	34.178	**0.321**	**0.263**	**0.378**	0.338	-2.091	2.767	**-0.614**	**5.583**

Figure A1

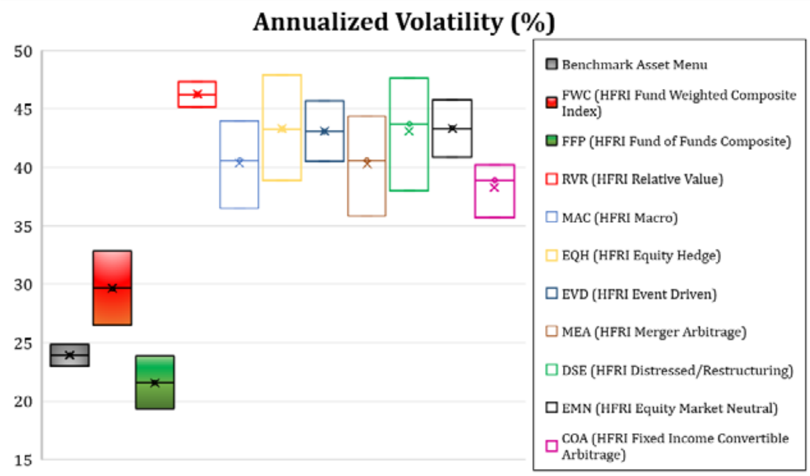
Realized Portfolio Return Volatility Across Alternative Asset Allocation Models

The plots represent the mean (as a solid horizontal line), median (as a cross), and realized 90% range (as a bin) of OOS volatility obtained with reference to a recursive, portfolio exercise for the sample 2004:01 – 2019:12. The calculations are performed assuming the investor assess realized performance with a 5-year horizon (H = 60 months) and using the predictive density from a Bayesian estimate of the predictability model for excess asset returns, when the priors are non-informative. Monthly rebalancing applies and it is taken into account by a long-horizon investor.

Best Model: Frequentist Methods



Median Model: Frequentist Methods



Best Model: Bayesian Methods